Measurement of the Photonic de Broglie Wavelength of Entangled Photon Pairs Generated by Spontaneous Parametric Down-Conversion

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Using a basic Mach-Zehnder interferometer, we demonstrate experimentally the measurement of the photonic de Broglie wavelength of entangled photon pairs (biphotons) generated by spontaneous parametric down-conversion. The observed interference manifests the concept of the photonic de Broglie wavelength. We also discuss the phase uncertainty obtained from the experiment.

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The nature of entanglement or quantum correlation between two or more particles has attracted great interest and produced many applications in quantum information processing [1], such as quantum computation [2], quantum cryptography [3], and quantum teleportation [4–7]. Especially, a number of novel experiments have used parametric down-converted photons because they produce a superior environment for the realization of ideas concerning quantum entanglement. In addition to such applications to quantum informatics, the genuine quantum optical properties of parametric down-converted photons are also very attractive. Jacobson *et al.* [8] proposed the concept of the "photonic de Broglie wavelength" in multiphoton states. They argued that the photonic de Broglie wavelength of an ensemble of photons with wavelength λ and number of photons n can be measured to be λ/n using a special interferometer that does not split the multiphoton states into constituent photons. Following this concept, Fonseca *et al.* [9] used a kind of Young's double slit interferometer to measure the photonic de Broglie wavelength of entangled photon pairs ("biphotons") generated by parametric down-conversion. Boto *et al.* [10] proposed the principle of "quantum lithography," utilizing the reduced interferometric wavelength of nonclassical n photon states for optical imaging beyond the classical diffraction limit. Recently, a proof-of-principle experiment in quantum lithography was demonstrated by D'Angelo *et al.* [11] utilizing parametric down-converted biphotons. In this Letter, we propose and demonstrate the measurement of the photonic de Broglie wavelength for the $n = 2$ state in a very simple and straightforward manner, utilizing biphotons generated by parametric down-conversion and a basic Mach-Zehnder (MZ) interferometer. We show that not only the "wavelength" but also the coherence length of the biphoton is different from those of a single photon. We also discuss the uncertainty of phase measurements obtained from our experiment, and show that the phase uncertainty is below the standard quantum limit expected in classical one-photon interference.

The schematic view of the apparatus used in our experiment is shown in Fig. 1. Pairs of photons were generated by spontaneous parametric down-conversion (SPDC) in a 5-mm-long $KNbO_3$ (KN) crystal, the temperature of which is controlled and stabilized to within 0.01 °C. The pump source of the SPDC was the second harmonic light of a single longitudinal mode Ti:sapphire laser operating at $\lambda_0 = 861.6$ nm (linewidth $\Delta\nu_0 \sim$ 40 MHz). We selected correlated photon pairs traveling along two paths (P0 and P1) by two symmetrically placed pinholes. The central wavelength of the photon pair was tuned to degenerate at 861.6 nm by controlling the KN temperature. The MZ interferometer was composed of two 50%/50% beam splitters (BS1 and BS2). A biphoton was generated at either one (P2 or P3) of the interferometer arms when a pair of down-converted photons simultaneously entered at both input ports of a beam splitter (BS1), as a result of Hong-Ou-Mandel (HOM) interference [12]. By observing the HOM interference, ΔL_1 was adjusted so that the coincidence rate detected at both output ports (P2 and P3) of BS1 was minimized (see Fig. 2). Thus we can prepare the entangled biphoton state

$$\frac{1}{\sqrt{2}}(|2\rangle_{P2}|0\rangle_{P3} + |0\rangle_{P2}|2\rangle_{P3}) \tag{1}$$

in the MZ interferometer, where $|n\rangle_i$ denotes n photons' travel along the arm i. The path-length difference (ΔL_2) between the two arms of the MZ interferometer was controlled by a piezoelectric positioner, which was capable of controlling ΔL_2 with nanometer resolution. The two paths were combined at the output beam splitter (BS2), and the biphoton interference was measured at one of the output ports (P5) of BS2. The biphoton

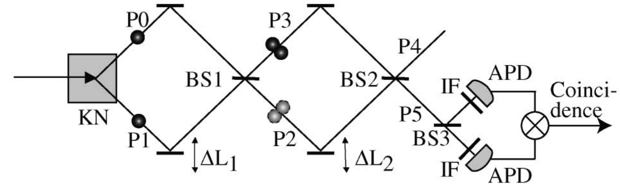

FIG. 1. Schematic experimental setup of the biphoton interference. KN: $KNbO_3$ crystal, BS1 ~ 3: beam splitters, IF: interference filters, APD: avalanche photodiodes.

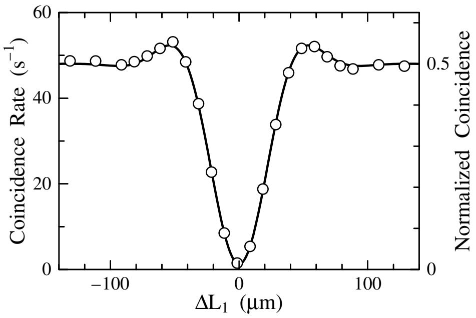

FIG. 2. Coincidence-photon-counting rate detected at the two output ports of BS1 as a function of the optical path-length difference (ΔL_1) between the two paths from KN and BS1 in Fig. 1. Open circles indicate experimental data, and the solid curve is a fitted function that assumes the observed photons have a rectangular spectral shape.

interference pattern was recorded by a two-photon detector, consisting of a 50%/50% beam splitter (BS3) and two avalanche photodiodes (APD) followed by a coincidence counter. We put an interference filter (IF: center wavelength $\lambda_c = 860$ nm, bandwidth $\Delta\lambda = 10$ nm) in front of each APD to eliminate undesired background light. In general, the interferometer was designed to measure the photonic de Broglie wavelength of the biphoton without using any unreal optical components. For comparison, we also measured the usual single-photon interference by using a single detector and blocking one of the input ports.

It is worth discussing the interference patterns we expected to obtain in our experiment. For simplicity, we consider only degenerate single-frequency photons. The single-frequency treatment is adequate for predicting most distinct properties of interference patterns for both one-photon and two-photon detection, although it will be necessary to consider multifrequency treatment in order to discuss more-detailed phenomena such as the coherence length of the interference. The one- and two-photon counting rates (R_i and R_{ij}, respectively) at an output port P_i and P_j ($i, j = 4, 5$) of the interferometer are described by

$$R_i(|\psi_0, \psi_1\rangle) = \langle\psi_0, \psi_1|\hat{a}_i^\dagger \hat{a}_i|\psi_0, \psi_1\rangle, \tag{2}$$

$$R_{ij}(|\psi_0, \psi_1\rangle) = \langle\psi_0, \psi_1|\hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_j \hat{a}_i|\psi_0, \psi_1\rangle, \tag{3}$$

where $\hat{a}_{i(j)}^\dagger$ and $\hat{a}_{i(j)}$ are photon creation and annihilation operators at the output port $P_{i(j)}$, and ψ_0 and ψ_1 denote the quantum states of light at the two input ports P0 and P1, respectively. The photon operators at the output ports are connected to those of the input ports through the scattering matrices of the beam splitters and the optical path-length difference [13]

$$\begin{pmatrix} \hat{a}_4 \\ \hat{a}_5 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} e^{i\phi_2} & 0 \\ 0 & e^{i\phi_3} \end{pmatrix}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} \hat{a}_0 \\ \hat{a}_1 \end{pmatrix} \tag{4}$$

and their Hermitian conjugates, where ϕ_2 and ϕ_3 are phase retardations along the interferometer arms P2 and P3, respectively. Here we assume the MZ interferometer consists of lossless 50%/50% beam splitters. Using (4), we can calculate the counting rates (2) and (3) for arbitrary input states of light. The resultant two-photon interference patterns for the case of $|\psi_0, \psi_1\rangle = |1, 1\rangle$, i.e., where both inputs are $n = 1$ Fock states, are

$$R_{55}(|1, 1\rangle) = \tfrac{1}{2}(1 - \cos 2\phi), \tag{5}$$

$$R_{45}(|1, 1\rangle) = \tfrac{1}{2}(1 + \cos 2\phi), \tag{6}$$

where $\phi \equiv \phi_2 - \phi_3 = 2\pi\Delta L_2/\lambda$ is the optical phase difference between the two arms, ΔL_2 the path-length difference, and λ the wavelength of the input light. Here the constant coefficients of the right sides have been omitted. The corresponding one-photon interference for the case of $|\psi_0, \psi_1\rangle = |0, 1\rangle$ becomes

$$R_5(|0, 1\rangle) = \tfrac{1}{2}(1 - \cos\phi). \tag{7}$$

From Eqs. (5)–(7), we see that both R_{55} and R_{45} will have the oscillation period $\lambda/2$, while R_5 has the period λ. This oscillation period $\lambda/2$ for the two-photon counting rate R_{55} is attributable to the photonic de Broglie wavelength λ/n for the biphoton ($n = 2$) state. Although R_{45} will also have the oscillation period $\lambda/2$, it is not attributable to the photonic de Broglie wavelength of the biphoton because in this case the two photons are split from each other by BS2. Furthermore, the oscillation period $\lambda/2$ in R_{45} could be observed even for classical states, while that in R_{55} could not. In fact, for the coherent input state $|0, \alpha\rangle$,

$$R_{55}(|0, \alpha\rangle) = R_5^2 = \frac{|\alpha|^4}{4}(1 - \cos\phi)^2, \tag{8}$$

$$R_{45}(|0, \alpha\rangle) = R_4 R_5 = \frac{|\alpha|^4}{8}(1 - \cos 2\phi). \tag{9}$$

Although the coherent state is classified as a classical state of light, the oscillation period of $R_{45}(|\alpha, 0\rangle)$ becomes $\lambda/2$. Consequently, the oscillation period $\lambda/2$ of R_{45} does not necessarily originate from the quantum nature of light. On the other hand, in the classical treatment, R_{55} should have the same period as R_5 since $R_{55} = R_5^2$ and $R_5 \geq 0$. Thus, the oscillation period $\lambda/2$ of $R_{55}(|1, 1\rangle)$ reflects the quantum nature of the input field and is consistent with the photonic de Broglie wavelength of the biphoton. Hence, the intent of our experiment is to measure the interference pattern of $R_{55}(|1, 1\rangle)$.

Before observing the interference pattern of the MZ interferometer, we measured the HOM interference to check the position of the zero path-length difference ΔL_1 and to ensure that the biphotons were generated at either arm of the interferometer. The observed HOM interference, i.e., the coincidence counting rate between the two output ports (P2 and P3) of BS1 as a function of the optical path-length difference (ΔL_1), is presented in

Fig. 2. The visibility of the HOM interference was 0.97, guaranteeing that the photon pair was traveling together almost perfectly along either arm of the interferometer when $\Delta L_1 = 0$. To our knowledge, this is one of the best visibilities ever obtained in a HOM interference experiment. After the measurement of the HOM interference, ΔL_1 was fixed at 0, where the coincidence rate was minimized. Thus we prepared the entangled biphoton state (1) in our interferometer.

Figure 3 shows the measured interference pattern for both one-photon [$R_5(|0, 1\rangle)$: upper graph] and two-photon [$R_{55}(|1, 1\rangle)$: lower graph] detection, as a function of path-length difference (ΔL_2) around $\Delta L_2 \sim 0$ μm. Note that one of the input ports of the interferometer was blocked during measurement of the one-photon counting rate; otherwise, no interference was expected. For both one- and two-photon counting rates, clear interference fringes were observed. The fringe visibilities of the one- and two-photon interferences were 0.88 and 0.75, respectively. It can be seen in the figure that the one-photon interference has a period of approximately 860 nm, whereas the period in the two-photon interference is approximately 430 nm. This result clearly indicates that the biphoton state exhibits the interference as a "wave" whose length is half that of the one-photon state. This is consistent with the prediction that the photonic de Broglie wavelength of the biphoton state would be $\lambda_c/2$. Thus, we have clearly measured the photonic de Broglie wavelength of the biphoton generated by SPDC.

We have also observed the difference in the coherence length between the one- and two-photon counting rates, as demonstrated in Fig. 4. Although the interference oscillation in the one-photon counting rate disappears at $\Delta L_2 \sim 400$ μm, the interference of the two-photon counting rate remains until the path-length difference is much larger, indicating that biphotons have much longer coherence lengths than do single photons. Since the spontaneous parametric down-converted photons have considerably wide spectral widths, the coherence length of the one-photon counting rate is governed by the spectral bandwidth $\Delta\lambda$ of the interference filters placed in front of the detectors. Thus, the coherence length of the one-photon counting rate becomes very short ($\lambda_c^2/\Delta\lambda \sim 70$ μm). On the other hand, the coherence length of the two-photon counting rate is governed by the spectral width of the sum frequency of signal (ν_s) and idler (ν_i) photons. This sum frequency is identical to the frequency of pump photons ($2\nu_0$) of the SPDC. Since we used the second harmonic light of the single longitudinal mode continuous laser as a pump source, its coherence length is very long ($c/\Delta\nu_0 \sim 400$ cm). As a result, a clear interference fringe was observed for the two-photon counting rate even at $\Delta L_2 \sim 400$ μm [14], whereas almost no fringe was observed for the one-photon counting rate.

Thus far, a number of works concerning two-photon interference have used parametric down-converted photons and either Mach-Zehnder or Michelson interferometer [15–18]. However, those previous experiments did not intend to observe the photonic de Broglie wave. Most of them [15,16,18] detected two split photons at each of the output ports of the interferometer. In our experiment, by detecting the two-photon counting rate at one of the output ports, we directly showed that the observed biphoton interference manifests the concept of the photonic de Broglie wavelength. In this context, it is worth discussing the uncertainty of phase measurements in our experiment. There are a number of works concerning accurate phase measurements beyond the standard quantum limit utilizing quantum interferometry [19–22]. We have evaluated the phase uncertainty $\Delta\phi$ using the least-squares analysis of the experimental data. Assuming that we have N data points of the interference fringes measured at $\phi = \phi_0 + 2m\pi/N$ ($m = 1, \ldots, N$), and that the data statistics follow the Poisson distribution, we obtain the phase uncertainty for the perfect classical one-photon interference:

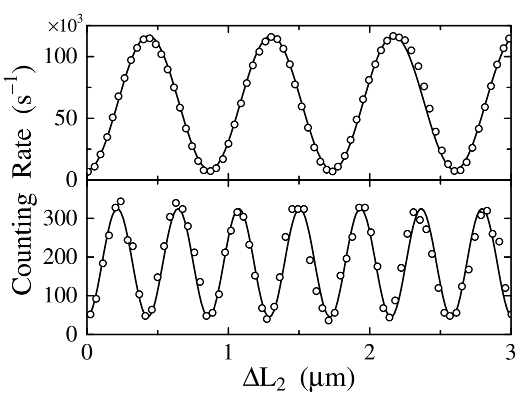

FIG. 3. Interference patterns in the one-photon (upper) and two-photon (lower) counting rates at a path-length difference of around 0 μm.

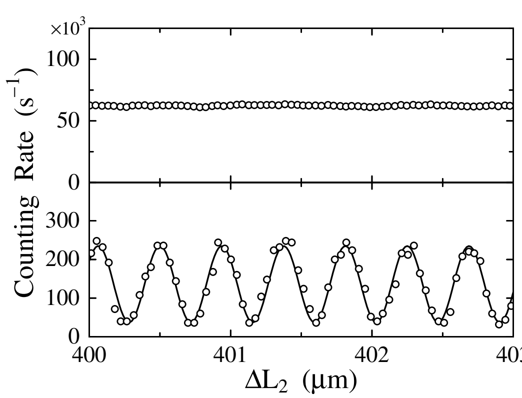

FIG. 4. Interference patterns in the one-photon (upper) and two-photon (lower) counting rates at a path-length difference of around 400 μm.

$$\Delta\phi^{(1)} = \sqrt{\frac{2}{Na}} \equiv \Delta\phi^{(1)}_{\mathrm{SQL}}, \tag{10}$$

where a is the interference amplitude. Equation (10) corresponds to the standard quantum limit (SQL) of the phase uncertainty for one-photon interference $\Delta\phi^{(1)}_{\mathrm{SQL}}$; note that $Na/2$ equals the total photon number detected over the interference fringe. In contrast, we find for perfect biphoton interference

$$\Delta\phi^{(2)} = \sqrt{\frac{1}{2Na}} = \frac{1}{2}\Delta\phi^{(1)}_{\mathrm{SQL}}. \tag{11}$$

The factor 1/2 of $\Delta\phi^{(2)}$ from $\Delta\phi^{(1)}_{\mathrm{SQL}}$ is the direct consequence of the reduced interference period, i.e., reduced de Broglie wavelength of the biphoton. The reduction factor is also attributable to the Heisenberg limit $(1/n)$ of the phase measurement using n photons (in our case $n = 2$) [19,20]. The phase uncertainty obtained from the experimental biphoton interference shown in Fig. 3 was $0.81\Delta\phi^{(1)}_{\mathrm{SQL}}$, indicating that it was indeed smaller than the one-photon SQL. It was, however, still larger than the expected Heisenberg limit $\frac{1}{2}\Delta\phi^{(1)}_{\mathrm{SQL}}$, or even larger than the SQL using two photons: $\Delta\phi^{(2)}_{\mathrm{SQL}} = \frac{1}{\sqrt{2}}\Delta\phi^{(1)}_{\mathrm{SQL}}$. The main reason for the incomplete reduction of the phase uncertainty is the imperfect visibility of the measured interference; our analysis indicated that the visibility should have been higher than 7/9 for $\Delta\phi^{(2)}$ to exceed $\Delta\phi^{(2)}_{\mathrm{SQL}}$. The phase uncertainty would have reached the Heisenberg limit only if the visibility had been unity [22].

Finally, we note the relationship between our experiment and the nonlocal nature of the entangled photon pairs. As previously demonstrated [23–25], two-photon quantum interference occurs for biphotons even using two spatially separated interferometers. With these results taken together, we can understand that the interferometric properties of the biphoton originate from its nonlocal quantum correlation in frequency between the constituent photons, but not from the spatial closeness of the two photons. The concept of photonic de Broglie wavelength is attributable to a special case of more general treatments [26,27] of biphoton interference, i.e., the case where the two photons travel the same paths and are detected at the same place. Furthermore, the concept is valid for the cases where more than two photons travel together [8]. In this sense, the photonic de Broglie wavelength reminds the close similarity to the usual de Broglie wavelength of matter. It is interesting that we observe the reduced de Broglie wavelength regardless of any physical binding between the two constituent particles; they exhibit the de Broglie wavelength identical to a virtual compound particle by means of the artificially controlled pathway and detection procedure. Needless to say, the concept of photonic de Broglie wavelength is not inconsistent with the standard quantum treatment of light; rather, it provides an intuitive and essential way to understand the interferometric properties of the entangled multiphoton states.

In conclusion, we have successfully measured the photonic de Broglie wavelength of the biphotons generated by spontaneous parametric down-conversion utilizing a Mach-Zehnder interferometer. Our results will encourage novel applications, such as quantum lithography, that utilize quantum-mechanical interference of entangled photons.

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