# Separation of Time Scales and Reparametrization Invariance for Aging Systems

Claudio Chamon,[1] Malcolm P. Kennett,[2] Horacio E. Castillo,[1] and Leticia F. Cugliandolo[3,4]

[1]*Department of Physics, Boston University, Boston, Massachusetts 02215*

[2]*Department of Physics, Princeton University, Princeton, New Jersey 08544*

[3]*Laboratoire de Physique Théorique de l'Ecole Normale Supérieure, 75231 Paris Cedex 05, France*

[4]*Laboratoire de Physique Théorique et Hautes Energies, Jussieu, 75252 Paris Cedex 05, France*

(Received 14 September 2001; published 31 October 2002)

We show that the generating functional describing the slow dynamics of spin-glass systems is invariant under reparametrizations of the time. This result is general and applies for both infinite and short-range models. It follows simply from the assumption that a separation between short time scales and long time scales exists in the system, and from the constraints of causality and unitarity. Global-time reparametrization invariance suggests that the low action excitations in a spin-glass may be smoothly spatially varying time reparametrizations. These Goldstone modes may provide the basis for an analytic dynamical theory of short-range spin glasses.

DOI: 10.1103/PhysRevLett.89.217201 PACS numbers: 75.10.Nr, 05.30.–d, 75.10.Hk, 75.10.Jm

Much progress has been made in recent years in understanding the nonequilibrium dynamics of glassy systems. Late after a quench to low temperatures, two distinct dynamic regimes develop [1,2]. At short time differences the dynamics is similar to that in the disordered phase. The correlations depend only on time differences [time translation invariance (TTI)] and decay towards a nonvanishing Edwards-Anderson (EA) order parameter. The correlations and their associated responses are related by the fluctuation-dissipation theorem (FDT). At long time differences, for a freely relaxing system, physical quantities relax very *slowly* and depend on the waiting time, so that correlations depend on two times rather than on time differences. The separation of time scales exists also when a glassy system is gently driven, whether the full dynamics becomes stationary or not. In the slow regime, the FDT is replaced by an out-of-equilibrium fluctuation-dissipation relation (OEFDR) between each bulk response $R$ and its associated bulk correlation $C$:

$$R(t_1, t_2) = \beta_{\rm eff}[C(t_1, t_2)] \frac{\partial}{\partial t_2} C(t_1, t_2), \tag{1}$$

where $t_1 \geq t_2$, and $\beta_{\rm eff}[C]$ is the inverse effective temperature measuring deviations from the FDT (e.g., [3]). These properties, observed in experiments and simulations [1], are embodied in the concepts of weak ergodicity breaking and weak long term memory [2,4,5]. This scenario has been demonstrated analytically for mean-field glassy models, such as the Sherrington-Kirkpatrick model [5]. However, for short-range models such as the Edwards-Anderson model, no analytic solution is known for either the statics or the dynamics. The work here presented uncovers a symmetry that strongly constrains the properties of such a dynamic solution.

The OEFDR is consistent with the observation that the equations of motion of a large class of glassy models are invariant under time reparametrizations $t \to h(t)$ in the slow regime [3,5–8]. In particular,

$$\tilde{C}(t_1, t_2) = C[h(t_1), h(t_2)], \tag{2}$$

$$\tilde{R}(t_1, t_2) = \left(\frac{\partial h}{\partial t_2}\right) R[h(t_1), h(t_2)], \tag{3}$$

are related by the same OEFDR as in Eq. (1). Equations (2) and (3) are particular cases of the general correlator reparametrization group (RpG) transformation

$$\tilde{G}(t_1, t_2) = \left(\frac{\partial h}{\partial t_1}\right)^{\Delta_A^G} \left(\frac{\partial h}{\partial t_2}\right)^{\Delta_R^G} G[h(t_1), h(t_2)], \tag{4}$$

with $\Delta_A^G$ and $\Delta_R^G$ defined in Ref. [7] as, respectively, the advanced and retarded scaling dimensions of $G$ under the reparametrization $t \to h(t)$ of $t_{1,2}$. In particular, $\Delta_{A,R}^C = 0$, $\Delta_A^R = 0$, and $\Delta_R^R = 1$ [see Eqs. (2) and (3)].

In this Letter we show that: (i) RpG invariance exists at the level of the generating functional for the long-time dynamics, and (ii) it holds for *short-range* models and their *local,* and not only bulk, quantities. These are much stronger results than those of Refs. [3,5–8], which only applied to mean-field models and to global quantities. Furthermore, the existence of a continuous symmetry in the action allows us to predict the presence of a Goldstone mode controlling the dynamics, which may be the basis for developing a systematic analytical theory of dynamical fluctuations in short-range spin glasses.

In order to prove our result, we take three steps: (i) we determine the long-time action by using the renormalization group (RG) in the *time* variables, assuming that there is a separation between short and long time scales, (ii) we analyze the surviving terms in the action, showing that they are RpG invariant, and (iii) we show that the measure in the functional integral is also RpG invariant.

For the sake of concreteness, we discuss the general disordered spin model defined by

$$H = \sum_{ij} J_{ij} S_i S_j + H_{\rm loc}, \tag{5}$$

where $H_{\rm loc}$ contains arbitrary self-interactions that, for example, place soft or hard ($\pm S$) constraints on the spins. Typically, the random couplings $J_{ij}$ are Gaussian distributed according to $P(J_{ij}) \propto \exp[-J_{ij}^2/(2K_{ij})]$, with $K_{ij}$ the connectivity matrix. In the EA model, $K_{ij} = \tilde{J}^2/z$ if $i, j$ are nearest neighbors or zero otherwise ($z$ is the coordination of the lattice). We consider a quantum extension [7,9], with quantization rules imposed via a kinetic term, $\sum_{i=1}^N p_i^2/(2M)$, where the momenta $p_i$ are canonically conjugate to the "coordinates" $S_i$, $[p_i, S_j] = -i\hbar\delta_{ij}$. The use of bosonic variables to represent the spins (e.g., quantum rotors, which lack Berry phases) simplifies the presentation considerably, although the arguments below can be extended to other glassy systems such as the SU(N) Heisenberg model [10]. The system is linearly coupled to an equilibrated environment.

We study the dynamics with the Schwinger-Keldysh closed time-path formalism [7,9–13] in which the spin variables acquire another index labeling the two Keldysh branches, $S_i \to S_i^a$, $a = 0, 1$. We work in the rotated basis $\sigma_i^0 = \hat{\sigma}_i = (S_i^0 - S_i^1)/\sqrt{2}$ and $\sigma_i^1 = \sigma_i = (S_i^0 + S_i^1)/\sqrt{2}$.

Once the disorder has been integrated out, we introduce a *local* Hubbard-Stratonovich field $Q_i(t_1, t_2)$ [14]

$$Q_i = \begin{pmatrix} Q_i^{00} & Q_i^{01} \\ Q_i^{10} & Q_i^{11} \end{pmatrix} = \begin{pmatrix} Q_i^K & Q_i^R \\ Q_i^A & Q_i^D \end{pmatrix}, \tag{6}$$

that decouples the four spin term generated by the average over the $J_{ij}$. The choice of indices 0, 1 anticipates the result that we prove, namely, invariance of the action under the transformation $Q_i^{ab}(t_1, t_2) \to \tilde{Q}_i^{ab}(t_1, t_2)$, with

$$\tilde{Q}_i^{ab}(t_1, t_2) = \left(\frac{\partial h}{\partial t_1}\right)^a \left(\frac{\partial h}{\partial t_2}\right)^b Q_i^{ab}[h(t_1), h(t_2)] \tag{7}$$

[cf. Eq. (4)]. This ansatz for how the $Q$ fields transform is motivated as follows. The $Q^K$ component is a time-dependent local measure of freezing (analogous to the EA order parameter for the static case) since it is related to the same-site correlator at two different times; hence, the choice of vanishing dimensions $\Delta_{A,R}^{Q^K} = 0$ for this slowly decaying quantity. The scaling dimensions for the retarded and advanced components $Q^{R,A}$ are suggested by the FDT, once the dimensions of $Q^K$ are fixed.

One usually keeps only the three nonvanishing components, $K$, $R$, and $A$, since Keldysh propagators are related to expectation values $\langle Q_i^K \rangle$, $\langle Q_i^R \rangle$, and $\langle Q_i^A \rangle$, while $\langle Q_i^D \rangle = 0$ [11,12]. Here we include the fourth component because we are also interested in the fluctuations of the $Q_i$'s. The generating functional reads

$$Z = \int [DQ] \exp(-S_K[Q] - S_{\rm nl}[Q]), \tag{8}$$

$$S_K[Q] = \frac{1}{2}\sum_{ij} K_{ij}^{-1} \int dt_1 dt_2 \sum_{ab} Q_i^{ab}(t_1, t_2) Q_j^{\bar{b}\bar{a}}(t_2, t_1),$$

$$S_{\rm nl}[Q] = -\ln \int [D\sigma][D\hat{\sigma}] \exp(iS[\sigma, \hat{\sigma}; Q] + iS_{\rm spin}), \tag{9}$$

$$S[\sigma, \hat{\sigma}; Q] = \sum_{i,a,b} \int dt_1 dt_2 \sigma_i^a(t_1) Q_i^{ab}(t_1, t_2) \sigma_i^b(t_2). \tag{10}$$

$S_{\rm spin}$ includes all the *local* spin dynamics, including the kinetic term, the self-interactions in $H_{\rm loc}$, and the coupling to the bath. The overline is a shorthand notation such that $\bar{0} = 1$ and $\bar{1} = 0$. All integrals start at $t = 0$. In order to observe nonequilibrium glassy dynamics, we first take the thermodynamic limit, $N \to \infty$, consider times such that the upper limit in the time-integrals diverges subsequently [3], and use a weak coupling to the bath.

Upon integration over the spin variables $\sigma_i^a$ in Eq. (9):

$$S_{\rm nl}[Q] = \sum_{n=1}^{\infty} \frac{1}{n!} \sum_{i_1,\ldots,i_n} \int dt_1 \ldots dt_{2n} G_{a_1,\ldots,a_{2n}}^{i_1,\ldots,i_n}(t_1, \ldots, t_{2n}) \times Q_{i_1}^{a_1 a_2}(t_1, t_2) \cdots Q_{i_n}^{a_{2n-1}a_{2n}}(t_{2n-1}, t_{2n}), \tag{11}$$

with $G_{a_1,\ldots,a_{2n}}^{i_1,\ldots,i_n}(t_1, \ldots, t_{2n}) \equiv \langle \sigma_{i_1}^{a_1}(t_1) \cdots \sigma_{i_n}^{a_{2n}}(t_{2n}) \rangle$. [$\langle \cdot \rangle$ denotes an average over the fields $\sigma, \hat{\sigma}$ with the weight $\exp(iS_{\rm spin})$]. The nonlinear terms in Eq. (11) are generated because $H_{\rm loc}$ is nonquadratic. This expansion contains only connected terms in the sense that none of the powers of $Q$ can be factored. Each $Q$ serves as a source for a pair of $\sigma$'s; hence, the $2n$-point correlations $G^{(2n)}$ are not necessarily connected in terms of *individual* $\sigma$'s. In general, the contribution at any order $n$ in $Q$ can be evaluated by rewriting the $G^{(2n)}$ into sums over products $\prod_{\alpha=1}^M G_c^{(2m_\alpha)}$ of connected Green's functions $G_c^{(2m_\alpha)}$, with $m_1 + \cdots + m_M = n$. An example, to order $Q^3$, is shown in Fig. 1; notice that the three $Q$'s form a connected bubble diagram, though at the same time it contains both a 2-point and a 4-point connected Green's function for the $\sigma$'s. Later we use this example to illustrate some of the steps in our arguments.

(i) *RG in the time variable.*—We assume there is an initial short-time cutoff $\tau_0 = \Omega^{-1}$. We then separate the fast and slow components of the field $Q_i$,

$$Q_i^{ab}(t_1, t_2) = \begin{cases} Q_{i\rm fast}^{ab}(t_1, t_2) & \text{if } \tau_0 < |t_1 - t_2| \leq b\tau_0, \\ Q_{i\rm slow}^{ab}(t_1, t_2) & \text{if } b\tau_0 < |t_1 - t_2|, \end{cases}$$

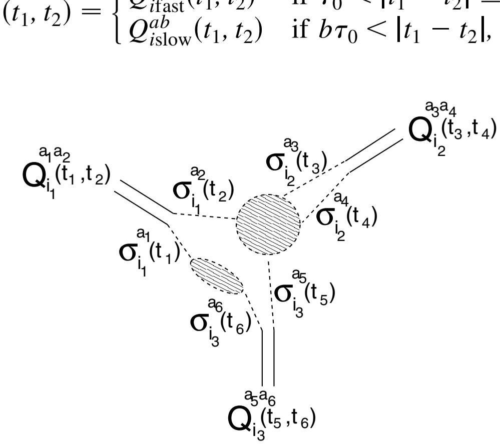


FIG. 1. A cubic term contributing to $S_{\rm nl}[Q]$. The dashed lines are $\sigma$'s, which together with the shaded regions represent connected Green's functions for the $\sigma$'s.

for a rescaling $b > 1$ (and $\delta\ell = \ln b > 0$). The couplings that flow in the RG are the coefficients $G^{i_1,\ldots,i_n}_{a_1,\ldots,a_{2n}}(t_1,\ldots,t_{2n})$ in Eq. (11). As usual, each RG transformation involves two operations. The first is the integration over the fast modes $Q_{i\mathrm{fast}}$, which we perform by representing $S_{\mathrm{nl}}$ as a path integral over spin variables $\sigma_i^a$ [Eqs. (9) and (10)]. The second is rescaling time $t \to t/b$ to restore the cutoff to its original scale $\tau_0 = \Omega^{-1}$, accompanied by a rescaling of the $Q$ fields, performed on the expression in Eq. (11). Each of these operations generates a change of order $\delta\ell$ in the action. The quadratic term in the action, $S_K[Q]$, plays a role analogous to the kinetic energy term in a usual RG calculation: it does not mix fast and slow modes, and the rescaling of fields is chosen so as to keep it invariant.

The integration over fast modes yields a nonlocal four spin interaction similar to the one obtained by performing the disorder average, but fundamentally different in that it is *short*-ranged in time. This extra spin-spin interaction leads to a change in the $2n$-point spin correlation functions, $\delta G^{(2n)} = \delta\ell[dG^{(2n)}/d\ell]|_{\mathrm{fast}}$.

As mentioned above, each coefficient $G^{(2n)}$ can be expressed as a sum of products of connected Green's functions. Consider one of these connected Green's functions, which plays the role of a coupling for the $Q$'s. In the same way that in a usual RG calculation (in real space) nonlocal interactions flow into local interactions (i.e., the only couplings that are left are those connecting fields at the same point), in our case all couplings connecting different times flow into couplings that connect only *equal* times (i.e., other terms are irrelevant compared to these terms). Using this property, the term in Fig. 1 reads:

$$\int d\tau G^{i_1,i_3}_{ca_1,a_6}(\tau,0)\int d^3\tau G^{i_1,i_2,i_2,i_3}_{ca_2,a_3,a_4,a_5}(\tau',\tau'',\tau''',0)\int dt_1dt_2 Q^{a_1a_2}_{i_1}(t_1,t_2)Q^{a_3a_4}_{i_2}(t_2,t_2)Q^{a_5a_6}_{i_3}(t_2,t_1).$$

Here and in the next equation, the $\tau$ variables represent time differences (time can be shifted because of TTI) and their range of integration is extended to be $(-\infty,+\infty)$. For a general term in the slow action, we obtain an integral of a product of $Q$ fields over time variables (one time variable per connected spin Green's function), with prefactors given by the integrals of the connected spin Green's function over all possible time differences that define the coupling constants:

$$g^{i_1,\ldots,i_{2m}}_{a_1,\ldots,a_{2m}} \equiv \int d^{2m-1}\tau G^{i_1,\ldots,i_{2m}}_{ca_1,\ldots,a_{2m}}(\tau_1,\ldots,\tau_{2m-1},0) = \lim_{\{\omega\}\to 0}\chi^{i_1,\ldots,i_m}_{ca_1,\ldots,a_{2m}}(\omega_1,\ldots,\omega_{2m-1}). \tag{12}$$

They correspond to physical zero-frequency (dc) generalized correlators $\chi_c^{\mathrm{dc}}$ of the spin variables.

Let us now turn to the effect of restoring the cutoff to the original scale by rescaling the times, $t \to t' = t/b = te^{-\delta\ell}$, and the fields, $Q_i^{a_1a_2}(t_1,t_2) \to Q_i'^{a_1a_2}(t_1',t_2') = b^{a_1+a_2}Q_i^{a_1a_2}(t_1,t_2)$. It is easy to see that it satisfies two conditions: it produces the same effect as an RpG transformation with $h(t) = bt$ [cf. Eq. (7)], and it leaves the quadratic term $S_K$ invariant. In the example:

$$\begin{aligned}I_{\mathrm{ex}}[Q] &\equiv \int dt_1dt_2Q^{a_1a_2}_{i_1}(t_1,t_2)Q^{a_3a_4}_{i_2}(t_2,t_2)Q^{a_5a_6}_{i_3}(t_2,t_1)\\ &= b^2\int dt_1'dt_2'b^{-(a_1+a_6)}b^{-(a_2+a_3+a_4+a_5)}Q'^{a_1a_2}_{i_1}(t_1',t_2')Q'^{a_3a_4}_{i_2}(t_2',t_2')Q'^{a_5a_6}_{i_3}(t_2',t_1') = b^{1-(a_1+a_6)}b^{1-(a_2+a_3+a_4+a_5)}I_{\mathrm{ex}}[Q'].\end{aligned}$$

In a general term, for each time variable $t_\alpha$ there is a coupling $g_\alpha \equiv g^{i_1,\ldots,i_{2m_\alpha}}_{a_1,\ldots,a_{2m_\alpha}}$ and also an exponent $\Delta_\alpha \equiv a_1 + \cdots + a_{2m_\alpha} \geq 0$ originating from the rescaling of fields. In the example, $\Delta_1 = a_1 + a_6$ and $\Delta_2 = a_2 + a_3 + a_4 + a_5$.

Combining the contributions from both the integration of fast modes and the rescaling, we obtain the following flow equation for the coupling constant $g_\alpha$:

$$\frac{dg_\alpha}{d\ell} = (1-\Delta_\alpha)g_\alpha + \frac{dg_\alpha}{d\ell}\bigg|_{\mathrm{fast}} \tag{13}$$

Terms with $\Delta_\alpha = 0$ occur only when all the $a$'s are 0. These terms would be naively relevant (according to their engineering dimensions), but in fact they vanish identically due to the constraints of normalization and causality for $G_c^{(2n)}$ [11,12]:

$$G^{i_1,\cdots,i_{2n_\alpha}}_{0,0,\ldots,0}(t_1,\ldots,t_{2n_\alpha}) = \langle\hat\sigma_{i_1}(t_1)\cdots\hat\sigma_{i_{2n_\alpha}}(t_{2n_\alpha})\rangle = 0.$$

There remain only terms which are either marginal (for $\Delta_\alpha = 1$) or irrelevant (for $\Delta_\alpha \geq 2$) according to their engineering dimensions. Under the assumption of a separation between short and long time scales, at some point in the RG flow all of the fluctuations associated with short time scales will have been integrated over (but the flow will not have yet reached a point were the long time scales are probed), and therefore any new integrations of fast modes produce no change in the coupling constants; i.e., the second term in the right-hand side of Eq. (13) is zero. This assumption is used in an analogous way in the solution of mean-field spin-glass models [1–3]. Physically, the separation between short and long time scales corresponds to a saturation of the generalized dc correlators for the spins $\chi_c^{\mathrm{dc}}$ defined in Eq. (12) to a *finite* value. Therefore, the existence of a separation of time scales implies that the engineering dimension actually determines the long-time behavior, and only marginal ($\Delta_\alpha = 1$) terms are left in the effective long-time action.

(ii) *RpG invariance of the effective long-time action.*—For a RpG transformation applied to a generic term in $S_{\rm nl}$, each integral over time gives rise to a factor $[\partial h(t_\alpha)/\partial t_\alpha]^{\Delta_\alpha}$, and since only terms with $\Delta_\alpha = 1$ are present, the derivative factor yields exactly the Jacobian necessary to make the term RpG invariant:

$$\int \prod_\alpha dt_\alpha \ldots \rightarrow \int \prod_\alpha dt_\alpha \left(\frac{\partial h}{\partial t_\alpha}\right) \ldots = \int \prod_\alpha dh_\alpha \ldots .$$

To illustrate this point, let us perform a RpG transformation on our example term:

$$\begin{aligned} I_{\rm ex}[\tilde{Q}] &= \int dt_1 dt_2 \tilde{Q}_{i_1}^{a_1 a_2}(t_1, t_2) \tilde{Q}_{i_2}^{a_3 a_4}(t_2, t_2) \tilde{Q}_{i_3}^{a_5 a_6}(t_2, t_1) \\ &= \int dt_1 dt_2 [\partial h(t_1)/\partial t_1]^{a_1 + a_6} [\partial h(t_2)/\partial t_2]^{a_2 + a_3 + a_4 + a_5} Q_{i_1}^{a_1 a_2}[h(t_1), h(t_2)] Q_{i_2}^{a_3 a_4}[h(t_2), h(t_2)] Q_{i_3}^{a_5 a_6}[h(t_2), h(t_1)] \\ &= \int dh_1 dh_2 Q_{i_1}^{a_1 a_2}(h_1, h_2) Q_{i_2}^{a_3 a_4}(h_2, h_2) Q_{i_3}^{a_5 a_6}(h_2, h_1) = I_{\rm ex}[Q]; \end{aligned}$$

where we have used that $a_1 + a_6 = \Delta_1 = 1$ and $a_2 + a_3 + a_4 + a_5 = \Delta_2 = 1$. For the $S_K$ term, the same argument applies with $\Delta = 1$ replaced by $a + \bar{a} = 1$ and $b + \bar{b} = 1$. Therefore, the long-time action is RpG invariant.

(iii) *RpG invariance of the measure.*—Under the RpG transformation of Eq. (7), the Jacobian for the functional integral over the $Q$ fields is simply $J[\frac{D\tilde{Q}}{DQ}] = \prod_x \prod_{t_1, t_2} |\frac{\partial h}{\partial t_2} \frac{\partial h}{\partial t_1}|^2 = e^{\int d^d x dt_1 dt_2 \ln|(\partial h/\partial t_1)(\partial h/\partial t_2)|^2}$. The Jacobian depends on $h(t)$, but not on the fields $Q$, and therefore the generating functional is RpG invariant.

The RpG invariance implies that the long-time dynamical action of a spin-glass is basically a "geometric" random surface theory [15], with the $Q$'s being the natural coordinates. The original two times parametrize the surface. Physical quantities as the bulk integrated response $\chi(t_1, t_2) = \int_{t_2}^{t_1} dt' \langle Q^R(t', t_2) \rangle$ and correlation $\langle Q^K \rangle(t_1, t_2)$ have scaling dimension zero under $t \rightarrow h(t)$ [5,7] as well as their local counterparts that are directly related to the $Q_i$'s. A possibly related gaugelike symmetry has also been noted in the replica approach [16].

The emergence of the RpG invariance may provide a novel, completely dynamical, angle to address the still poorly understood short-range spin-glass problem analytically. We have shown that the *global* reparametrization, $t \rightarrow h(t)$, is a symmetry of the slow dynamical action. The particular scaling function $h(t)$ selected by the system is determined by matching the fast and the slow dynamics. It depends on several details—the existence of external forcing, the nature of the microscopic interactions, etc. In other words, the fast modes which are absent in the slow dynamics act as symmetry breaking fields for the slow modes. The *global* RpG invariance of the slow action suggests that the low energy physics of the glassy phase could be described by slowly spatially varying reparametrizations $t \rightarrow h(\vec{x}, t)$. Basically, we propose that there are Goldstone modes for the glassy action which can be written as slowly varying, spatially inhomogeneous time reparametrizations. Comparing with the $O(N)$ nonlinear sigma model [17], *global* time reparametrizations are analogous to uniform spin rotations, while local $t \rightarrow h(\vec{x}, t)$ reparametrizations describe the spin waves. Numerical tests in the 3D EA model are consistent with this conjecture [18].

This work was supported by NSF Grants No. DMR-98-76208 and No. INT-01-28922, the A. P. Sloan Foundation (C. C.), and ACI France and CNRS-12931 (L. F. C.). C. C. thanks the LPTHE and M. P. K. thanks BU for their hospitality.

[1] J. P. Bouchaud *et al.*, in *Spin Glasses and Random Fields*, edited by A. P. Young (World Scientific, Singapore, 1998).
[2] L. F. Cugliandolo and J. Kurchan, Phys. Rev. Lett. **71**, 173 (1993); Philos. Mag. B **71**, 501 (1995).
[3] L. F. Cugliandolo and J. Kurchan, Physica (Amsterdam) **263A**, 242 (1999).
[4] J. P. Bouchaud, J. Phys. I (France) **2**, 1705 (1992).
[5] L. F. Cugliandolo and J. Kurchan, J. Phys. A **27**, 5749 (1994).
[6] S. Franz and M. Mézard, Physica (Amsterdam) **210A**, 48 (1994).
[7] M. P. Kennett and C. Chamon, Phys. Rev. Lett. **86**, 1622 (2001); M. P. Kennett, C. Chamon, and J. Ye, Phys. Rev. B **64**, 224408 (2001).
[8] H. Sompolinsky, Phys. Rev. Lett. **47**, 935 (1981); V. S. Dotsenko *et al.*, in *Spin Glasses and Related Problems*, Soviet Scientific Reviews Vol. 15 (Harwood Academic, Chur, Switzerland, 1990).
[9] L. F. Cugliandolo and G. Lozano, Phys. Rev. Lett. **80**, 4979 (1998); Phys. Rev. B **59**, 915 (1999).
[10] G. Biroli and O. Parcollet, Phys. Rev. B **65**, 094414 (2002).
[11] U. Weiss, *Quantum Dissipative Systems* (World Scientific, Singapore, 1999).
[12] K.-C. Chou *et al.*, Phys. Rep. **118**, 1 (1985).
[13] C. Chamon, A. W. W. Ludwig, and C. Nayak, Phys. Rev. B **60**, 2239 (1999); A. Kamenev and A. Andreev, *ibid.* **60**, 2218 (1999).
[14] H. Sompolinsky and A. Zippelius, Phys. Rev. Lett. **50**, 1297 (1983).
[15] A. M. Polyakov, *Gauge Fields and Strings* (Harwood Academic, Chur, Switzerland, 1987).
[16] I. Kondor and C. De Dominicis, Europhys. Lett. **2**, 617 (1986); T. Temesvári *et al.*, Eur. Phys. J. B **18**, 493 (2000).
[17] E. Brézin and J. Zinn-Justin, Phys. Rev. B **14**, 3110 (1976).
[18] H. E. Castillo *et al.*, Phys. Rev. Lett. **88**, 237201 (2002).