Vacuum Induced Spin-1/2 Berry's Phase

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We calculate the Berry phase of a spin-1/2 particle in a magnetic field considering the quantum nature of the field. The phase reduces to the standard Berry phase in the semiclassical limit and the eigenstate of the particle acquires a phase in the vacuum. We also show how to generate a vacuum induced Berry phase considering two quantized modes of the field which has an interesting physical interpretation.

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Geometric phases in quantum theory attracted great interest since Berry [1] showed that the state of a quantum system acquires a purely geometric feature (called the Berry phase) in addition to the usual dynamical phase when it is varied slowly and eventually brought back to its initial form. The Berry phase has been extensively studied [2,3], generalized in various directions [4], and has very interesting applications, such as the implementation of quantum computation by geometrical means [5–7]. In a strict sense, however, the Berry phase has been studied only in a *semiclassical* context until now. This means that the geometric evolution of a quantum system is studied under the effect of a time varying classical field. However, this *field itself has never been quantized* [8]. Thus, the effects of the vacuum field on the geometric evolution are unknown. Many effects in quantum optics such as quantum jumps, collapses, and revivals of the Rabi oscillations [9], can be explained only by considering a quantum field, showing the importance of field quantization in the complete description of physical systems. Moreover, in quantum mechanics several interesting effects are observed due to the interaction of quantum systems with the vacuum (spontaneous emission, lamb shift) [10].

The canonical experiment that demonstrates the existence of the Berry phase involves a spin-1/2 particle interacting with an external magnetic field whose direction is slowly changed in a cyclic fashion [5,11]. In this Letter we analyze such an experiment in a fully quantized context and give an expression for the geometric phase of a joint state of the particle and field that, as expected, reduces to the standard Berry phase in the high amplitude limit of a coherent state of the field. The relevant differences between this phase and the semiclassical version of the Berry phase become evident when states with low photon number are considered. We show that in the fully quantized scheme it is possible to produce nontrivial geometric phases that have no correspondence to phases that can be generated in the semiclassical scenario. The main difference arises from the interaction of the spin-1/2 system and the vacuum field. In fact, we show that even if the field is in the vacuum state, an adiabatic evolution of the field can be engineered to induce a nontrivial geometric phase in the system.

We calculate the deviations from the semiclassical model when a spin-1/2 particle interacts with a single mode quantum field. In addition, we design a scheme to generate a vacuum induced phase considering the interaction of the particle with two modes of the field in the vacuum state which are mixed adiabatically to generate the phase. This phase has a very interesting physical interpretation which we discuss in this Letter. Using a scheme designed to detect the Berry phase of the joint state of a harmonic oscillator and a two level system using an ion trap [12], it is possible to measure this vacuum induced phase.

In the semiclassical scenario we consider a spin 1/2 particle, or more generally, a two level system, coupled to a external classical oscillating field with frequency ν not far from the Bohr frequency ω of the two level system. In this case, it is convenient to work in a frame of reference rotating with frequency ν. The two level system is described in terms of Pauli operators σ_z, $\sigma_\pm = (\sigma_x \pm i\sigma_y)/2$ and its dynamics is characterized by the following Hamiltonian:

$$H = \frac{\Delta}{2}\sigma_z + \lambda(\sigma_+ \alpha e^{-i\phi} + \sigma_- \alpha e^{i\phi}), \quad (1)$$

where $\Delta = \omega - \nu$ is the detuning, λ is the coupling constant between the system and the field, and α represents the amplitude of the oscillating field. This Hamiltonian can be expressed in terms of an effective vector field $\mathbf{B} = (\lambda\alpha\cos\phi, \lambda\alpha\sin\phi, \Delta/2)$ as $H = \mathbf{B}\cdot\boldsymbol{\sigma}$ where $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$. When the state of the two level system is initially prepared in an eigenstate of the Hamiltonian and the direction of the effective field $\mathbf{B}$ is changed adiabatically, the state of the system will follow the field and after a complete cycle it will acquire a geometric phase equal to $\gamma = \pm\frac{1}{2}\Omega$. The $\pm$ sign depends on whether the state was initially aligned or against the direction of the field, and Ω is the solid angle subtended by the path followed by $\mathbf{B}$ in parameter space.

In the simplest case, where the detuning and strength of the coupling are fixed and the phase ϕ varies from 0 to 2π, the eigenstates of the particle describe a loop C in the bloch sphere and Berry's adiabatic geometric phase is (up to a $\pm$ sign)

$$\gamma = \pi(1 - \cos\theta), \tag{2}$$

where $\cos\theta = \Delta/\sqrt{\Delta^2 + 4(\alpha\lambda)^2}$.

We will now calculate the phase taking into account that a more rigorous description of the oscillating field that drives the spin-1/2 particle involves a quantum field. This means that the field cannot be considered anymore an external variable, it is part of the system itself, and its state can be manipulated by varying the parameters of the Hamiltonian. In the initial situation we consider the Hamiltonian of a spin-1/2 particle interacting with a single quantized mode of the field in the rotating wave approximation [13]

$$H_o^q = \nu a^\dagger a + \frac{\omega}{2}\sigma_z + \lambda(\sigma_+ a + \sigma_- a^\dagger), \tag{3}$$

where ν is the frequency of the field described in terms of the creation and annihilation operators a and $a^\dagger$, ω is the transition frequency between the eigenstates of the particle, and λ is the coupling constant. It can be seen by replacing the operators a and $a^\dagger$ by the classical amplitude α that this Hamiltonian corresponds to the semiclassical Hamiltonian (1) before the rotation involving ϕ has been applied. In the standard semiclassical experiment previously introduced, the variation of the state was induced by an adiabatic change of the phase of classical field. In the fully quantized context we need a procedure capable of generating an analogous phase change in the state of the field. To this end we introduce the phase shift operator $U(\phi) = \exp(-i\phi a^\dagger a)$ that, applied adiabatically to the Hamiltonian of the system, is capable of changing the state of the field. Changing ϕ slowly from 0 to 2π the Berry phase generated is calculated as follows:

$$\gamma_\pm^q = i\int_c d\phi \left\langle \Phi_n^\pm \middle| U(\phi)^\dagger \frac{d}{d\phi} U(\phi) \middle| \Phi_n^\pm \right\rangle, \tag{4}$$

where $|\Phi_n^\pm\rangle$ are the eigenstates of the Hamiltonian (3). Substituting the expression of $|\Phi_n^\pm\rangle$ leads to

$$\gamma_+^q = \pi(1 - \cos\theta_n) + 2\pi n, \tag{5}$$

$$\gamma_-^q = -\pi(1 - \cos\theta_n) + 2\pi(n+1), \tag{6}$$

with $\cos\theta_n = \Delta/\sqrt{\Delta^2 + 4\lambda^2(n+1)}$. It is important to note that for $n = 0$ the phase is different from zero, which means that the vacuum field introduces a correction in Berry's phase. Moreover, if we consider a coherent state with large amplitude we recover the semiclassical result ($\cos\theta_n \approx \cos\theta$). We now have a general expression for the Berry phase considering the quantum nature of light. This expression is relevant when systems are driven by fields with few photons and Berry's result is recovered when the photon number grows.

In order to study the physical meaning of the second term we need a nontrivial contribution that is different from an integer multiple of 2π. Therefore, we now describe a scheme were the particle interacts with two modes of the field. Consider the initial Hamiltonian

$$H_o^{2q} = \nu a^\dagger a + \nu b^\dagger b + \frac{\nu}{2}\sigma_z + \lambda(\sigma_+ a + \sigma_- a^\dagger), \tag{7}$$

describing a spin-1/2 particle interacting with one mode of the field with creation and annihilation operators a and $a^\dagger$ through a Jaynes-Cummings resonant interaction with coupling constant λ and a second mode of the field with creation and annihilation operators b and $b^\dagger$ and frequency ν which initially does not interact with the particle nor the first mode of the field. The eigenstates of this Hamiltonian are

$$|\Psi_{n,n'}^\pm\rangle = \frac{1}{\sqrt{2}}(|e,n\rangle \pm |g,n+1\rangle)|n'\rangle. \tag{8}$$

The state vector is a product state of the states $|n'\rangle$ of the field with modes b and $b^\dagger$ and the Jaynes-Cummings eigenstates of joint state of the field with modes a and $a^\dagger$ and the particle. Now we are allowed to exploit the second mode to perform a more general class of transformations, using the mode b as an "ancilla." Instead of changing the phase of the field and detuning between the two level system and the field, we consider the possibility of linearly mixing the two modes in a cyclic way, without any direct action on the degrees of freedom of the two level system. Before considering this transformation let us introduce some notation. The operation of linear mixing of two modes can be represented in a suitable way employing the Schwinger angular momentum [SU(2)] operators

$$J_z = \frac{1}{2}(a^\dagger a - b^\dagger b), \qquad J_x = \frac{1}{2}(a^\dagger b + ab^\dagger),$$
$$J_y = \frac{1}{2i}(a^\dagger b - ab^\dagger).$$

The physical meaning of this operator can be easily understood if we look at the two modes as two different polarizations of the same spatial mode. The Schwinger operators can then be interpreted as the generators of the polarization transformations, or in more abstract terms they determine rotations in the Poincaré's sphere $S^2 \sim SU(2)/U(1)$. In order to generate a Berry phase we perform a cyclic loop in the Poincaré's sphere, changing adiabatically the Hamiltonian by means of the unitary transformation:

$$U(\theta, \phi) = \exp(-i\phi J_z)\exp(-i\theta J_y), \tag{9}$$

where θ and ϕ are slowly varying parameters. The transformed Hamiltonian is

$$\begin{aligned} H^{2q} &= U H_o U^\dagger \\ &= \nu(\sigma_z/2 + a^\dagger a + b^\dagger b) + \lambda(\cos\theta/2\sigma_+ a e^{-i\phi/2} \\ &\quad + \sin\theta/2\sigma_+ b e^{i\phi/2} + \text{H.c.}). \end{aligned} \tag{10}$$

This Hamiltonian describes spin-1/2 particle interacting simultaneously with two modes of the field through a Jaynes-Cummings Hamiltonian where the mode phases and the coupling between spin-1/2 particle and each of the modes are adiabatically varied through the parameters θ and ϕ. Both eigenstates (8) acquire the phase

$$\gamma_{n,n'} = \frac{1}{2}\Omega(n - n' + 1/2), \tag{11}$$

where Ω is the solid angle subtended by the cyclic loop in the Poincaré's sphere. The phase dependence on the photon number is not trivial and can be measured by using an interference procedure between any of the eigenstate $|\Psi^{\pm}_{n,n'}\rangle$ and the ground state $|g, 0, 0\rangle$, which is the only state that acquires no geometric phase. The most remarkable case is obtained with the initial state $|e, 0, 0\rangle = \frac{1}{\sqrt{2}}(|\Psi^{+}_{0,0}\rangle + |\Psi^{-}_{0,0}\rangle)$. Indeed, even though the field is in a vacuum state the geometric operations performed on the degrees of freedom of the field determine a Berry's phase

$$\gamma_{\text{zero}} = \frac{\Omega}{4}. \tag{12}$$

Clearly, this result has no semiclassical correspondence, on account of the absence of a classical interpretation of a vacuum state. However, it is worth investigating the physical meaning of Eq. (11). An interpretation of the first part of Eq. (11), namely $\Omega(n - n')/2$, can be provided in terms of a polarized field not interacting with the two level system, subjected to a rotation of its polarization [2,14]. Indeed, this term has a classical origin that can be understood as follows. Suppose a beam of classical polarized light traverses an anisotropic dielectric such that its polarization slowly rotates and performs a closed loop in the Poincaré's sphere due to the variation of dielectric properties along the direction of propagation. According to Maxwell's equations, if the variation in the medium is slow enough, the beam of light acquires a geometric phase. Therefore, if α and β are the complex amplitudes of the two eigenmodes of the dielectric, under the cyclic rotation in the Poincaré's sphere they become $\alpha e^{i\Omega/2}$ and $\beta e^{-i\Omega/2}$, respectively. If the same experiment is analyzed in a context where the electromagnetic field is quantized, we expect to see the same effect. If we quantize the two modes by substituting the complex amplitude α and β with the corresponding annihilation operators a and b, the effect of the geometric evolution on the Fock states of the field is

$$|n, n'\rangle \rightarrow e^{i(n-n')\Omega/2}|n, n'\rangle. \tag{13}$$

Clearly, if we consider coherent states of the field the classical result is recovered: $|\alpha\rangle|\beta\rangle \rightarrow |\alpha e^{i\Omega}\rangle|\beta e^{-i\Omega}\rangle$. Therefore, if we look at the two modes a and b in Eq. (7) as two different polarizations of the same spatial mode, $\Omega(n - n')/2$ is the geometric phase corresponding to the polarization rotation of an electromagnetic field not interacting with the two level system. Then the question is: what is the physical origin of the term (12)? Clearly the interaction with the two level system is responsible for the appearance of this term. To have a picture of how this term comes into play, we can consider the semiclassical limit of the two level system interacting with the field. Suppose that modes a and b of the field initially are in a coherent state $|\alpha\rangle$ and $|\beta\rangle$, respectively, and the particle is in the linear combination $(|e\rangle \pm |g\rangle)$. Now, the state is adiabatically transformed by means of the operator (9) whose parameter are slowly varied along a close loop in the parameter space θ, ϕ. In the limit of large amplitude coherent state $|\alpha\rangle$ ($|\alpha| \gg 1$) the state is transformed as

$$(|e\rangle \pm |g\rangle)|\alpha, \beta\rangle \rightarrow (e^{i\Omega/4}|e\rangle \pm e^{-i\Omega/4}|g\rangle)|e^{i\Omega/2}\alpha, e^{-i\Omega/2}\beta\rangle. \tag{14}$$

After the adiabatic evolution, the amplitudes α and β of the coherent states acquire a phase $\Omega/2$ and $-\Omega/2$, respectively. According to the result (13) obtained in the case of a field noninteracting with the two level system, this phase is associated with the polarization rotation, and originates from the term $\Omega(n - n')/2$ of Eq. (11). Since in the large amplitude coherent state the two level system is approximately not entangled with the field, the term (12) appears in the last equation as a phase associable to the state of the two level system only. Under this condition exist a possible explanation of the phase (12) in terms of a semiclassical model. Consider the semiclassical Hamiltonian (1) in the resonant case ($\Delta = 0$). Suppose to rotate the polarization direction of the classical field. Coherently with the previous notation we describe this polarization with the two dimensional complex unit vector

$$\boldsymbol{\epsilon}(\theta, \phi) = \begin{pmatrix} e^{i\phi/2}\cos\theta/2 \\ e^{-i\phi/2}\sin\theta/2 \end{pmatrix} = A\begin{pmatrix} \alpha \\ \beta \end{pmatrix}, \tag{15}$$

where $A = 1/\sqrt{|\alpha|^2 + |\beta|^2}$. During the rotation the field acquires a geometric phase $\gamma(s)$ that depends on the path s followed by the vector $\boldsymbol{\epsilon}$ on the Poincaré's sphere. Since this phase is not integrable, the state of the field cannot be specified completely in terms of the parameters θ and ϕ only, but it needs to be expressed also as a function of the path s. Then, in a more complete description, the state of polarization has to be expressed as $\tilde{\boldsymbol{\epsilon}}(\theta, \phi) = e^{i\gamma(s)}\boldsymbol{\epsilon}(\theta, \phi)$. Since the field is a parameter of the semiclassical Hamiltonian, in the expression (1) the field must be considered together with its phase factor $e^{i\gamma(s)}$. Therefore, the Hamiltonian is no longer a function of θ and ϕ only, but depends, through the field, also on its previous "history." Taking into account this further phase in the expression (1) leads to

$$H = \lambda A \sigma_+ e^{i\phi/2}\cos\theta/2 e^{i\gamma(s)} + \text{H.c.} \tag{16}$$

This means that the eigenstates of (16) are functions also of the geometric phase $\gamma(s)$. The presence of the geometric phase in the Hamiltonian becomes nontrivial when the field performs a closed loop in its parameter phase. At the end of this loop the states have experienced a global transformation due to the geometric phase accumulated by the field in its evolution. For example, starting from parameters $\theta = 0$ and $\phi = 0$ after a closed loop the Hamiltonian transforms into $H = \lambda A(\sigma_+ e^{i\Omega/2} + \text{H.c.})$ and accordingly, its eigenstates transform as $|e\rangle \pm |g\rangle \rightarrow e^{i\Omega/4}|e\rangle \pm e^{-i\Omega/4}|g\rangle$, which is the result expected from the quantum description [see (14)].

On the other hand, the classical picture fails to explain the phase $\Omega/4$ when the system involve a field with low number of photons. The entanglement in the eigenstates (8) cannot be neglected in this case, and the expression (12) cannot be interpreted as a phase of the two level system only. A remarkable explanation of the origin of the phase $\Omega/4$ can be provided in terms of the vacuum fluctuation of the field. Because of the entanglement of the eigenstates (8) the field is not in a pure state, and must be considered as an incoherent combination of $|n, n'\rangle$ and $|n+1, n'\rangle$. It is still possible to provide an operationally well defined generalization of geometric phase when a system is in a mixed state. According to the definition introduced by Sjöqvist *et al.* [15], the geometrical phase for a mixed state $\rho = \sum_k w_k |k\rangle\langle k|$ evolving under a closed, adiabatic transformation U can be expressed in terms of an average connection form

$$e^{i\gamma} = \sum_k w_k e^{i\gamma_k}, \tag{17}$$

where γ_k is Berry's phase related to the eigenstate $|k\rangle$. Applying this concept to the state of the field $\rho = 1/2(|n\rangle\langle n| + |n+1\rangle\langle n+1|) \otimes |n'\rangle\langle n'|$, evolving under the transformation (9) leads to the geometric phase given by the expression (11). In the case of $n = n' = 0$ the phase $\Omega/4$ is obtained. This phase is therefore determined by the vacuum photon fluctuation due to the interaction with the two level system. The average number of photons in the the state of the field determines the noninteger number "$n - n' + 1/2$" multiplying the classic geometric phase $\Omega/2$ in the expression (11).

The Berry phase is usually described as the phase acquired by the state of a system when an adiabatic and cyclic change in the state is generated by means of variations on external classical fields acting on a quantum system. However, in physics there are many striking effects of field quantization. We have investigated the canonical example of the Berry phase giving a rigorous description of the field. We considered the quantum nature of the field in the geometric evolution for a spin-1/2 particle rotating in a cyclic fashion by means of a slowly varying magnetic field and found a general expression for the phase which reduces to the standard result in the semiclassical limit and presents vacuum induced effects. In addition, we have shown how to generate a vacuum induced phase in the state of a spin-1/2 particle using the most general rotation in the space of the system, which corresponds to a two mode displacement operator that mixes the modes of the field. This result opens up a new arena to the study of the consequences of field quantization in the geometric evolution of states. We are investigating possible applications of this effect and its connections to other quantum effects in different systems.

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