

### Comment on “Low-Density Spin Susceptibility and Effective Mass of Mobile Electrons in Si Inversion Layers”

In a recent Letter [1], the effective  $g$  factor and the effective mass,  $m$ , have been studied by Pudalov *et al.* in a dilute 2D electron system in silicon. By analyzing Shubnikov–de Haas oscillations in superimposed parallel and perpendicular magnetic fields, the authors reproduce the strong increase of  $gm$  with decreasing electron density previously reported in Ref. [2]. However, they contrast their data with the data obtained by our group [3] and claim that the spin susceptibility (or the product  $gm$ ) “increases gradually with decreasing density,” which “does not support the occurrence of spontaneous spin polarization and divergence of  $gm$  at  $n_s = n_c$ ” (here  $n_s$  is the electron density and  $n_c$  is the critical density for the metal-insulator transition, MIT). The purpose of this Comment is to show that all available experimental data, including those of Pudalov *et al.*, are consistent with each other and are in favor of a spontaneous spin polarization in this 2D system and a divergence of  $gm$  at a finite electron density.

Spin polarization and spin susceptibility were recently studied by measuring the (parallel) magnetic field  $B_c$ , required to fully polarize the electrons’ spins, using scaling of magnetoresistance [3] and magnetoconductivity [4]. To compare the data of the above two groups with the newer data of Pudalov *et al.* [1], in Fig. 1 we plot all three sets of data (two sets in the inset for better visibility). To convert  $gm$  from Ref. [1] into  $B_c$ , we use the condition for the full spin polarization:  $g\mu_B B_c/2 = \pi\hbar^2 n_s/2m$ , where  $\mu_B$  is the Bohr magneton. (The factor of 2 on the right side of the equation reflects the valley degeneracy.) The agreement between all three sets of data is remarkable, especially if one takes into account that different groups used different methods, different

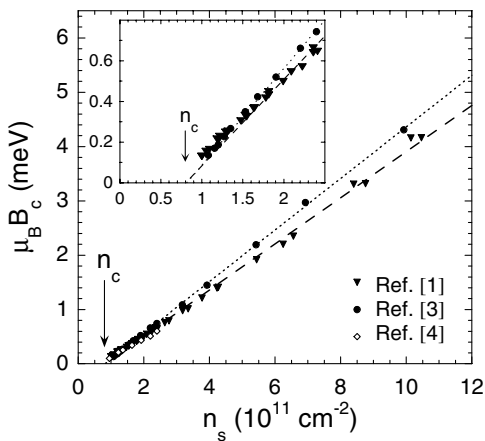


FIG. 1.  $B_c(n_s)$  calculated using the data from Refs. [1,3,4]. The dashed and dotted straight lines are fits to the data from Refs. [1,3], respectively.

samples, and different field/spin-polarization ranges.  $B_c$  is a linear function of  $n_s$  and extrapolates to zero at a *finite* electron density which we will designate  $n_\chi$ . The linear fit of the data from Ref. [1] yields  $n_\chi = 8 \times 10^{10} \text{ cm}^{-2}$ , which is identical with ours suggesting that  $n_\chi$  is sample independent.

Contrary to the claim made in Ref. [1] of a *gradual* increase of  $\chi$  with decreasing  $n_s$ , the linear dependence of  $B_c \propto n_s/gm$  (Fig. 1) points to the *critical* behavior of the spin susceptibility:  $\chi \propto n_s/(n_s - n_\chi)$ . The divergence of  $\chi$  should occur at the sample-independent electron density  $n_\chi$ , which in the samples studied in Ref. [3] coincides with the critical density  $n_c$  for the MIT indicated by the arrow. In more disordered samples, however,  $n_c$  may be noticeably higher than  $n_\chi$ . This is the case for the samples from Ref. [1] with  $n_c \approx 1 \times 10^{11} \text{ cm}^{-2}$  being well above the expected ferromagnetic transition point  $n_\chi \approx 8 \times 10^{10} \text{ cm}^{-2}$ . This explains why no divergence of  $gm$  is seen by Pudalov *et al.* at electron densities down to  $n_c$  in their samples. Note that even in the least disordered samples  $gm$  is still expected to be finite near  $n_\chi$ , as it normally occurs for any ferromagnetic transition due to nonzero temperature, inhomogeneous broadening, etc.

Of course, for the spin susceptibility to diverge at  $n_s = n_\chi$ , the extrapolation of  $B_c(n_s)$  to zero must be valid. To verify its validity, accurate data at lower densities, lower temperatures, and on much less disordered samples are needed. We emphasize that, in contrast to their claim, the method used by Pudalov *et al.* [1] certainly cannot be applied “down to and across the 2D MIT” because at  $n_s \lesssim 10^{11} \text{ cm}^{-2}$  (i) the amplitude of oscillations is too large (and even diverges as  $T \rightarrow 0$ ) [5], which is inconsistent with the Lifshitz-Kosevich formula they use, and (ii) there are too few oscillations [5] to study the beating pattern. We also note that the Lifshitz-Kosevich formula was deduced for the case of weak electron-electron interactions, and its application to strongly correlated system is not justified.

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[1] V. M. Pudalov *et al.*, Phys. Rev. Lett. **88**, 196404 (2002).

[2] S. V. Kravchenko *et al.*, Solid State Commun. **116**, 495 (2000).

[3] A. A. Shashkin *et al.*, Phys. Rev. Lett. **87**, 086801 (2001).

[4] S. A. Vitkalov *et al.*, Phys. Rev. Lett. **87**, 086401 (2001).

[5] V. M. Pudalov *et al.*, Surf. Sci. **305**, 107 (1994).