Limiting Temperatures and the Equation of State of Nuclear Matter

J. B. Natowitz, K. Hagel, Y. Ma, M. Murray, L. Qin, R. Wada, and J. Wang

Cyclotron Institute, Texas A&M University, College Station, Texas 77845

From experimental observations of limiting temperatures in heavy ion collisions we derive the critical temperature of infinite nuclear matter $T_c = 16.6 \pm 0.86$. Theoretical model correlations between T_c, the compressibility modulus K, the effective mass m^*, and the saturation density ρ_s are then exploited to derive the quantity $(K/m^*)^{1/2}\rho_s^{-1/3}$. This quantity together with calculations employing Skyrme and Gogny interactions indicates a value of K in moderately excited nuclei that is in excellent agreement with the value determined from giant monopole resonance data.

PACS numbers: 25.70.Gh, 21.65.+f, 24.10.–i

Improved knowledge of the nuclear equation of state and a coherent picture of the relationship between the properties of finite nuclei and bulk nuclear matter remains a key requirement to understanding nuclear structure, heavy ion collisions, supernova explosions, and neutron star properties [1–3]. Significant effort has been devoted to the development of microscopic theoretical models which can provide reliable mathematical formulations of this equation of state [4–20]. Such calculations are usually specified for symmetric nuclear matter, a hypothetical system of equal numbers of neutrons and (uncharged) protons interacting through nuclear forces. Driven by the astrophysical problems and recent laboratory excursions into the region of exotic nuclei, the dependence of the equation of state on neutron-proton asymmetry has also become a subject of significant interest [21–23]. In this Letter we employ data from experimental measurements of caloric curves in nuclear collisions, together with systematic trends and correlations derived from a number of theoretical investigations of nuclear matter, to derive the critical temperature and incompressibility of symmetric nuclear matter. The techniques employed offer a natural method to extend such investigations to more asymmetric systems.

In a recent paper measurements of nuclear specific heats from a large number of experiments were employed to construct caloric curves for five different regions of nuclear mass [24]. Within experimental uncertainties each caloric curve exhibits a plateau region at higher excitation energy. As indicated in Ref. [24] the plateau temperatures can be interpreted as representing the limiting temperatures resulting from Coulomb instabilities of expanded and heated nuclei. In Fig. 1 these limiting temperatures from Ref. [24] are presented as a function of mass. As previously noted, they are observed to decrease with increasing mass. This decrease with increasing mass has long been predicted [25–34].

The results employed in Ref. [24] were based upon temperature determinations derived from double isotope yield ratios and from slope measurements of particle spectra. More recently the TAPS Collaboration has re-

ported temperatures determined from a new technique, observations of "second chance" bremsstrahlung gamma ray emission [35,36]. There are not yet sufficient data of this latter type to construct caloric curves. However, in each case studied with this technique the collisions lead to excitation energies which are above those identified as the starting points of the plateau regions identified in Ref. [24]. Thus it is reasonable to compare the temperatures determined from the thermal bremsstrahlung measurements with the earlier limiting temperature values. As seen in Fig. 1, the gamma temperatures and their mass dependence are in excellent agreement with the earlier results.

A large number of theoretical calculations of the critical temperature of semi-infinite nuclear matter (nuclear matter with a surface) have been reported in the literature [25–34]. The different nuclear interactions employed in these calculations lead to large differences in the critical temperatures derived. Values from 13 to 24 MeV are reported in Refs. [25–34]. The limiting temperatures plotted in Fig. 1 are well below these calculated critical temperatures. This difference reflects finite size effects,
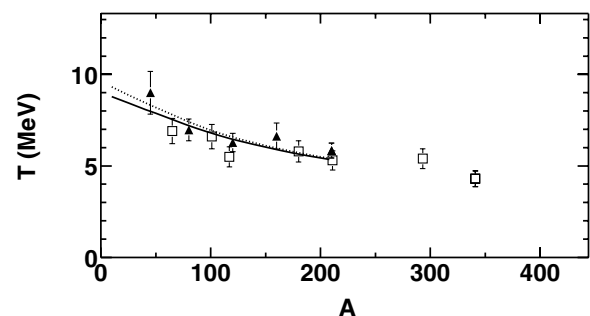

FIG. 1. Limiting temperatures vs mass. Limiting temperatures derived from double isotope yield ratio measurements are represented by solid triangles. Temperatures derived from thermal bremsstrahlung measurements are represented by open squares. Lines represent limiting temperatures calculated using interactions proposed by Gogny (dashed line) [29] and Furnstahl *et al.* [10,34] (solid line).

Coulomb effects, and isospin asymmetry effects for the finite nuclei studied.

A first order estimate of the magnitude of these combined effects suggests that limiting temperatures in nuclei should be ≈ 0.5 times the critical temperature of nuclear matter [5]. Given the wide variation in the calculated values of T_c it is not surprising that large variations result for the absolute values of limiting temperatures calculated for finite nuclei.

Employing a variety of Skyrme-type interactions Song and Su have previously noted a mass dependent scaling of the Coulomb instability temperatures with the critical temperature of nuclear matter (see Fig. 6 of Ref. [28]). A similar scaling exists when other model interactions are employed.

Mean values of $T_{\lim}/T_c$ for five different masses which result from averaging the results of different calculations [25–34] are shown in Fig. 2. The estimated uncertainties are relatively small, $\approx 6\%$. A useful parametrization of $T_{\lim}/T_c$, valid for $10 \leq A \leq 208$, is $(T_{\lim}/T_c) = 0.597 - 0.001\,95A + 3.45 \times 10^{-6}A^2$. For comparison, the figure also presents ratios of $T_{\lim}/T_c$ which are expected to result assuming only finite size effects as derived from a lattice calculation [37] and the ratio of the nuclear binding energy per nucleon along the line of beta stability to the bulk binding energy per nucleon, 16 MeV. We have employed the mean variation of $T_{\lim}/T_c$ with A, determined from commonly used microscopic theoretical calculations, together with the five experimental limiting temperatures reported in Ref. [24], to extract the critical temperature of nuclear matter. In doing so we treat the theoretical variation as if it were an experimental uncertainty. Since the various interactions employed have been "tuned" to other nuclear properties, we consider this a reasonable approach. The results are presented
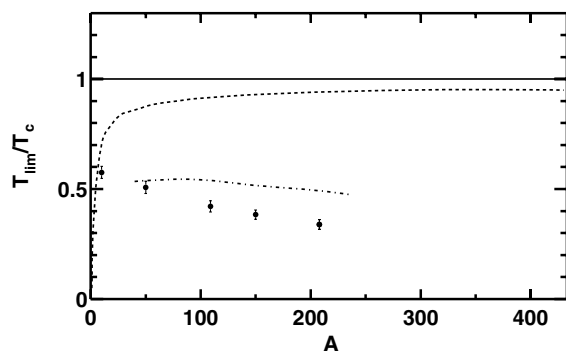
in Fig. 3. Averaging the individual results we find 16.6 ± 0.86 MeV, where the error represents 1 standard deviation.

It is interesting to ask whether additional equation of state information can be extracted from this result. Blaizot et al. [9] have argued that the most effective way to extract the incompressibility modulus of nuclear matter from experimental data is by comparison with microscopic calculations using giant monopole resonance (GMR) data. The generally accepted best current value of $K = 231 \pm 5$ MeV has been determined in such a fashion [38] by comparison of experimental centroids with those calculated with Gogny interactions [9]. We have adopted a similar comparison procedure using the present result for T_c determination.

We began by using a relation suggested by the work of Kapusta [39] and Lattimer and Swesty [40] who have pointed out that correlations between parameters used to describe nuclear matter are such that a relationship between the critical temperature, T_c, the incompressibility, K, the effective mass, m^* ($= m_{\mathrm{eff}}/m$, where m_{eff} is the nucleon effective mass and m is the nucleon mass), and the saturation density, ρ_s, may be written as

$$T_c = C_T (K/m^*)^{1/2} \rho_s^{-1/3}, \qquad (1)$$

where C_T is a constant. Using this relationship, we have determined the constant C_T in this equation using published theoretical values for T_c, calculated utilizing a number of different microscopic interactions [4–20]. Results of calculations using interactions with $155 < K < 384$ MeV are depicted in Fig. 4. A least squares fit to these data suggests a very slight decrease of C_T with increasing $(K/m^*)^{1/2} \rho_s^{-1/3}$. Using the present value for T_c together with Eq. (1) and the fit equation for the solid line in Fig. 4 leads to

$$C_T = 0.484 \pm 0.074 \ \mathrm{MeV}^{1/2}\,\mathrm{fm}^{-1}$$

and

FIG. 2.   Theoretical variation of the ratio $T_{\lim}/T_c$ with mass along the line of beta stability. The solid line indicates the reference value of T_c. The short dashed line shows the effect of finite size scaling derived from an Ising model [37]. The line with alternating short and long dashes depicts the ratio of the nuclear binding energy per nucleon to the bulk binding energy per nucleon, 16 MeV. Points with uncertainties are derived from the model calculations in Refs. [25–34].
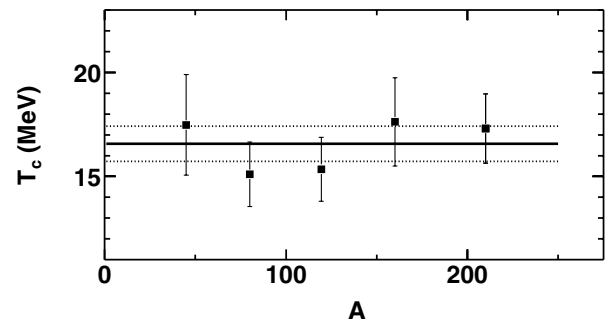

FIG. 3.   Derived values of the critical temperature of symmetric nuclear matter. The mean value of 16.6 MeV is indicated by the horizontal solid line. The range corresponding to ± 1 standard deviation from this mean value is shown by the thin dotted lines.
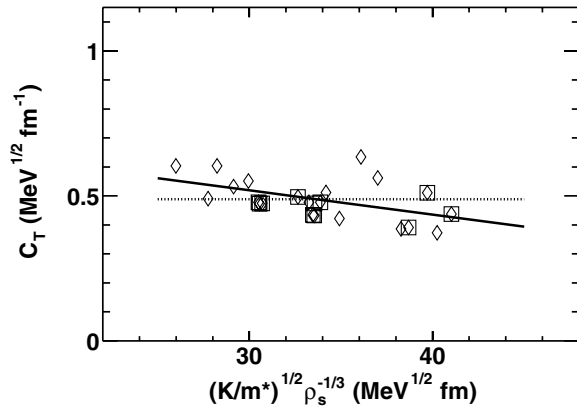
FIG. 4. The constant C_T of Eq. (1), evaluated from various microscopic calculations. C_T is plotted against $(K/m^*)^{1/2}\rho_s^{-1/3}$. Derived values of C_T are indicated by open diamonds. The dotted horizontal line indicates the mean value of C_T. The solid line represents the linear least squares fit to the derived values. Values of C_T obtained from Skyrme and Gogny interactions are further identified by open squares placed around the diamonds.

$$(K/m^*)^{1/2}\rho_s^{-1/3} = 34.2 \pm 5.34 \text{ MeV}^{1/2} \text{ fm}.$$

The saturation density, ρ_s, is well established by charge density measurements to be 0.16 ± 0.005 fm^{-3} [13]. The standard deviation of the model values of ρ_s from 0.16 fm^{-3}, calculated with the different interactions, is 3.4%. Either of these uncertainties is very small compared to other uncertainties in the determination. Therefore T_c is a good measure of K/m^*. K and m^* are not independent variables but are correlated [9,13]. For Skyrme effective interactions Chabanat *et al.* have discussed the correlation between K and m^* [13]. In that work m^* is written as

$$m^* = \left(1 + \frac{m}{8\hbar^2}\rho\Theta_s\right)^{-1}, \qquad (2)$$

where

$$\Theta_s = \frac{K - B - C\sigma}{D(1 - \frac{3}{2}\sigma)} \qquad (3)$$

and σ is a parameter which ranges from zero to 1 and controls the density dependence of the interaction. B, C, D are parameters directly related to e_∞, the energy per nucleon in infinite nuclear matter and e_F, the Fermi energy of infinite nuclear matter.

$$B = -9e_\infty + \frac{3}{5}e_F, \qquad (4)$$

$$C = -9e_\infty + \frac{9}{5}e_F, \qquad (5)$$

$$D = \frac{3}{20}\rho k_F^2. \qquad (6)$$

Here k_F is the Fermi momentum.

For a given K the ratio K/m^* in Eq. (1) depends on the choice of σ, the parameter of the density dependent term. (In the Gogny interactions of Ref. [9] this parameter controlling the density dependent term is designated α.) As a result, determination of K from K/m^* is sensitive to the choice of this parameter. For example, in Ref. [13] the relation between K and m^* is such that small values of σ dictate lower values of K. Also, for smaller values of σ, m^* decreases as K increases while for larger σ, m^* increases with increasing K (see Fig. 2 of Ref. [13]).

For comparison to the data we present in Fig. 5 a plot of K vs $(K/m^*)^{1/2}\rho_s^{-1/3}$, obtained using the Gogny interactions from Ref. [9] and various Skyrme interactions. The dashed lines in the plot show the trend of the generalized Skyrme interactions for $\sigma = \frac{1}{6}, \frac{1}{3}$, and 1 [13]. The solid lines connect results for Gogny interactions with $\alpha = \frac{1}{3}$ and $\frac{2}{3}$ [9]. As seen in the figure, a higher value of σ leads to a higher apparent K. It has been pointed out that maintaining K in a "reasonable" range of 200–300 MeV requires low values of σ (or α) [5,9,13]. In particular, the value of $K = 231 \pm 5$ MeV derived from the GMR data [38] was obtained by comparison of data for the breathing mode energy of five different nuclei with energies calculated employing the Gogny D1 ($\alpha = \frac{1}{3}$), D1S ($\alpha = \frac{1}{3}$), and D250 ($\alpha = \frac{2}{3}$) interactions [9]. For three of these nuclei only the D1S interaction results were used. For the other two a fit to the trend in energies calculated from the three interactions was employed.

The value of $(K/m^*)^{1/2}\rho_s^{-1/3}$ derived from this work is also indicated on the figure by the vertical line. We note that this line intersects the calculated values essentially at a point where the $\frac{1}{6}$ Skyrme and $\frac{1}{3}$ Gogny lines intersect. The different slopes of the Skyrme and Gogny lines in the
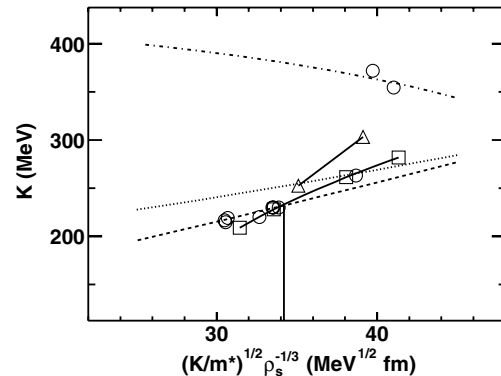

FIG. 5. Compressibility modulus, K, as a function of $(K/m^*)^{1/2}\rho_s^{-1/3}$. The values obtained for Gogny interactions of Ref. [9] are represented by open squares ($\alpha = \frac{1}{3}$) and open triangles ($\alpha = \frac{2}{3}$). Symbols for each set are connected by thin solid lines. Results using different Skyrme interactions are represented by open circles. The other lines represent generalized calculations using Skyrme interactions [13] with $\sigma = \frac{1}{6}$ (short dashed line), $\sigma = \frac{1}{3}$ (dotted line), and $\sigma = 1$ (alternating dashes and dots).

figure lead to different uncertainties in the K value. Thus employing Skyrme interactions with the $\sigma = \frac{1}{6}$ parametrization [13], $K = 232 \pm 22$ MeV. Using Gogny interactions with $\alpha = \frac{1}{3}$ [9] leads to $K = 233 \pm 39$ MeV. These results for K lead, respectively, to m^* values of $0.674^{+0.18}_{-0.13}$ or $0.674^{+0.11}_{-0.09}$. The compressibility modulus determined from the critical temperature in this manner is then entirely consistent with that determined from the GMR measurements. Higher values of σ (or α) will lead to higher apparent K. Thus for the Skyrme $\sigma = \frac{1}{3}$ line a value of $K = 252$ would result. For the extension of the Gogny $\alpha = \frac{2}{3}$ line, $K = 242$ would be obtained. The calculated breathing mode energies are apparently less sensitive to the value of the parameter of the density dependent interaction.

In summary, from limiting temperature values obtained in five different mass regions we have determined a critical temperature of 16.6 ± 0.86 MeV for symmetric infinite nuclear matter. This has been used to derive both K, the incompressibility, and m^*, the effective mass. Extracted by comparison with the same interactions as were employed to determine K from observations of the giant monopole resonance at low excitation energy, the value of K, obtained here from properties of nuclei at moderate excitation energies, is found to be in excellent agreement with that GMR result [38]. The precision for the T_c measurement could be improved. However, given the relative complexity of the collision dynamics involved, the breathing mode measurements should remain as the standard. Nevertheless, using newly available radioactive beams the determination of limiting and critical temperatures may play a significant role in providing a means to establish the N/Z asymmetry dependence of the compressibility modulus and other important nuclear properties [41].

---

[1] B. A. Li et al., Int. J. Mod. Phys. E **7**, 147 (1998).

[2] J. M. Lattimer and M. Prakash, Phys. Rep. **333–334**, 121 (2000).

[3] P. G. Reinhard et al., RIKEN Rev. **26**, 23 (2000); D. Vautherin and D. M. Brink, Phys. Rev. C **5**, 626 (1972).

[4] J. P. Blaizot, Phys. Rep. **64**, 171 (1980).

[5] H. Jaqaman et al., Phys. Rev. C **27**, 2782 (1983); **29**, 2067 (1984).

[6] W. D. Myers and W. J. Swiatecki, Phys. Rev. C **57**, 3020 (1998).

[7] J. Zimanyi and S. A. Moszkowski, Phys. Rev. C **42**, 1416 (1990).

[8] P. G. Reinhard et al., Z. Phys. A **323**, 13 (1986).

[9] J. P. Blaizot et al., Nucl. Phys. **A591**, 435 (1995).

[10] R. J. Furnstahl et al., Nucl. Phys. **A615**, 441 (1997).

[11] G. Lalazissis et al., Phys. Rev. C **55**, 540 (1997).

[12] M. Malheiro et al., nucl-th/9606014.

[13] E. Chabanat et al., Nucl. Phys. **A627**, 710 (1997).

[14] M. Farine et al., Nucl. Phys. **A615**, 135 (1997).

[15] M. Abd-Alla and S. A. Hager, Phys. Rev. C **61**, 044313 (2000).

[16] A. L. Goodman et al., Phys. Rev. C **30**, 851 (1984).

[17] J. Boguta and A. R. Bodmer, Nucl. Phys. **A292**, 413 (1977).

[18] D. Von-Eiff et al., Phys. Lett. B **324**, 279 (1994).

[19] M. Onsi and J. M. Pearson, Phys. Rev. C **65**, 047302 (2002).

[20] J. S. Sá Martins and A. Delfino, Phys. Rev. C **61**, 044615 (2000).

[21] H. Müller and B. D. Serot, Phys. Rev. C **52**, 2072 (1995).

[22] K. C. Chung et al., Eur. Phys. J. A **10**, 27 (2001).

[23] S. Haddad, Acta Phys. Pol. B **32**, 1333 (2001).

[24] J. B. Natowitz et al., Phys. Rev. C **65**, 034618 (2002).

[25] P. Bonche et al., Nucl. Phys. **A436**, 265 (1985).

[26] S. Levit and P. Bonche, Nucl. Phys. **A437**, 426 (1985).

[27] J. Besprovany and S. Levitt, Phys. Lett. B **217**, 1 (1989).

[28] H. Q. Song and R. K. Su, Phys. Rev. C **44**, 2505 (1991).

[29] Y.-J. Zhang et al., Phys. Rev. C **54**, 1137 (1996).

[30] A. Das et al., J. Phys. G **18**, 869 (1992).

[31] H. Q. Song et al., Phys. Rev. C **47**, 2001 (1993).

[32] H. Q. Song et al., Phys. Rev. C **49**, 2924 (1994).

[33] M. Baldo and L. S. Ferreira, Phys. Rev. C **59**, 682 (1999).

[34] L. L. Zhang et al., Phys. Rev. C **59**, 3292 (1999).

[35] D. G. d'Enterria et al., Phys. Rev. Lett. **87**, 022701 (2002).

[36] D. G. d'Enterria et al., Phys. Lett. B **538**, 27 (2002).

[37] J. B. Elliott et al., Phys. Rev. C **62**, 064603 (2000).

[38] D. H. Youngblood et al., Phys. Rev. Lett. **82**, 691 (1999).

[39] J. Kapusta, Phys. Rev. C **29**, 1735 (1984).

[40] J. M. Lattimer and F. D. Swesty, Nucl. Phys. **A535**, 331 (1991).

[41] *Isospin Physics in Heavy-Ion Collisions at Intermediate Energies*, edited by B. A. Li and W. U. Schroeder (Nova Science Publishers, New York, 2001).