Scalable Solid-State Quantum Processor Using Subradiant Two-Atom States

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We propose a realization of a scalable, high-performance quantum processor whose qubits are represented by the ground and subradiant states of effective dimers formed by pairs of two-level systems coupled by resonant dipole-dipole interaction. The dimers are implanted in low-temperature solid host material at controllable nanoscale separations. The two-qubit entanglement either relies on the coherent excitation exchange between the dimers or is mediated by external laser fields.

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The main stumbling blocks en route to the realization of useful quantum computers, comprised of many qubits, are [1] (i) *fidelity loss* due to decoherence, which grows with the amount of single- and two-qubit operations and requires large redundancy for the application of error-correction methods; (ii) *scalability* of the quantum processor (QP), which restricts the choice of candidate systems and gives preference to solid-state structures. QP proposals and realizations have thus far predominantly involved optical manipulations of atoms in ion traps [2–4], high-Q cavities [5], and optical lattices [6]. Yet, the decoherence caused by radiative (spontaneous emission) and nonradiative processes, as well as difficulties with the scalability, cast doubts on the suitability of these schemes for truly large-scale quantum computation [7]. Solid-state QP realizations [8–11] appear to be more promising, both principally and technologically.

Here we propose a combined optical/solid-state approach that can significantly enhance the speed, fidelity, and scalability of a QP. The crux of this approach is the hitherto unexplored concept of a "subradiant dimer" (SD) qubit: two similar two-level systems (atoms or quantum dots) that are separated by a few nanometers and interact via the resonant dipole-dipole interaction (RDDI) [12], thereby forming an effective "dimer," whose ground and subradiant ("dark") states serve as the qubit basis. *All the basic ingredients of quantum computation* (state preparation, universal logic gates, and qubit readout) [13] are shown to be realizable by high-speed optical manipulations of these dimers with very small error probability, due to strong inhibition of radiative decay. A scalable QP is envisioned in a low-temperature solid host material doped with such dimers at controllable nanoscale separations.

Let us recall the cooperative properties of two identical two-level atoms (TLAs), 1 and 2, at fixed positions $\mathbf{r}_1$ and $\mathbf{r}_2$, whose ground and excited states are labeled as $|g_{1,2}\rangle$ and $|e_{1,2}\rangle$, respectively [Fig. 1(a)]. The effective (non-Hermitian) Hamiltonian of the system can be cast in a form [12]

$$H = H_{\mathrm{A}} + V_{\mathrm{RDDI}}, \tag{1}$$

where $H_{\mathrm{A}} = \hbar(\omega_{eg} - i\gamma/2)(|e_1\rangle\langle e_1| + |e_2\rangle\langle e_2|)$ represents the atomic Hamiltonian, with ω_{eg} being the resonant frequency and γ the radiative decay rate on the atomic transition $|e\rangle \rightarrow |g\rangle$, and $V_{\mathrm{RDDI}} = \hbar(\Delta - i\gamma_{12}/2)(|e_1 g_2\rangle\langle g_1 e_2| + |g_1 e_2\rangle\langle e_1 g_2|)$ describes the interatomic RDDI potential, whose real part Δ is equal to the rate of coherent excitation exchange (hopping) between the atoms, and the imaginary part γ_{12} is responsible for the cooperative radiative decay of the system. Both Δ and γ_{12} are functions of the normalized distance between the atoms $\zeta = qr_{12}$, with $q = \omega_{eg}/c$ and $r_{12} = |\mathbf{r}_1 - \mathbf{r}_2|$. The diagonalization of Hamiltonian (1) yields the dimer eigenstates $|G\rangle = |g_1 g_2\rangle$, $|\pm\rangle = \frac{1}{\sqrt{2}}(|e_1 g_2\rangle \pm |g_1 e_2\rangle)$, and $|E\rangle = |e_1 e_2\rangle$, with the energy eigenvalues $\lambda_G = 0$, $\lambda_\pm = \omega_{eg} \pm \Delta - i\Gamma_\pm/2$, and $\lambda_E = 2\omega_{eg} - i\Gamma_E/2$, respectively [Fig. 1(b)]. At small separations $\zeta \ll 1$, the symmetric $|+\rangle$ and doubly excited $|E\rangle$ eigenstates are *superradiant,* having the corresponding decay rates $\Gamma_+ = \gamma + \gamma_{12} \approx \Gamma_E = 2\gamma$, while the antisymmetric eigenstate $|-\rangle$ is *subradiant,* with the decay rate $\Gamma_- = \gamma - \gamma_{12} \approx \gamma\zeta^2/5 \ll \gamma$ [12]. The energy levels of states

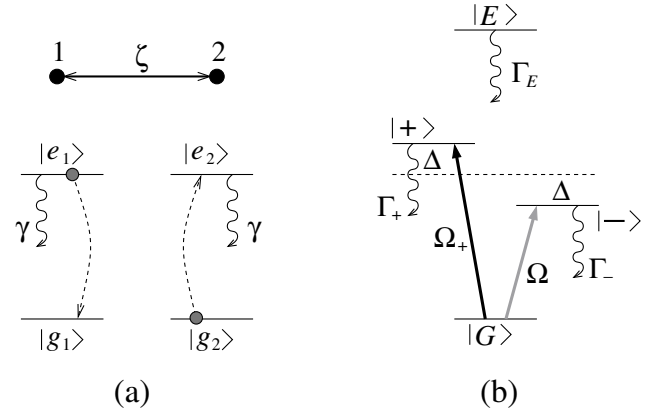

FIG. 1. (a) Two TLAs 1 and 2, separated by normalized distance ζ, interact via RDDI and exchange a single excitation. (b) Energy level diagram of the resulting dimer states of the system.

$|\pm\rangle$ are then shifted from that of state $|e\rangle$ by $\pm\Delta$, with $|\Delta| \approx 3\gamma/(4\zeta^3) \gg \gamma$.

The coupling strength of a laser field $\mathcal{E}$, having frequency $\omega \sim \omega_{eg}$ and wave vector $\mathbf{k}$, with the dimer is expressed by its Rabi frequencies, which are equal to $\pm\Omega_-$ on the transitions $|G\rangle \rightarrow |-\rangle$ and $|-\rangle \rightarrow |E\rangle$, respectively, and to Ω_+ on the transitions $|G\rangle \rightarrow |+\rangle$ and $|+\rangle \rightarrow |E\rangle$, where $\Omega_\pm = 2^{-1/2}\Omega[1 \pm e^{-i\mathbf{k}\mathbf{r}_{12}}]$ and $\Omega = \mu\mathcal{E}/\hbar$ is the Rabi frequency of the field for a single isolated atom, with μ being the dipole matrix element for the atomic transition $|g\rangle \rightarrow |e\rangle$. In the limit of small interatomic separations, $\Omega_+ \simeq 2^{1/2}\Omega$ and $\Omega_- \simeq i2^{-1/2}\Omega\zeta\cos\phi$, where ϕ is the angle between the vectors $\mathbf{k}$ and $\mathbf{r}_{12}$. Hence, Ω_- identically vanishes if the propagation direction of the field is perpendicular to the interatomic axis, $\mathbf{k} \perp \mathbf{r}_{12}$, while it is maximized in the $\mathbf{k} \parallel \mathbf{r}_{12}$ configuration, for $\zeta \ll 1$. In physical terms, the subradiant $|G\rangle \rightarrow |-\rangle$ transition exhibits a *quadrupolar* behavior and dipole-moment suppression, due to destructive interference of the two-atom interactions with the field, as opposed to their constructive interference in the superradiant $|G\rangle \rightarrow |+\rangle$ transition.

Now we are in a position to introduce the concept of the subradiant dimer (SD) qubit. The two-qubit states correspond to the ground $|G\rangle$ and subradiant $|-\rangle$ states of the dimer. An arbitrary single-qubit operation (rotation) can be performed by the laser field $\mathcal{E}_r$ with wave vector $\mathbf{k}_r \parallel \mathbf{r}_{12}$ and frequency $\omega_r = \omega_{eg} - \Delta$ that is resonant with the qubit transition $|G\rangle \rightarrow |-\rangle$ [Fig. 1(b)]. During the qubit flip-time $T_{\text{flip}} = \pi/(2|\Omega_-^{(r)}|)$, the probability of error P_-^{sp} due to spontaneous emission from the subradiant state $|-\rangle$ has the upper bound $P_-^{\text{sp}} \leq \Gamma_- T_{\text{flip}} = \pi\gamma\zeta/(5\sqrt{2}\Omega_r)$, while the probability of error due to population transfer from the ground state $|G\rangle$ to the superradiant state $|+\rangle$ satisfies $P_+^{\text{tr}} \leq \Gamma_+|\Omega_+^{(r)}|^2 T_{\text{flip}}/(2\Delta)^2 = 8\sqrt{2}\pi\Omega_r\zeta^5/(9\gamma)$. As an example, for the parameters $\zeta \simeq 0.02$ and $\Omega_r/\gamma \simeq 30$, the decay rate of the antisymmetric state is $\Gamma_- \approx 8 \times 10^{-5}\gamma$ and the error probabilities during the flip-time of a SD qubit are $P_-^{\text{sp}} \simeq 3 \times 10^{-4}$ and $P_+^{\text{tr}} \ll P_-^{\text{sp}}$, as compared to the corresponding error probability for a single atom, $P_{\text{atom}}^{\text{sp}} \leq \pi\gamma/(2|\Omega_r|) \simeq 0.05$. Such small errors of the SD qubit are amenable to error correction [1].

In order to *readout* (measure) the state of the qubit, we may use a modification of the electron-shelving technique [14]. Let us apply for a time T_{rout} a probe field $\mathcal{E}_p$ at a frequency $\omega_p = \omega_{eg} + \Delta$ that is resonant with the dimer transition $|G\rangle \rightarrow |+\rangle$. Since the Rabi frequency on that transition is much larger than on the qubit transition $|G\rangle \rightarrow |-\rangle$, from which the probe field is detuned by 2Δ, the presence or absence of fluorescence from $|+\rangle$ would indicate whether the qubit state is $|G\rangle$ or $|-\rangle$, respectively. However, since the frequency ω_p exactly matches that of the transition $|-\rangle \rightarrow |E\rangle$ [Fig. 1(b)], the dimer in state $|-\rangle$ can first be excited to $|E\rangle$ by absorbing a probe photon, then decay to $|+\rangle$, subsequently producing the same fluorescence signal as if it were initially in state $|G\rangle$. Therefore, for a reliable measurement, the condition $\gamma_{-+}T_{\text{rout}} < 1$ should be satisfied, where $\gamma_{-+} = |\Omega_p|^2\zeta^2/\gamma$ is the rate of transition $|-\rangle \rightarrow |+\rangle$. This leads to the condition $\Omega_p/\gamma < \sqrt{2\eta}/\zeta$, where $\eta < 1$ is the detector efficiency. With $\eta \simeq 0.3$, $\Omega_p/\gamma \simeq 5$, and $\zeta \simeq 0.02$, we obtain 98% measurement reliability. If, however, the probe laser is applied for a time $T_{\text{rout}} \geq \gamma_{-+}^{-1}$, it will *initialize* the state of the qubit to its ground state $|G\rangle$.

We next consider the RDDI-induced entanglement between two neighboring dimers of size ζ, labeled as A and B, whose normalized separation $\xi = qr_{AB}$ satisfies the condition $\zeta < \xi \ll 1$ [Fig. 2(a)]. The rate of coherent excitation exchange between the dimers on the qubit transitions $|G\rangle_{A,B} \rightarrow |-\rangle_{A,B}$ is given by $\Delta_{AB}^{(-)} \simeq 3\Gamma_-/(4\xi^3) = 3\gamma\zeta^2/(20\xi^3)$. If the difference in the qubit transition frequencies of the two dimers exceeds $\Delta_{AB}^{(-)}$ (as is usually the case in a solid host), then their excitation exchange is effectively switched off. To switch their interaction on, one can apply an off-resonant, intense, standing-wave field, such that dimers A and B are exposed to different field amplitudes and therefore undergo different ac Stark shifts [Fig. 2(a)]. The standing-wave pattern is then shifted along the $A - B$ axis until the qubit transitions of the two dimers become resonant. Then, during the time $T_{\text{SWAP}} = \pi/[2\Delta_{AB}^{(-)}]$, the SWAP transformation takes place, $|-\rangle_{A(B)}|G\rangle_{B(A)} \rightarrow -i|G\rangle_{A(B)}|-\rangle_{B(A)}$, while other initial states of the two qubits, $|-\rangle_A|-\rangle_B$ and $|G\rangle_A|G\rangle_B$, remain unaffected [Fig. 2(b)]. In the same way, one can realize the *square root of* SWAP ($\sqrt{\text{SWAP}}$) gate between two qubits. By switching on the interaction for time $T_{\sqrt{\text{SWAP}}} = \pi/[4\Delta_{AB}^{(-)}]$, one fully entangles the two qubits, attaining an equally weighted superposition of SWAP and no-SWAP,

$$|-\rangle_{A(B)}|G\rangle_{B(A)} \rightarrow \frac{1}{\sqrt{2}}[|-\rangle_{A(B)}|G\rangle_{B(A)} - i|G\rangle_{A(B)}|-\rangle_{B(A)}]. \tag{2}$$

The main source of error in this scheme is the cooperative

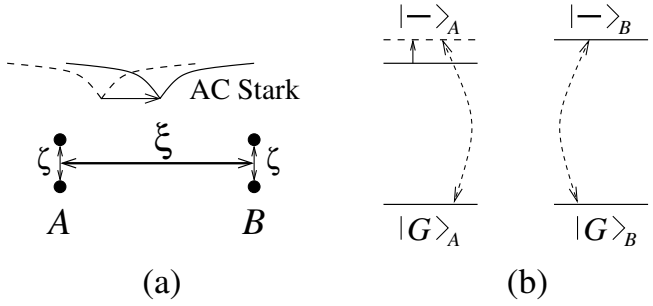

FIG. 2. (a) Dimers A and B are separated by normalized distance $\xi > \zeta$. An external ac Stark field can switch on and off the RDDI between the dimers. (b) When the qubit transitions of dimers A and B are brought to resonance, they start swapping a single excitation.

spontaneous decay of the excited states of the qubits, $P^{\rm sp}_{\rm SWAP} \le 2\Gamma_- T_{\rm SWAP} = 4\pi\xi^3/3$. With interdimer separation $\xi \simeq 0.1 \gg \zeta$, this leads to $P^{\rm sp}_{\rm SWAP} \le 4 \times 10^{-3}$, which can be taken care of by error-correction schemes [1].

A fast *controlled-phase* (CPHASE) logic gate between two closely spaced SD qubits can be realized by a laser field acting on the auxiliary transition $|G\rangle \to |+\rangle$ and thereby populating the state $|+\rangle$. This will induce the RDDI between the dimers, causing an excitation exchange between state $|+\rangle_A$ of dimer A and state $|G\rangle_B$ of dimer B and vice versa [Fig. 3(a)]. From the above analysis, the rate of this exchange is given by $\Delta^{(+)}_{AB} \simeq 3\Gamma_+/(4\xi^3) = 3\gamma/(2\xi^3)$, which is much larger than $\Delta^{(-)}_{AB}$, since $\Gamma_+/\Gamma_- \simeq 10/\zeta^2 \gg 1$. Therefore, during a time interval that is small compared to $|\Delta^{(-)}_{AB}|^{-1}$, we can neglect the RDDI between the dimers on the qubit transitions $|G\rangle_{A,B} \to |-\rangle_{A,B}$ in comparison to that on the auxiliary transitions $|G\rangle_{A,B} \to |+\rangle_{A,B}$. To the same accuracy, the eigenstates of the two-dimer system are $|G_A G_B\rangle$, $|M\rangle = \frac{1}{\sqrt{2}}(|+_A G_B\rangle - |G_A +_B\rangle)$, $|P\rangle = \frac{1}{\sqrt{2}}(|+_A G_B\rangle + |G_A +_B\rangle)$, and $|+_A +_B\rangle$. The singly excited states $|M\rangle$ and $|P\rangle$, having the decay rates $\Gamma_M \simeq \Gamma_+ \xi^2/5$ and $\Gamma_P \simeq 2\Gamma_+$, correspond, respectively, to the antisymmetric and symmetric combinations of the superradiant states of the two dimers [Fig. 3(b)]. To perform the CPHASE gate, we irradiate the system with the coupling field $\mathcal{E}_c$ having the wave vector $\mathbf{k}_c \parallel \mathbf{r}_{AB}$ and frequency $\omega_c = \omega_{eg} + \Delta - \Delta^{(+)}_{AB}$ that is resonant with the transition $|G_A G_B\rangle \to |M\rangle$ (Fig. 3, inset). The Rabi frequencies of this field on the transitions $|G_A G_B\rangle \to |M\rangle$ and $|G_A G_B\rangle \to |P\rangle$ are equal, respectively, to $\Omega^{(c)}_M = \Omega_c \xi$ and $\Omega^{(c)}_P = 2\Omega_c$. Since $\mathbf{k}_c \perp \mathbf{r}^{A,B}_{12}$, this field does not couple to the qubit transitions of the dimers. During the time $T_{\rm CPHASE} = \pi/\Omega^{(c)}_M$, the system of two dimers, being initially in the state $|G_A G_B\rangle$, undergoes the Rabi cycle from $|G_A G_B\rangle$ to $|M\rangle$ and back, resulting in the π-phase-shift

$$|G_A G_B\rangle \to -|G_A G_B\rangle. \tag{3}$$

This transformation corresponds to the CPHASE logic gate, since all other initial states, such as $|-\rangle_A|-\rangle_B$ and $|-\rangle_{A(B)}|G\rangle_{B(A)}$, remain unaffected, due to the fact that the RDDI between the dimers is present only if their combined state is either $|G\rangle_A|+\rangle_B$ or $|+\rangle_A|G\rangle_B$, otherwise the coupling field is off-resonant with the system. The error during this gate operation is due to the spontaneous emission from the state $|M\rangle$, with the probability $P^{\rm sp}_{\rm CPHASE} \le \Gamma_M T_{\rm CPHASE} = 2\pi\gamma\xi/5\Omega_c$, as well as due to population transfer from the state $|G_A G_B\rangle$ to the state $|P\rangle$, with the probability $P^{\rm tr}_{\rm CPHASE} \le \Gamma_P |\Omega^{(c)}_P|^2 T_{\rm CPHASE}/(2\Delta^{(+)}_{AB})^2 = 16\pi\Omega_c\xi^5/(9\gamma)$. With $\Omega_c/\gamma \simeq 30$ and $\xi \simeq 0.1 > \zeta$, we obtain $P^{\rm sp}_{\rm CPHASE} \le 4 \times 10^{-3}$ and $P^{\rm tr}_{\rm CPHASE} \ll P^{\rm sp}_{\rm CPHASE}$. This error probability is exactly the same as for a SWAP gate with similar ξ, but the CPHASE gate is then 25 times faster than the SWAP gate, since $T_{\rm SWAP}/T_{\rm CPHASE} = 10\Omega_c\xi^4/(3\gamma\zeta^2)$.

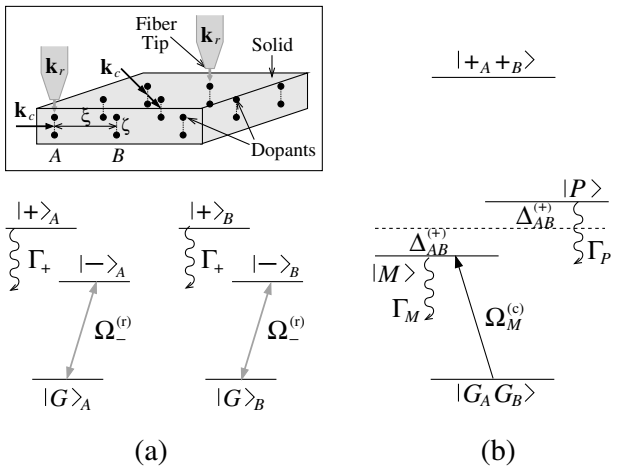

FIG. 3. Inset: Schematic drawing of the proposed QP and the geometry for the CPHASE gate: The interatomic axis of each dimer (SD qubit) is perpendicular to the interdimer axis, $\mathbf{r}^{A,B}_{12} \perp \mathbf{r}_{AB}$. Each SD qubit can be separately addressed by a laser field with $\mathbf{k}_r \parallel \mathbf{r}_{12}$. Two-qubit interaction is mediated by a coupling field with $\mathbf{k}_c \parallel \mathbf{r}_{AB}$. (a) Internal level structure of two dimers. (b) Eigenstates of the combined system of two dimers.

Having established all the basic principles of the proposed QP, we now describe its possible realization. We envision a solid-state host doped with active atoms having a *nondegenerate* ground state, so as to avoid mixing of various degenerate atomic states, which would invalidate our simple two-level atomic model. Possible candidate systems include sulphur-doped silicon, rare-earth (Yb or Nd) doped crystals [15], or semiconductor based nanostructures (quantum dots) [16]. The implantation of dopants and dots with controllable separations of a few nanometers is achievable with reasonable accuracy [10,17].

With the arrangement of dopants shown in the inset in Fig. 3, our scheme is capable of implementing arbitrary one-qubit rotations and two-qubit logic gates, so as to obtain any desired unitary transformation [1]. (a) *Individual SD qubits* would be rotated or read out (and initialized) by laser fields with frequency ω_r or ω_p, respectively (see above), and a wave vector parallel to the interatomic axis, using the "near-field" technique (Fig. 3, inset). The polarization of these fields can be chosen such that they act on only the atomic transition from the nondegenerate ground state to one of the magnetic sublevels of the excited state, consistent with our two-level description of the atoms. (b) *The* CPHASE *gate* between a chosen pair of qubits A and B is executed by a coupling field with frequency ω_c and wave vector $\mathbf{k}_c \parallel \mathbf{r}_{AB}$ that are specific for that pair. (c) *The* SWAP *action* between neighboring qubits can be used to convey the information in the QP, step by step, over large distances for which the direct RDDI vanishes. To neutralize the SWAP, one can flip the qubits at time intervals short compared to $[\Delta^{(-)}_{AB}]^{-1}$, which is equivalent to the spin

echo technique used in NMR [18]. Alternatively, the $\sqrt{\text{SWAP}}$ gate between two qubits A and B can be switched on and off via external ac Stark fields.

Throughout this Letter, we have dealt with only the *radiative relaxation* of the excited atomic state $|e\rangle$. This is adequate provided the competing *nonradiative relaxation processes* are strongly *suppressed* by working below the liquid helium temperature [19] and/or using fast ac Stark modulation of the vibrationally relaxing levels [20]. Another important consideration is the *inhomogeneous broadening* of the atomic resonances. Consider two atoms having slightly different resonant frequencies, $\omega_{eg}^{(2)} - \omega_{eg}^{(1)} = \delta$, due to the host inhomogeneity. This frequency mismatch results in an increase of the decay rate Γ_- of the SD qubit in the amount $\gamma\delta^2/(8\Delta^2)$. If we require that this additional relaxation rate does not exceed Γ_- for two resonant atoms, we obtain that the inhomogeneous width δ must be less than γ/ζ^2, which, for $\zeta \simeq 0.02$, yields $\delta \leq 2.5 \times 10^3 \gamma$.

It is instructive to compare our scheme with previously considered optically controlled single- and two-qubit quantum gates:

(i) In a commonly used optical scheme [5,6,11], a Raman qubit is represented by two metastable ground states $|g_1\rangle$ and $|g_2\rangle$ that are manipulated by two laser fields detuned by the amount $\delta_e \gg \gamma_e$ from the intermediate excited state $|e\rangle$ having the spontaneous decay rate γ_e. The error probability during the qubit flip $P_e^{\text{sp}} \leq \pi\gamma_e/(2\delta_e)$ is then an order of magnitude larger than for the SD qubit, given *similar* values of the single-photon Ω_R and the effective two-photon Ω_R^2/δ_e Rabi frequencies.

(ii) A CPHASE logic gate between two closely spaced Raman qubits [see (i) above], A and B, trapped in an optical lattice [6], is realized by an off-resonant "catalysis" field with Rabi frequency Ω_C, which induces a RDDI-dependent ac Stark shift of the two-qubit states. One then can show that during the gate operation, the probability of error due to spontaneous decay of the excited states $|e\rangle_{A,B}$ is given by $P_{\text{CPHASE}}^{(R)} \simeq 8\pi\xi^3/3$, where ξ is the Lamb-Dicke parameter. With $\xi \simeq 0.1$, we obtain that $P_{\text{CPHASE}}^{(R)} \simeq 8 \times 10^{-3}$, which is twice worse than in our scheme with the same ξ. More dramatically, for *similar* field strengths, e.g., $\Omega_C/\gamma_e \simeq 30$ and $\delta_e \simeq 5\Delta_{AB}^{(R)} \gg \Omega_C$, where $\Delta_{AB}^{(R)}$ is the RDDI coupling strength between the atoms on the transitions $|g_2\rangle_{A,B} \rightarrow |e\rangle_{A,B}$, we find that the SD qubit implementation of the CPHASE gate is $\sim$30 times faster.

(iii) A CPHASE gate in an ion trap [2] operates with speed and error probability similar to our scheme. The error in the ion trap QP is caused by the radiative decay of the auxiliary excited state, but one must also reckon with error due to the phonon-mode decoherence [3,4]. The main limitations of ion trap schemes are related to difficulties with their scalability.

To conclude, our proposal for an optically manipulated, solid-state quantum processor has no principal limitations on scalability. It allows us to suppress radiative decoherence and enhance the speed of photon-mediated quantum-logic gates, owing to the use of the ground and subradiant states of effective dimers formed by resonant dipole-dipole interacting two-level systems. These states constitute a physically realistic, simple, and robust "decoherence-free subspace" [21], whose implementation draws efficiently upon the system resources (only two atoms per qubit). The highly challenging experimental realization of such a quantum computer requires nanofabrication techniques with nanometer precision of dopant or quantum dot implantation [10,17].

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