Transverse Sound in Liquid-^{3}He–Aerogel System

S. Higashitani, M. Miura, T. Ichikawa, M. Yamamoto, and K. Nagai

Faculty of Integrated Arts and Sciences, Hiroshima University, Kagamiyama 1-7-1, Higashi-Hiroshima 739-8521, Japan

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Transverse sound in normal liquid ^{3}He in aerogel is studied on the basis of the Landau transport equation taking into account the simultaneous oscillation of the aerogel. We show that the nature of transverse sound is strongly modified if aerogel is immersed in the liquid. Scattering of ^{3}He quasiparticles by the aerogel molecules causes friction between the liquid and the aerogel, giving rise to coupled motion of the two systems. There exists a low-attenuation transverse sound mode in this coupled system. The high-temperature behavior of its attenuation α is given by $\alpha \propto \text{const} + T^{-2}$, which is in contrast to $\alpha \propto T$ in pure liquid ^{3}He in the hydrodynamic regime.

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Liquid ^{3}He has been known to be an extremely clean Fermi liquid which undergoes a transition to p-wave pairing superfluid state. Recently, liquid ^{3}He in highly porous silica aerogel has attracted much attention. The aerogel is a tenuous random network of interconnected silica strands of approximately 3 nm diam. For a typical aerogel with a porosity 98%, a geometrical consideration [1] gives an estimate for the mean free path of ^{3}He quasiparticles to be 150 nm, which is comparable to the bulk coherence length of superfluid ^{3}He. This system thus provides a unique model system for studying impurity effects on unconventional Cooper pairing states.

The impurity effect of aerogel on superfluid ^{3}He was first observed by Porto and Parpia [2] using a torsion pendulum. They found that the superfluid density and the superfluid transition temperature are both substantially suppressed in the aerogel. Sprague *et al.* [3] reported magnetization measurements suggestive of an equal-spin pairing state in the aerogel. These works triggered a number of experimental attempts [4–12] to probe the superfluid state of ^{3}He in aerogel. However, the order parameter structure in the p-wave manifold is not as yet identified.

Acoustics has been a powerful tool in the development of our understanding of pure liquid ^{3}He. For example, the observation [13] of the first-to-zero sound transition in the longitudinal sound attenuation provided a clear verification of Landau's Fermi liquid theory, ultrasound absorption has played a key role in identifying the order parameter collective modes (see, e.g., [14]), and the confirmation of the acoustic Faraday effect [15] proved the existence of a propagating transverse sound predicted by Landau. In recent studies of ^{3}He in aerogel, Gervais *et al.* [12] succeeded in detecting multiple phase transitions of this system under magnetic fields using a transverse acoustic technique. At present, however, the nature of sound propagation in dirty Fermi liquids such as ^{3}He in aerogel is less well known even in the normal state.

Recent longitudinal sound experiment by Nomura *et al.* [16] (see also [17]) aroused a renewed interest in the normal phase of the dirty Fermi liquid. The sound attenuation that they observed does not exhibit the first-to-zero sound transition. On the other hand, the magnitude of the attenuation is of the same order as that of pure liquid ^{3}He in spite of the presence of impurity scattering. In this experiment the sound wavelength ($\sim 20\ \mu$m) is so long that the detailed structure of the aerogel is expected not to play a significant role. Nonetheless, the observed attenuation is surprisingly smaller than the value expected from the estimate [18,19] using the homogeneous impurity model [1,18–20] that regards aerogel as a collection of randomly distributed scattering centers. The present authors [18] pointed out that the small attenuation is recovered when simultaneous oscillation of the aerogel due to the collision drag effect [21] is taken into account. The collision drag model also explains quite nicely the temperature and the pressure dependence of the attenuation [18].

In this Letter, we discuss the collision drag effect on transverse sound in the normal liquid-^{3}He–aerogel system. We begin with the linearized Landau transport equation including two collision integrals: one is for the impurity scattering and the other is for the mutual collisions between quasiparticles. An important difference between the two collision integrals is that unlike the mutual quasiparticle collisions the impurity scattering does not conserve quasiparticle momentum.

We consider the space and time dependent quasiparticle distribution function $n_{\mathbf{k}}(\mathbf{r}, t) = f(\epsilon^0_{\mathbf{k}}) + \delta n_{\mathbf{k}}(\mathbf{r}, t)$ and the quasiparticle energy $\epsilon_{\mathbf{k}}(\mathbf{r}, t) = \epsilon^0_{\mathbf{k}} + \delta\epsilon_{\mathbf{k}}(\mathbf{r}, t)$, where $\mathbf{k}$ is the quasiparticle momentum, $f(\epsilon^0_{\mathbf{k}})$ the equilibrium distribution function (the Fermi function), $\epsilon^0_{\mathbf{k}}$ the equilibrium value of the quasiparticle energy, and $\delta\epsilon_{\mathbf{k}}$ the mean field due to the Fermi liquid interaction. When the motion of the aerogel is taken into account, the local-equilibrium distribution function $\delta n^{\text{l.e.}}_{\mathbf{k}}$ is given by [18,21]

$$\delta n^{\text{l.e.}}_{\mathbf{k}}(\mathbf{r}, t) = f'(\epsilon^0_{\mathbf{k}})[\delta\epsilon_{\mathbf{k}}(\mathbf{r}, t) - \mathbf{k} \cdot \mathbf{v}_a(\mathbf{r}, t)], \quad (1)$$

where $\mathbf{v}_a$ is the local velocity field of the scatterers, i.e., the aerogel molecules. Equation (1) implies that in local

equilibrium the liquid and the aerogel move with the same velocity. We shall denote the deviation of $\delta n_{\mathbf{k}}$ from the local-equilibrium value $\delta n_{\mathbf{k}}^{\text{l.e.}}$ by $\delta n'_{\mathbf{k}}$.

Using a function $\nu_{\mathbf{k}}$ defined by $\delta n_{\mathbf{k}} = f'(\epsilon_{\mathbf{k}}^0)\nu_{\mathbf{k}}$, the linearized Landau equation can be written as

$$\partial_t \nu_{\mathbf{k}} + v_F \hat{\mathbf{k}} \cdot \nabla_{\mathbf{r}}(\nu_{\mathbf{k}} - \delta\epsilon_{\mathbf{k}}) = I_c + I_i, \tag{2}$$

where $v_F = k_F/m^*$ is the Fermi velocity and $\hat{\mathbf{k}} = \mathbf{k}/k$. Collision integrals for the mutual quasiparticle collisions and for the impurity scattering are denoted by I_c and I_i, respectively. For I_c, we use a relaxation-time approximation in which the backscattering effect is included to guarantee the conservation laws (see, e.g., [22,23]). This relaxation-time expression has the same form as the impurity collision integral I_i; those together can be written in the form [18]

$$I_{c,i} = -\frac{1}{\tau_{c,i}}\left[\nu'_{\mathbf{k}} - \sum_{\ell}(2\ell+1)\lambda_\ell^{c,i}\langle P_\ell(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}')\nu'_{\mathbf{k}'}\rangle'\right], \tag{3}$$

where τ_c and τ_i are the relaxation times due to mutual collisions and impurity scattering, respectively, $\langle\cdots\rangle'$ denotes the angle average over $\hat{\mathbf{k}}'$, and $\nu'_{\mathbf{k}} = \nu_{\mathbf{k}} - \delta\epsilon_{\mathbf{k}} + \mathbf{k}\cdot\mathbf{v}_a$ represents the deviation of $\nu_{\mathbf{k}}$ from the local equilibrium. In Eq. (3), some of the parameters $\lambda_\ell^{c,i}$ of the backscattering effect are fixed by conservation laws: the particle number conservation requires that $\lambda_0^c = \lambda_0^i = 1$; the momentum conservation in mutual collision processes yields $\lambda_1^c = 1$. Unlike λ_1^c, λ_1^i is not equal to unity because the impurity scattering does not conserve quasiparticle momentum. The parameter λ_1^i is related to the transport mean free time τ_{tr} via $1/\tau_{\text{tr}} = (1-\lambda_1^i)/\tau_i$. Using I_c with $\lambda_\ell^c = 0$ for $\ell \geq 2$, Lea *et al.* [22] discussed the dispersion relations of transverse sound in pure ^{3}He. The parameter λ_2^c was taken into account by Flowers and Richardson [23] in the study of transverse acoustic impedance.

Now we turn to the motion of the aerogel. Since the wavelength of the sound of interest is still much longer than the mean distance between the aerogel strands, the aerogel motion is expected to be governed by an wave equation characterized by a sound velocity c_a. (c_a is either the longitudinal or the transverse sound velocity according to whether the longitudinal or the transverse response is considered.) Let the local displacement vector of the aerogel be $\mathbf{u}_a$; then $\mathbf{v}_a = \partial_t \mathbf{u}_a$. When the aerogel is immersed in liquid ^{3}He, $\mathbf{u}_a$ is expected to obey

$$\rho_a \partial_t^2 \mathbf{u}_a = \rho_a c_a^2 \nabla_{\mathbf{r}}^2 \mathbf{u}_a + \mathbf{F}, \tag{4}$$

where ρ_a is the mass density of the aerogel and $\mathbf{F}$ is the drag force density exerted on the aerogel from ^{3}He quasiparticles. Since $\mathbf{F}$ is equivalent to the momentum density lost by the liquid ^{3}He per unit time during the scattering processes, it can be deduced from the impurity collision integral

$$\mathbf{F} = N_F k_F \langle \hat{\mathbf{k}} I_i\rangle = -\frac{1}{\tau_{\text{tr}}} N_F k_F \langle \hat{\mathbf{k}}\, \nu'_{\mathbf{k}}\rangle, \tag{5}$$

where $N_F = m^* k_F/\pi^2$ is the density of states of ^{3}He quasiparticle at the Fermi level. Note that the drag force is finite only when the distribution function deviates from the local equilibrium.

The Landau transport equation and the equation of motion of the aerogel form a closed set to describe the sound propagation in the ^{3}He-aerogel system. Let all the space and time dependent quantities be proportional to $\exp(i\mathbf{q}\cdot\mathbf{r} - i\omega t)$. We take the direction of $\mathbf{q}$ as the polar axis to specify the direction of $\mathbf{k}$ by the polar angle θ and the azimuthal angle ϕ. Propagation of transverse sound is described by $\nu_{\mathbf{k}}$ with a form $\nu_{\mathbf{k}} \propto e^{i\phi}$. We expand $\nu_{\mathbf{k}}$ in terms of the spherical harmonics $Y_\ell^m(\theta,\phi)$ with $m = 1$:

$$\nu_{\mathbf{k}} = \sum_{\ell} \nu_\ell Y_\ell^1(\theta,\phi). \tag{6}$$

Keeping the Landau parameters F_ℓ^s and the backscattering parameters $\lambda_\ell^{c,i}$ up to $\ell = 2$, we can write the Landau transport equation, including the effect of the drag force $\mathbf{F}$, in the following form:

$$(\tilde{s} - \cos\theta)\tilde{\nu}_{\mathbf{k}} = \sum_{\ell=1}^{2}(\tilde{s} - s\beta_\ell)\tilde{\nu}_\ell Y_\ell^1(\theta,\phi), \tag{7}$$

where $s = \omega/v_F q$, $\tilde{s} = (1 + i/\omega\tau)s$, $1/\tau = 1/\tau_c + 1/\tau_i$,

$$\beta_\ell = \frac{1}{1 + F_\ell^s/(2\ell+1)} + \frac{i}{\omega\tau_\ell}, \tag{8}$$

$$\frac{1}{\tau_1} = \frac{1}{(1+i\kappa)\tau_{\text{tr}}}, \tag{9}$$

$$\frac{1}{\tau_2} = \frac{1-\lambda_2^c}{\tau_c} + \frac{1-\lambda_2^i}{\tau_i}, \tag{10}$$

and $\tilde{\nu}_\ell = [1 + F_\ell^s/(2\ell+1)]\nu_\ell$ is the partial wave component of $\tilde{\nu}_{\mathbf{k}} = \nu_{\mathbf{k}} - \delta\epsilon_{\mathbf{k}}$. The collision drag effect appears through κ given by

$$\kappa = \frac{(1 + F_1^s/3)\rho\omega}{\tau_{\text{tr}}\rho_a(\omega^2 - c_a^2 q^2)}, \tag{11}$$

where ρ is the mass density of ^{3}He. That κ vanishes in the limit $\rho_a \to \infty$ means that putting $\kappa = 0$ corresponds to assuming aerogel to be at rest.

From Eq. (7) and the orthonormal relation for Y_ℓ^1, the dispersion relation is obtained as

$$0 = \det\begin{bmatrix} 1 - L_{11}(\tilde{s} - s\beta_1) & -L_{12}(\tilde{s} - s\beta_2) \\ -L_{21}(\tilde{s} - s\beta_1) & 1 - L_{22}(\tilde{s} - s\beta_2)\end{bmatrix}, \tag{12}$$

where

$$L_{11} = \frac{3}{2}\left\langle \frac{\sin^2\theta}{\tilde{s} - \cos\theta}\right\rangle, \qquad L_{22} = \frac{15}{2}\left\langle \frac{\sin^2\theta\cos^2\theta}{\tilde{s} - \cos\theta}\right\rangle, \tag{13}$$

$$L_{12} = L_{21} = \frac{3\sqrt{5}}{2}\left\langle \frac{\sin^2\theta\cos\theta}{\tilde{s} - \cos\theta}\right\rangle. \tag{14}$$

Using the identities $L_{22} = \sqrt{5}\,\tilde{s}L_{12} = 5\tilde{s}^2(L_{11} - 1/\tilde{s})$, the dispersion relation (12) can be rewritten as

$$5\beta_1\left(\frac{\omega}{v_F q}\right)^2 = \frac{1}{\beta_2 - (1 + i/\omega\tau)(1 - L_{11}/L_{22})}. \quad (15)$$

In the collisionless regime, Eq. (15) has a solution corresponding to a propagating transverse zero sound when the Landau parameters fulfill a condition $F_1^s + 3F_2^s/(1 + F_2^s/5) > 6$ [23].

In the hydrodynamic regime $\omega\tau \ll 1$, we may expand $L_{\ell\ell}$ in Eq. (15) in powers of $1/\tilde{s}$. The right-hand side of Eq. (15) is then found to be dominated by $i/\omega\tau_2$ in β_2; as a result the dispersion relation in the hydrodynamic regime is given by

$$\rho[\omega + i(1 + F_1^s/3)/\tau_1] = -i\eta q^2, \quad (16)$$

where $\eta = \frac{1}{5}\rho v_F^2(1 + F_1^s/3)\tau_2$ is a viscosity coefficient generalized to impure Fermi liquid.

The hydrodynamic dispersion relation (16) is valid for $\omega\tau \ll 1$, namely, for $\omega\tau_c \ll 1$ and/or $\omega\tau_i \ll 1$. If we assume $\omega\tau_c \ll 1$ and take the clean limit $\tau_i \to \infty$ ($\tau_1 \to \infty$), then Eq. (16) gives the Navier-Stokes dispersion relation $\rho\omega = -i\eta q^2$, as expected. In the dirty limit $\tau_i \to 0$, on the other hand, τ_2 and therefore η vanish. However, the relaxation time τ_1 remains finite by virtue of the collision drag effect included in κ [see Eqs. (9) and (11)]. Using the limiting value of τ_1 and putting $\eta = 0$, we find from Eq. (16) the dirty-limit dispersion relation

$$(\rho + \rho_a)\omega^2 = \rho_a c_a^2 q^2. \quad (17)$$

This implies that, in the dirty limit, liquid ^{3}He is locked to the aerogel and they oscillate together feeling the restoring force of the aerogel. McKenna *et al.* [24] derived a similar equation for liquid ^{4}He by assuming such an interlocking motion and modifying the conventional hydrodynamic equations.

In actual ^{3}He-aerogel systems, the condition $\omega\tau_i \ll 1$ is satisfied over the wide frequency range. For example, using $l_{\rm tr} = v_F\tau_{\rm tr} = 50$ nm [18] and $v_F = 41.83$ m/s (at 15 bars) [14], we find that $\omega\tau_i < \omega\tau_{\rm tr} \sim 1$ (in general, $\tau_i < \tau_{\rm tr}$) is achieved at a rather high frequency $\omega/2\pi \sim 130$ MHz. We can say, therefore, that the recent "high-frequency" longitudinal sound experiment [16,17] ($\omega/2\pi \sim 15$ MHz) was done in the impurity-dominated hydrodynamic regime $\omega\tau_i \ll 1$.

When the condition $\omega\tau_i \ll 1$ is satisfied, Eq. (16) may be expanded in powers of $\omega\tau_{\rm tr}$. Keeping the leading order term, we obtain

$$(\rho + \rho_a)\frac{\omega^2}{q^2} = \rho_a c_a^2 - i\eta\omega - i\omega\tau_{\rm tr}^* \frac{\rho\rho_a(c_a^2 + i\eta\omega/\rho)^2}{(\rho + \rho_a)(c_a^2 - i\eta\omega/\rho_a)}, \quad (18)$$

where $\tau_{\rm tr}^* = \tau_{\rm tr}/(1 + F_1^s/3)$. In the limit $\omega\tau_i \to 0$, Eq. (18) is reduced to Eq. (17), as it should be. When τ_i is finite, there occurs a difference in the local velocity between the liquid and the aerogel. Such a velocity mismatch is the origin of the correction term ($\propto \omega\tau_{\rm tr}^*$) in Eq. (18). Another effect due to nonzero τ_i is the viscous effect that comes from the term $i\eta\omega$ on the right-hand side of Eq. (18). The viscosity coefficient η remains finite in the low-temperature limit. We find from Eq. (18) that the complex wave number q for a given frequency ω of the transverse sound in the dirty system is given by

$$q = \frac{\omega}{c_d\sqrt{1 - i\mu}}\left[1 + \frac{i\omega\tau_{\rm tr}^*}{2(1 + \rho_a/\rho)}\left(\frac{1 + i\mu\rho_a/\rho}{1 - i\mu}\right)^2\right], \quad (19)$$

where $c_d = c_a[\rho_a/(\rho + \rho_a)]^{1/2}$ is the velocity in the dirty limit and $\mu = \eta\omega/\rho_a c_a^2$ is a dimensionless measure of the viscous effect relative to the elastic effect of aerogel.

In Fig. 1, numerical results from Eq. (19) for the sound attenuation $\alpha = {\rm Im}\, q$ are plotted as a function of temperature T and are compared with the pure ^{3}He case. In this calculation we use the parameters for ^{3}He at a pressure of 15 bars; the viscous relaxation time $\tau_\eta = \tau_c/(1 - \lambda_2^c)$ is taken from Ref. [25] and the others from Ref. [14]. The parameters associated with the aerogel are chosen as $v_F\tau_{\rm tr} = 50$ nm [18], $v_F\tau_i/(1 - \lambda_2^i) = 100$ nm [18], and $\rho_a = 0.04$ g/cm^3 [7]. To the best of our knowledge, there has been no report on the transverse sound velocity c_a of aerogels, therefore, several values of c_a are used for illustration. The frequency $\omega/2\pi$ is taken to be 20 MHz, then we have $\omega\tau_{\rm tr} \simeq 0.15$. The only temperature dependent quantity in the present theory is $\tau_c \propto T^{-2}$. In pure ^{3}He the attenuation vanishes in the low-temperature limit $\omega\tau_c \to \infty$, since the Landau parameters

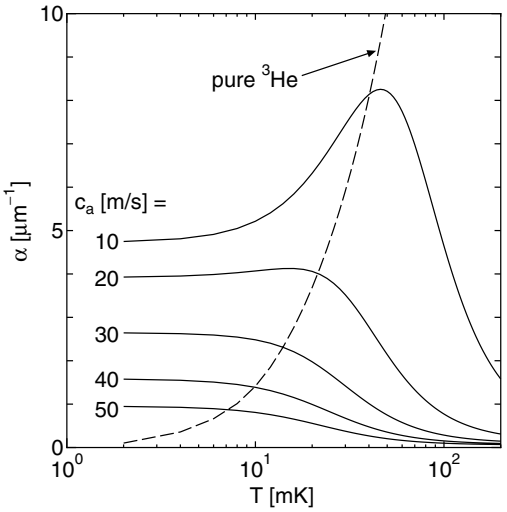

FIG. 1. Temperature dependence of the attenuation α of 20 MHz transverse sound in the ^{3}He-aerogel system at a pressure of 15 bars. The dashed line denotes the attenuation in pure ^{3}He. For parameters used in this calculation, see the paragraph below Eq. (19).

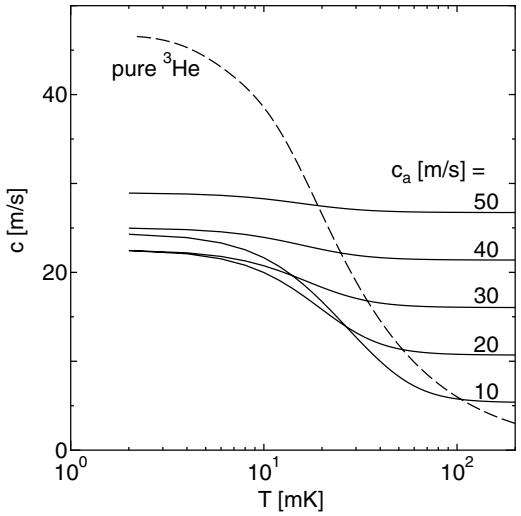

FIG. 2. Temperature dependence of the transverse sound velocity c corresponding to Fig. 1.

at 15 bars satisfy the condition for the existence of the zero sound. On the other hand, in the hydrodynamic regime at high temperatures $\omega\tau_c \ll 1$, the attenuation in pure ^{3}He shows the temperature dependence $\alpha \propto \tau_c^{-1/2} \propto T$. The attenuation in the dirty ^{3}He-aerogel system exhibits a quite different temperature dependence from the pure case. The impurity scattering gives rise to finite attenuation even in the low-temperature limit. As temperature increases, the viscosity decreases as $\eta \propto \tau_2 \propto 1/(\text{const} + T^2)$. If a condition $\eta\omega \ll \rho_a c_a^2$ ($\mu \ll 1$) is satisfied, Eq. (19) may be further approximated by

$$q = \frac{\omega}{c_d}\left[1 + \frac{i}{2}\left(\frac{\eta\omega}{\rho_a c_a^2} + \frac{\omega\tau_{\rm tr}^*}{1+\rho_a/\rho}\right)\right]. \qquad (20)$$

We see from Eq. (20) that the attenuation in the dirty Fermi liquid behaves like $\alpha \propto T^{-2} + \text{const} \times \omega\tau_{\rm tr}$ in the high-temperature limit, in contrast to the pure hydrodynamic case $\alpha \propto T$. This shows that the hydrodynamic transverse sound in the ^{3}He-aerogel system can propagate over a long distance with the aid of the elasticity of aerogel. In the intermediate temperature regime, a broad peak structure is found for small c_a's because of the relatively strong viscous effect.

In Fig. 2, we plot the temperature dependence of the sound velocity $c = \omega/\mathrm{Re}\, q$. From Eq. (20) we find that the transverse sound velocity takes a value $c_d = c_a[\rho_a/(\rho+\rho_a)]^{1/2}$ in the high-temperature limit. This property enables us to determine the unknown parameter c_a from the measurement of the sound velocity.

The high-temperature behavior of the attenuation will give an information on the transport mean free path $l_{\rm tr} = v_F\tau_{\rm tr}$, as can be seen from Eq. (20). At present, no agreement is found in the reported estimates of $l_{\rm tr}$. The values estimated from the T_c reduction [19], from the thermal conductivity [26] and from the longitudinal sound damping [18] are different. Since the transport mean free path is a key quantity to characterize the liquid-^{3}He–aerogel system, it is important to fix its value. Moreover, it is interesting to confirm the decrease in attenuation at high temperatures as shown in Fig. 1 because it is a clear evidence of the collision drag effect. Further experiments with transverse sound will, thus, provide useful information on the nature of the aerogel itself as well as of the liquid ^{3}He in aerogel.

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