Quantum Transport in Parallel Magnetic Fields: A Realization of the Berry-Robnik Symmetry Phenomenon

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We analyze the magnetoconductance of two-dimensional electron and hole gases subject to a *parallel* magnetic field. It is shown that, for confining potential wells which are symmetric with respect to spatial inversion, a temperature-dependent weak localization signal exists even in the presence of a magnetic field. Deviations from this symmetry lead to magnetoconductance profiles that contain information on both the geometry of the confining potential and characteristics of the disorder.

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Weak localization (WL) corrections to the conductivity [1] and magnetoresistance of two-dimensional systems in perpendicular magnetic fields [2] have been studied extensively for many years. These phenomena originate in the constructive interference of time-reversed electron trajectories. The magnetic field breaks time-reversal invariance and, therefore, suppresses the interference. Considerably less attention has been directed to the effect of an *in-plane* magnetic field on WL phenomena. Truly two-dimensional systems would not feel the orbital effect of an in-plane field at all—the paths within the plane enclose no flux. In *real* systems, however, the microscopic profile of the wave functions in the transverse, or z direction leads to a nonvanishing magnetic response. Early works on this phenomenon focused on disordered metallic films [3,4], where size quantization is absent, and two-dimensional electrons subject to *short-range* disorder [5,6]. A recent paper [7] considers systems with rough interfaces, as, e.g., Si metal-oxide-semiconductor field-effect transistors are believed to be [8].

In this Letter we analyze the complementary case where the motion of the carriers in the z direction is not completely stochastic. Such scenarios are realized, e.g., in a gas of electrons or holes on a GaAs/AlGaAs interface. The mobility in these systems is limited by a *long-range* random potential, $V(x, y, z)$, created by charged impurities located far from the interface. The z dependence of this potential is probably weak. In the approximation that neglects this dependence, $V = V(x, y)$, the in-plane motion can be separated from the motion in the z direction. Under these conditions, WL effects acquire nonuniversal features, depending on the structure of the confining potential, $W(z)$. Thus, monitoring WL signals one can reveal information on the microscopic structure of the potential well. Specifically, the temperature and in-plane magnetic field dependencies of the conductivity $\sigma(T, H)$ are sensitive to the symmetry of the confining potential under reflection, $\mathcal{P}_z: z \to -z$. Further, $\sigma(T, H)$ qualitatively depends on the number of occupied carrier subbands, M. The single subband ($M = 1$) case turns out to be special and is characterized by quite unusual magnetoresistance.

Let us first discuss the magnetotransport qualitatively. An in-plane magnetic field, H, manifests itself through the phase coherence time, $\tau_\phi(H)$ [3,4]:

$$\Delta\sigma = \frac{e^2}{\pi h}\ln\frac{\tau}{\tau_\phi(H)}; \quad \frac{1}{\tau_\phi(H)} = \frac{1}{\tau_\phi} + \frac{1}{\tau_H}, \qquad (1)$$

where $\Delta\sigma$ is the WL correction to the conductivity, τ is the elastic scattering time, and $\tau_H \propto H^{-2}$. Consider now the system displayed in Fig. 1. The finite motion in the z direction implies a splitting of the electronic spectrum into different subbands of size quantization. If $H = 0$ and the disorder is z independent, these subbands are decoupled and contribute separately to the conductivity, σ. Universality of the WL implies that in this case the correction, $\Delta\sigma$, Eq. (1), should be multiplied by the number of the occupied subbands: $\Delta\sigma = M(e^2/\pi h)\ln(\tau/\tau_\phi)$. Note that, as $\tau_\phi \propto T^{-p}$, the conductivity displays logarithmic temperature dependence.

The magnetic field plays *two* complementary roles: it breaks time-reversal ($\mathcal{T}$) symmetry and (together with the z dependence of the random potential) couples different subbands. In fact, the second role determines the

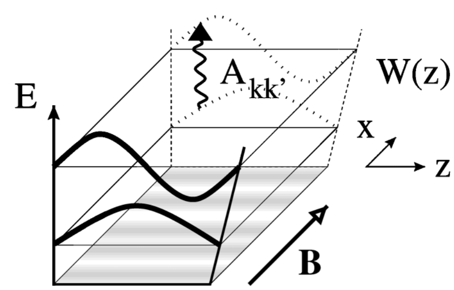

FIG. 1. Schematic picture of the quantum well. Two exemplary subband wave functions are shown. The profile of the impurity potential is sketched on the bottom of the well.

first one: $\mathcal{T}$ invariance is preserved as long as the subbands remain decoupled, since the vector potential of the parallel field can be gauged out in each particular subband. Therefore, the coupling governs the magnetoconductance. For strong intersubband coupling, we return to the disordered film situation, i.e., Eq. (1) for the WL effect. When the coupling is weak, the WL correction is determined by M different magnetic phase relaxation times τ_H^k:

$$\Delta\sigma = \frac{e^2}{\pi h}\sum_{k=0}^{M-1}\ln\frac{\tau}{\tau_\phi^k(H)}; \quad \frac{1}{\tau_\phi^k(H)} = \frac{1}{\tau_\phi} + \frac{1}{\tau_H^k}. \qquad (2)$$

It turns out that $1/\tau_H^{k\neq 0} > 0$, i.e., all WL corrections, except maybe one ($k = 0$), are temperature independent at $H \neq 0$ and low enough T. Whether $1/\tau_H^0$ vanishes or not depends on the $\mathcal{P}_z$ symmetry of the confining potential. A particularly interesting situation arises when the system is fully $\mathcal{P}_z$ symmetric: $W(z) = W(-z)$. Then, the original Hamiltonian is invariant under the combination of reversal of the magnetic field ($H \to -H$) and $\mathcal{P}_z$ inversions. This symmetry implies orthogonal rather than unitary level statistics [9]. As a result, $1/\tau_H^0 = 0$, and $\Delta\sigma \sim \ln(\tau/\tau_\phi)$ at arbitrary H [i.e., the logarithmic $\sigma(T)$ dependence persists]. All other relaxation times $\tau_H^{k\neq 0}$ are proportional to H^{-2} [10]. Accordingly, the WL correction reads $\Delta\sigma_s(H, T) = (e^2/\pi h)[p \ln T + 2(M-1)\ln H]$. By contrast, any violation of $\mathcal{P}_z$ symmetry (by either confining or disorder potentials) suppresses *all* T-dependent WL corrections, i.e., $\Delta\sigma_{as}(H, T) = 2M(e^2/\pi h)\ln H$ for $M \neq 1$. Therefore, the WL effects sensitively probe the symmetry properties of the confining (and disorder) potential. All in all, it is the interplay of the three factors — interband coupling, $\mathcal{T}$ invariance, and $\mathcal{P}_z$ invariance that determines the conductivity, $\sigma(T, H)$.

A special situation arises when just one subband is occupied, $M = 1$. In the absence of high-lying unoccupied bands, the parallel field has no effect whatsoever — a one-band system, being structureless in the z direction, cannot accommodate magnetic flux. Formally, the vector potential of the field can be removed by a gauge transformation (cf. the analysis below). Thus, $\mathcal{T}$ breaking at $M = 1$ requires virtual excursions into unoccupied subbands [5]. This fact substantially reduces the magnetoconductance: If the random potential is z independent, a residual effect exists, albeit of high order in the magnetic field, $\tau_{H,M=1} \sim H^{-6}$. This dependence can be understood as follows: The matrix elements controlling the interband hopping are proportional to H. This amounts to a hopping probability $\sim H^2$. Since the square of the field is $\mathcal{T}$ invariant, the virtual propagation *within* the empty bands must contribute another H and we arrive at $\sim H^3$ for the $\mathcal{T}$-breaking contribution to the self-energy. Finally, to obtain a quantum-mechanical intensity, the propagation amplitudes have to be squared which brings us to H^6. It is essential that the parallel field performs both $\mathcal{T}$ breaking *and* subband coupling. Sweeping the Fermi energy through the bottom of the second subband, a crossover $(\tau_H \sim H^{-2}) \leftrightarrow (\tau_H \sim H^{-6})$ in the WL profile should be observed. Table I displays a summary of the WL signals to be expected for a given subband population and symmetry configuration.

To derive these results we start from the Hamiltonian

$$\mathcal{H} = -\frac{1}{2m}(\partial + iHz\mathbf{e}_y)^2 + W(z) + V(x, y) \qquad (3)$$

of an electron subject to a confining potential, W, and lateral disorder, V. Later on, we will relax the condition of strict z independence of V. In the following, we consider only orbital coupling to the magnetic field; Zeeman splitting and spin-orbit scattering will be discussed elsewhere. Except for our final results, $\hbar = c = e = 1$.

It is convenient to project the Hamiltonian, Eq. (3), onto a basis of eigenfunctions, $\phi_k(z)$, of the transverse part of the Hamiltonian $[-\partial_z^2/(2m) + W(z)]\,\phi_k = \epsilon_k\phi_k$:

$$\mathcal{H}_{kk'} = \left[\frac{-\partial_x^2}{2m} + V(x, y) + \epsilon_k\right]\delta_{kk'} - \frac{1}{2m}(\partial_y - i\hat{A})^2_{kk'},$$

where the matrix elements

$$A_{kk'} = -H\int dz\phi_k(z)z\phi_{k'}(z) \equiv -Hd_{kk'} \qquad (4)$$

measure the degree of $\mathcal{P}_z$ violation. If the system is $\mathcal{P}_z$ symmetric, then $A_{kk} = 0$. Also notice that the explicit structure of the matrix $\hat{A}$ depends on the choice of gauge.

To assess transport properties we need to evaluate disorder averaged products of Green functions. In particular, WL corrections are determined by the two-particle Cooperon propagator. The key elements of this calculation, safe for the presence of a subband structure,

TABLE I. Magnetic phase relaxation times, $\tau_H^{(k)}$. Here d sets the scale for the width of the quantum well, Δ is the typical energy separation between subbands, D the diffusion constant, τ the mean scattering time, τ' the mean *transverse* scattering time, and v_F the Fermi velocity.

	$M = 1$	$M > 1$
$\mathcal{P}_z$ symmetry	$1/\tau_H = 0$	$1/\tau_H^0 = 0$, $1/\tau_H^{k\neq 0} \sim D/(\Delta\tau)^2\,(Hd)^2$
No $\mathcal{P}_z$ symmetry due to confining potential, $W(z) \neq W(-z)$	$1/\tau_H \sim D\,(v_F/\Delta)^4\,(Hd)^6$	$1/\tau_H^k \sim D/(\Delta\tau)^2\,(Hd)^2$
No $\mathcal{P}_z$ symmetry due to disorder, $V = V(x, y, z)$	$1/\tau_H \sim (v_F/\Delta)^2/\tau'\,(Hd)^2$	$1/\tau_H^0 \sim \min\{D/(\Delta\tau)^2\,(Hd)^2, 1/\tau'\}$, $1/\tau_H^{k\neq 0} \sim D/(\Delta\tau)^2\,(Hd)^2$

are largely standard [11]. The main result can be expressed through the Cooperon matrix, $(C_{\mathbf{q}})_{kk'}$:

$$\Delta\sigma = -\frac{2}{\pi}\sum_{k=0}^{M-1}\int (dq)(C_{\mathbf{q}})_{kk}; \tag{5}$$

$$(C_{\mathbf{q}}^{-1})_{kk'} = \left[(\mathbf{q}-2A_{kk}\mathbf{e}_y)^2 + \sum_{k''=0}^{M-1}\frac{\mathcal{X}_{kk''}H^2}{D_k}\right]\delta_{kk'} + \frac{\mathcal{X}_{kk'}H^2}{\sqrt{D_kD_{k'}}};$$
$$\mathcal{X}_{kk'} = (1-\delta_{kk'})d_{kk'}^2\frac{D_k+D_{k'}}{(\Delta_{kk'}\tau)^2+1}, \tag{6}$$

where D_k is the diffusion constant of the kth subband, and $\Delta_{kk'} = \epsilon_k - \epsilon_{k'}$. The decoherence rate, $1/\tau_\phi$, enters as a lower cutoff for the q integration. To understand the meaning of Eq. (6), let us compare it to the familiar equation for the Cooperon in the absence of a subband structure, $C_{\mathbf{q}}^{-1} \sim q^2 + (D\tau_H)^{-1}$. If the $\mathcal{P}_z$ asymmetry of the confining potential is weak, i.e., A_{kk} is small, then

$$(C_{\mathbf{q}}^{-1})_{kk} \sim q^2 + \lambda_k; \quad 1/\tau_H^k = \alpha_k\lambda_k, \tag{7}$$

where λ_k are the eigenvalues of the matrix $(C_{\mathbf{q}=0}^{-1})_{kk'}$ and α_k is a combination of the diffusion constants $\{D_0, \ldots, D_{M-1}\}$.

How do the eigenvalues depend on the magnetic field? For $H = 0$, all eigenvalues vanish trivially. Switching on a magnetic field leads to a coupling of the formerly independent subbands and, thereby, to a set of $M-1$ positive eigenvalues $\lambda_{k\neq 0}$. As a result $M-1$ Cooperon modes cease to contribute to the low temperature WL signal. However, the lowest eigenvalue, λ_0, plays a special role: For a perfectly symmetric potential, it remains zero implying that a single massless Cooperon mode survives application of a magnetic field. This is a direct manifestation of the Berry-Robnik phenomenon [9].

For a formal proof note that $A_{kk'} = 0$ for even $k + k'$, since $\mathcal{P}_z$ symmetry implies definite and alternating parity of the eigenstates. As a result the determinant of the Cooperon matrix, $C_{\mathbf{q}} = 0$, also vanishes. Indeed, it is straightforward to verify that $(C_{\mathbf{q}=0}^{-1})\mathbf{X} = 0$, where $\mathbf{X}$ is a M-component vector $\mathbf{X} \equiv \mathcal{N}^{1/2}\sum_k(-)^k\sqrt{D_k}\mathbf{e}_k$ and $\mathcal{N} = 1/\sum_k D_k$.

Now let $\mathcal{P}_z$ be slightly violated, either due to asymmetry of the confining potential or due to the impurity potential. In this case the matrix elements $A_{kk'}$, $k + k'$ even, become finite. To first order perturbation theory, the lowest eigenvalue shifts by $\delta\lambda_0(\mathbf{q}) = \mathbf{X}^T C_{\mathbf{q}}^{-1}\mathbf{X}$. With Eq. (6) this evaluates to $\delta\lambda_0^{(\mathrm{as})}(H) \sim \mathcal{N}[\sum_{k+k'\mathrm{even}}\mathcal{X}_{kk'} + \mathcal{N}\sum_{k,k'}D_kD_{k'}(d_{kk} - d_{k'k'})^2]H^2$.

Before considering concrete realizations of $W(z)$, let us explore how an additional weak z-dependent disorder potential, $\delta V(x, y, z)$, affects the Cooperon zero mode. z-dependent scattering leads to additional coupling between the subbands. At sufficiently high magnetic fields, where the field-induced coupling dominates the impurity-induced coupling, the lowest eigenvalue can again be evaluated perturbatively:

$$\delta\lambda_0^{(\mathrm{imp})} \sim \mathcal{N}\nu\int d^3\mathbf{r}\langle\delta V^2(\mathbf{r})\rangle\sum_{k+k'\mathrm{odd}}\int dz\phi_k^2\phi_{k'}^2,$$

or $\delta\lambda_0^{(\mathrm{imp})} \sim \mathcal{N}/\tau'$, where τ' can be understood as an intersubband scattering time. At small H, however, the dominating coupling mechanism is scattering in the z direction. In this case the lowest eigenvalue $\lambda_0^{(\mathrm{imp})}(H) \sim \mathcal{N}\sum_{k,k'}[\mathcal{X}_{kk'} + \mathcal{N}D_kD_{k'}(d_{kk} - d_{k'k'})^2]H^2$ corresponds to the vector $\mathbf{X}_l = \sum_k\sqrt{D_k}e_k$. Thus, $1/\tau_H^0$ increases as H^2 for small H and saturates at $H \sim H_c \sim \Delta/v_{\mathrm{F}}\sqrt{\tau/\tau'}/d$ (where Δ is the typical energy separation between subbands, v_{F} the Fermi velocity, and d the width of the quantum well) if the confining potential is $\mathcal{P}_z$ symmetric.

To illustrate our results on a simple and experimentally relevant example, let us consider a two-subband system, $M = 2$. Assuming for simplicity that $D_0 = D_1 \equiv D$, the 2×2 Cooperon takes the form

$$C^{-1} = \frac{1}{D}\begin{pmatrix} D(\mathbf{q}-\mathbf{A})^2 + \mathcal{X}_{01}H^2 + \frac{1}{\tau_\phi} & \mathcal{X}_{01}H^2 \\ \mathcal{X}_{01}H^2 & D(\mathbf{q}+\mathbf{A})^2 + \mathcal{X}_{01}H^2 + \frac{1}{\tau_\phi}\end{pmatrix},$$

where $\mathbf{A} = H(d_{00} - d_{11})\mathbf{e}_y$, and $\mathcal{X}_{01}$ obtains from Eq. (6).

At small fields, $H \ll H_\phi = (\mathcal{X}_{01}\tau_\phi)^{-1/2}$, the magnetoconductance is insensitive to the $\mathcal{P}_z$ symmetry and

$$\frac{\sigma(H)-\sigma(0)}{e^2/(2\pi^2\hbar)} \simeq \mathcal{X}_{01}\tau_\phi H^2 = \frac{2D(e/\hbar)^2 d_{01}^2\tau_\phi}{1+(\Delta_{10}\tau/\hbar)^2}H^2, \tag{8}$$

independent of the dipole elements d_{00} and d_{11}. [Notice that $\sigma(H) - \sigma(0)$ vanishes in the limit of infinitely separated bands, $\Delta_{10} \to \infty$, reflecting the behavior of isolated subbands.]

At large magnetic fields, $H \gg H_\phi$, and in the fully symmetric case, diagonalization of the Cooperon matrix yields $1/\tau_\phi^0(H) = 1/\tau_\phi$ and $1/\tau_\phi^1(H) = 2\mathcal{X}_{01}H^2$. While the second term leads to the usual logarithmic field dependence of $\Delta\sigma$ [cf. Eq. (2)], the field *independence* of the first term implies that the conductance continues to exhibit logarithmic scaling with temperature (through the T dependence of τ_ϕ) at these large fields. Slight violation of the symmetry results in a shift of both eigenvalues $\delta[1/\tau_\phi(H)] = D(e/\hbar)^2(d_{00} - d_{11})^2H^2$. Thus, the temperature dependence remains as long as $H < H_\phi^* = \hbar/[e(d_{00} - d_{11})\sqrt{D\tau_\phi}]$. For larger fields (or stronger asymmetry) the T dependence saturates and the slope of the

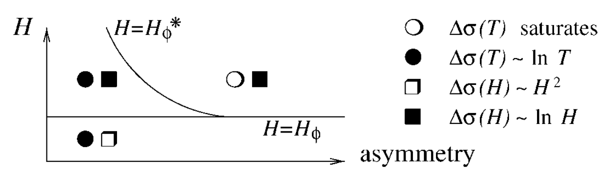

FIG. 2. Different regimes of T and H dependences.

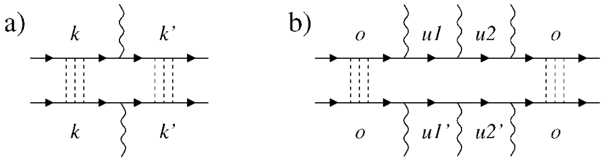

FIG. 3. Basic diagrams for (a) $M > 1$, and (b) $M = 1$. The wavy lines show interactions with the magnetic field while the dashed lines represent impurity scattering.

$\sigma(\ln H)$ dependence doubles [see discussion after Eq. (2)]. The regimes with different parameter dependences of the conductance are schematically shown in Fig. 2.

For concreteness, let us list the matrix elements $d_{kk'}$ for two common realizations of confining potentials: (i) For a symmetric box potential of width d, $d_{00} = d_{11} = 0$ and $d_{01} = -16d/(9\pi^2)$. Adding a small perturbation $\delta W(z) = wz$ to the confining potential yields the diagonal term $d_{00} - d_{11} = 4[16d/(9\pi^2)]^2 w/\Delta_{10}$. (ii) For an asymmetric triangular potential well, $W(z) = \infty$ for $z < 0$ and $W(z) = wz$ for $z > 0$, one obtains $d_{01} \approx 0.67(2mw)^{-1/3} < d_{00} - d_{11} \approx 1.17(2mw)^{-1/3}$.

What happens in the case of just one occupied subband? As discussed above, an in-plane magnetic field does not affect the single Cooperon mode in the $\mathcal{P}_z$-symmetric case. However, for broken $\mathcal{P}_z$ symmetry, virtual transitions into empty bands lead to a field and momentum dependent contribution $\Sigma(\hat{A}, \mathbf{p})$ to the self-energy of the Green functions of the *occupied* subband. The situation is depicted schematically in Fig. 3, where the relevant contribution to the Cooperon (the two-particle Green function) is shown for $M > 1$ (left panel) and $M = 1$ (right panel). In the latter case, sixth order scattering off the vector potential is needed to generate a field-dependent contribution. The self-energy can be presented in the form $\Sigma(\mathbf{p}) = \mathcal{D}_v(\mathbf{p}) + p_y \mathcal{A}_v(\mathbf{p})$, where $\mathcal{D}_v(\mathcal{A}_v)$ contains only even (odd) terms in the magnetic field. In contrast to $\mathcal{D}_v$ the second term violates $\mathcal{T}$ invariance by shifting the vector potential: $A_{00} \to A_{00} + \mathcal{A}_v(\mathbf{p})$ [13]. The corresponding magnetic scattering rate equals

$$\frac{1}{\tau_H} = \frac{D_0}{16}\left(v_F^2 \sum_{k,k'>0} \frac{A_{0k}(A_{kk'} - A_{00}\delta_{kk'})A_{k'0}}{\Delta_{0k}\Delta_{0k'}} \right)^2.$$

z-dependent scattering modifies this result. The mixing of the subbands by the disorder, $\delta V(\mathbf{r})$, brings a finite contribution to $1/\tau_H$ already at second order in the magnetic field. Calculation, as performed in Ref. [5], gives

$$\frac{1}{\tau_H} \sim \nu v_F^2 \int d^3\mathbf{r} \langle \delta V^2(\mathbf{r}) \rangle \sum_{k,k'>0} \frac{A_{0k}A_{k'0}}{\Delta_{k0}\Delta_{k'0}} \int dz \phi_0^2 \phi_k \phi_{k'}.$$

This leads to a $H^2 \to H^6$ crossover at the characteristic field $H_c^{M=1} \sim \sqrt{\Delta/D}(\tau/\tau')^{1/4}/d$.

To summarize, we have shown that the magnetoresistance of two-dimensional electron gases in an in-plane field responds sensitively to both the geometric structure of the confining potential and the nature of the impurity scattering. Those phenomena are intimately related to the Berry-Robnik symmetry mechanism [9]. We believe that the response in the magnetoconductance profile should be visible in experiment.

We thank V. I. Fal'ko for many valuable discussions and for sending us Ref. [14] (which discusses the H^6-type magnetoresponse for the one-subband system in more detail) prior to publication. We also thank C. M. Marcus for discussing potential experimental realizations.

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[10] The difference to the perpendicular case, where $\tau_{H_\perp} \sim H_\perp^{-1}$, follows from the fact that the characteristic area threaded by one flux quantum, $dl_{H_\parallel} \sim H_\parallel^{-1}$, is linearly related to the fixed width d of the potential well.
[11] Although fully perturbative, the intricate interplay of various scattering mechanisms in the problem suggests to employ the formalism of functional integration (as an alternative to direct diagrammatic perturbation theory). Straightforward adaption of the standard scheme [12] of deriving a field theory for disordered conductors to the structure of the present problem produces a model with effective action
$$S = \sum_{k=0}^{M-1} S_0[Q_k] - \frac{\pi\nu}{4} \sum_{k,k'=0}^{M-1} \chi_{kk'} \mathrm{Str}(\sigma_3^{\mathrm{tr}} Q_k \sigma_3^{\mathrm{tr}} Q_{k'}).$$
Here the matrices Q_k describe diffusive motion in subband k, controlled by the standard (field-dependent) action $S_0[Q_k]$ [12]. The second term in the action describes the field-induced coupling between the subbands, where $\mathcal{X}_{kk'}$ as defined in Eq. (7). Quantitatively, second order expansion around the saddle point configurations $Q_k \equiv \Lambda$ readily produces the Cooperon propagators.
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