Structure-Driven Nonlinear Instability of Double Tearing Modes and the Abrupt Growth after Long-Time-Scale Evolution

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The new nonlinear destabilization process is found in the nonlinear phase of the double tearing mode (DTM). This process causes the abrupt growth of DTM and subsequent collapse after long-time-scale evolution in the Rutherford-type regime. The nonlinear growth of the DTM is triggered when the triangular deformation of magnetic islands with sharp current point at the X point exceeds a certain value. Hence, the mode can be called the structure-driven one. Decreasing the resistivity increases the sharpness of the triangularity and the spontaneous growth rate in the abrupt-growth phase is almost independent of the resistivity.

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The observation of improved confinement of tokamak plasmas with reversed magnetic shear profile stimulates the development of the advanced concept of a tokamak reactor with high plasma performance and steady state operation, and theoretical investigations are focused to macroscopic and microscopic behaviors of a tokamak plasma with a nonmonotonic safety factor profile. Among several issues, the stability of the resistive double tearing mode (DTM) is one of the crucial subjects to be solved for a stable steady state tokamak operation, because this mode can have large growth rate even in a low-beta plasma. The problem has been extensively studied by many authors as the possible candidate of plasma disruption in fast current ramp-up experiments, and the nonlinear behavior of DTM has been categorized to two regimes, i.e., the case with the internal disruption due to pure DTM and the case with the magnetic island saturation for DTM with increasing distance between resonance surfaces [1,2]. Recently, we found a new phenomenon of the DTM in the middle of these two regimes; that is, in the case that two resonance surfaces are enough apart from each other, the DTM gently grows magnetic islands at each resonance surface such as in the Rutherford regime of the conventional tearing mode [3,4], but it suddenly changes to rapid growth after both magnetic islands grow enough to interact with each other [5]. This phenomenon has been observed in a simulation with helical symmetry, where all harmonics have the resonance surfaces at the same radius, so that the newly observed nonlinear destabilization of DTM is different from any theories presented so far, such as the nonlinear coupling among different helicities or the destabilization through the renormalized turbulence transport process [6,7]. The purpose of this Letter is to show the new feature and mechanism of this nonlinear destabilization process of DTM.

The linear stability and nonlinear behavior of the double tearing mode are studied by using the reduced set of resistive MHD equations in a cylindrical geometry [8]. In this Letter, ψ is the poloidal flux function, ϕ is the stream function, η is the resistivity, j is the toroidal current density, u is the vorticity, and B_0 is the toroidal magnetic field. The magnetic and velocity fields are related to the poloidal flux ψ and the stream function ϕ by $\vec{B} = B_0\vec{e}_\varphi + \nabla\psi \times \vec{e}_\varphi$ and $\vec{V} = \nabla\phi \times \vec{e}_\varphi$, where $\vec{e}_\varphi$ is the unit vector in the toroidal direction. In the following, we consider only the MHD activity with helical symmetry of $f(r, \theta, \varphi) = f[r, \zeta = \theta - (n/m)\varphi]$, where m and n are poloidal and toroidal mode numbers of the MHD mode, respectively, and we fix (m/n) to 3/1 in this Letter. These equations are solved by the conventional scheme of finite difference in the radial direction and Fourier expansion in the azimuthal direction [8]. In order to reproduce fine structures, the maximum numbers of equally spaced radial grids and of the Fourier components are taken to be 400 and 100, respectively.

In order to study the DTM stability, we employ the following type of safety factor profile:

$$q(r) = q_c\left\{1 + \left(\frac{r}{r_0}\right)^{2\lambda}\right\}^{1/\lambda}\left[1 + A\exp\left\{-\left(\frac{r - r_\delta}{\delta}\right)^2\right\}\right],$$

$$\psi(r) = -\frac{B_0}{R_0}\int_0^r \frac{rdr}{q(r)}, \tag{1}$$

with fixed parameters of $\lambda = 1$, $r_0 = 0.412$, $\delta = 0.273$, $r_\delta = 0$, and $A = 3$ throughout this Letter. The value q_c is used to change the distance between two resonance surfaces, Δr.

The linear stability analysis for the resistive mode in this q profile shows that, as increasing Δr, the resistivity (η) dependence of the linear growth rate γ changes from the resistive internal mode $\gamma \propto \eta^{1/3}$ in the limit of $\Delta r = 0$ to the conventional tearing mode $\gamma \propto \eta^{3/5}$ in the limit of $\Delta r = \infty$ [5,9]. Corresponding to this change of the linear stability, the nonlinear behavior of the mode also changes from the exponential growth with the linear growth rate to the quasilinear saturation of the magnetic islands around both resonance surfaces. The new type of nonlinear instability is found in the parameter range midway between those two limits. For the parameters used in this Letter, this range corresponds to $0.22 \leq \Delta r \leq 0.31$. The typical example of the temporal evolution of magnetic and kinetic energies of the nonlinear instability for $\Delta r = 0.31$ and $\eta = 5 \times 10^{-6}$ is shown in Figs. 1(a) and 1(b). After the exponential growth in the linear regime, the mode reduces its spontaneous growth rate and enters the slow growth phase of a magnetic island in the Rutherford-type regime. During this slowly growing phase, the inner islands are gradually deformed to a triangular shape, as seen in Fig. 1(c), and are pushed to the X point of the outer islands. Then, a nonlinear destabilization is suddenly triggered and the magnetic and kinetic energies abruptly grow. During this phase, the q profile averaged at the radius is flattened in the wide region, extending to the magnetic axis. This process may correspond to a plasma collapse or disruption for low-beta negative shear plasmas.

In order to study the mechanism of this abrupt growth of the DTM, we have performed several simulations for another parameter set of $\Delta r = 0.285$ and $\eta = 1 \times 10^{-5}$, where the duration of slow growth process is shorter than in the case of Fig. 1 due to the decreased Δr and the increased η. One of the possible candidates is the quasilinear modification of the q profile, which may cause the acceleration of the linear instability of fundamental and also higher harmonics. To check this, we performed the simulation by artificially resetting the perturbations to the small value for the main harmonics (i.e., $m/n = 3/1$) and to zero for all other higher harmonics (i.e., $\psi_{l\geq 2} = \phi_{l\geq 2} = 0$) on the way of the abrupt-growth phase. The result shows that the mode returns back to the linear growth phase and resumes the Rutherford-type evolution, suggesting that the quasilinear modification of the q profile is not the key factor of the nonlinear destabilization. Another possible candidate is nonlinear interaction between higher harmonics. In Figs. 1, simulation results for different numbers of $l_{\max}$ are also plotted. The temporal evolution up to the Rutherford-type regime is not sensitive so much on $l_{\max}$, while the behavior of the nonlinear destabilization, or specifically the triggered time, depends on $l_{\max}$. An increase of $l_{\max}$ accelerates the growth of the mode, showing that the mode coupling plays a key role of the nonlinear destabilization. Note that the nonlinear behavior does not depend on $l_{\max}$ larger than some critical number l_{crit}, which depends on Δr.

Figure 2 shows the energy spectrum at $t = 320$ in the nonlinear destabilization phase for $l_{\max} = 100$. The energy spectra clearly show a two stage structure; that is, the slope of the spectrum changes around l_0, and kinetic and magnetic energies exhibit a similar tendency in the higher harmonics regime of $l > l_0$. This indicates that the energy cascaded to higher harmonics through mode coupling does not flow back to the lower harmonics regime ($l < l_0$), suggesting that the higher harmonic regime ($l > l_0$) works only as the energy sink and does not play an essential role on the nonlinear behavior of the mode. Note that l_0 is close to the critical value l_{crit} discussed above, $l_0 \simeq l_{\mathrm{crit}}$. On the other hand, the intermediate harmonics have close coupling with each other and are essential for the nonlinear behavior of the mode. It is noted that the present simulation is based on the helical symmetry and does not cause any stochastization of

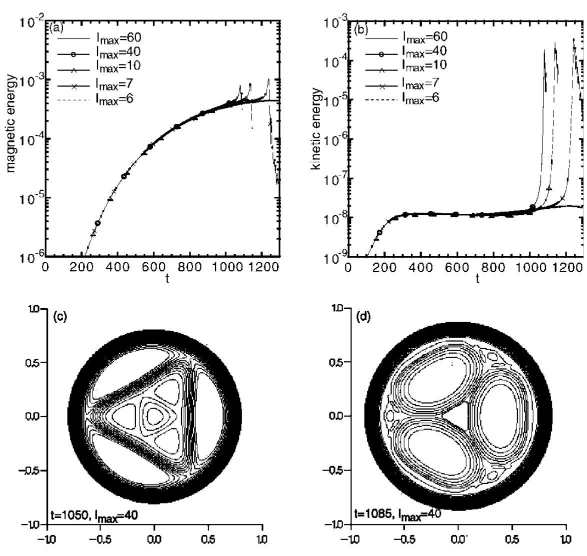

FIG. 1. Time evolutions of (a) magnetic and (b) kinetic energies of 3/1 mode with the different harmonics number, $l_{\max}$. The contours in (c) and (d) show those of the helical flux function ψ^* just before and after the abrupt growth phase. The curves for $l_{\max} = 60$ and 40 are almost overlapped and difficult to distinguish from each other.

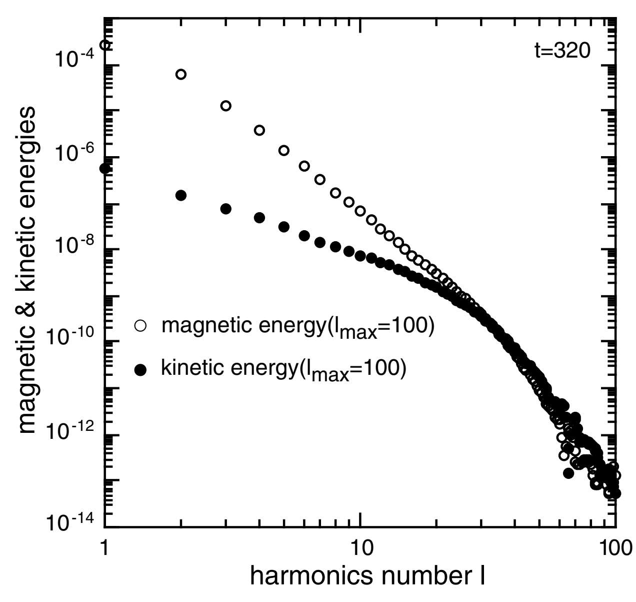

FIG. 2. Power spectrums of the magnetic and kinetic energies for $l_{\max} = 100$ at $t = 320$.

magnetic field lines. Instead of that, the highly developed power spectrum enhances the coherent spatial structure of the mode and leads to the triangular deformation of magnetic islands, as shown in the flux contour of Fig. 1. In the case of Fig. 1, the nonlinear destabilization is triggered for $l_{\max} = 7$, but not for $l_{\max} = 6$. This suggests that the degree of the island deformation is important for triggering the nonlinear destabilization.

It is interesting to know whether the magnetic perturbations $\psi_{l\geq 1}$ or the kinetic ones $\phi_{l\geq 1}$ are the key factor of the nonlinear destabilization. For this purpose, we reset magnetic or kinetic perturbations to zero on the way of the abrupt growth and investigated the subsequent phenomena. Results are shown in Figs. 3. In simulations resetting the kinetic perturbations to zero, the kinetic perturbations recovered to the same level as the original ones in a very short time and shows the abrupt growth [Fig. 3(a)], while in the case of resetting the magnetic perturbations to zero, the abrupt growth is not reproduced [Fig. 3(b)]. For the case retaining fundamental magnetic perturbation (i.e., $\psi_{l\geq 2} = 0$), the mode resumes the abrupt growth after the higher harmonics of magnetic perturbations grow up to sufficient amplitudes through the mode coupling. This comparison confirms that the nonlinear destabilization originates from the coupling among magnetic perturbations through $J \times B$, not from the driven reconnection type instability.

The difference of the nonlinear behaviors between the standard DTM and the nonlinearly destabilized DTM is clearly shown in Figs. 4, where the contours of the helical flux, ψ^*, the flow potential, ϕ, and the toroidal current excluding the fundamental harmonics, $j_{l\geq 1}$, in the nonlinear phase are plotted. In the case of standard DTM, the mode grows exponentially with the linear growth rate and the convective force pushes the magnetic flux to the resonance surface faster than the magnetic reconnection rate. Then, the shape of the reconnection region changes from the X-point type to the Y type with skin current flowing along the finite distance, which is known as the Sweet-Parker type current sheet, as shown in Figs. 4(a) and 4(b) [10]. On the other hand, in the case of the nonlinear destabilization of DTM, quadruple vortices and the X-point structure of the magnetic flux are sustained in the abrupt-growth phase, as shown in Fig. 4(c). In this case, the mode enters the Rutherford-type regime after the linear growth phase and grows slowly in proportion to the resistivity, η. During this phase, the convective force to push the magnetic flux into the outer reconnection region is weak compared with the magnetic reconnection rate, and the triangular deformation is highly developed through the mode coupling. As a result, the toroidal current concentrates in the small region and forms the current point, which is a current sheet with very short length, as shown in Fig. 4(d). Because of the relatively low kinetic energy, the sharp X-point structure is sustained even during the nonlinear growth phase. Thus, it is concluded that the formation and sustenance of the X-point structure is the key factor of the new destabilization process. The numerical accuracy of the above process was confirmed by increasing the radial mesh number N_r up to 1600 and the Fourier mode number $l_{\max}$ up to 100. The half width of the current point is $\delta r/a \simeq 0.008$ and $\delta\theta/2\pi \simeq 0.015$ in the typical case shown in Fig. 4, and the simulation with $N_r > 400$ and $l_{\max} > 40$ gives the same result both in the spatial structure and the time evolution. When the degree of the triangular deformation and/or the current concentration exceed some critical value, DTM enters a nonlinear destabilization phase. The critical value is not so clear, but we see the evidence

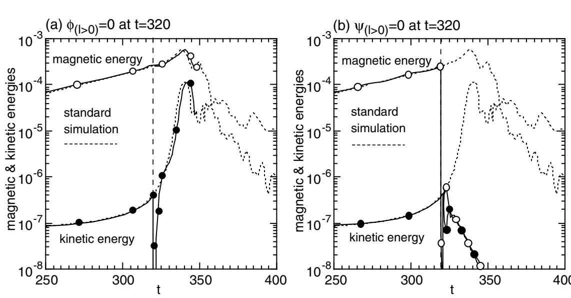

FIG. 3. Time evolutions of magnetic and kinetic energies of 3/1 mode for the standard and restarted simulations: (a) perturbations of $\phi(l > 0)$ are set to zero, (b) perturbations of $\psi(l > 0)$ are set to zero.

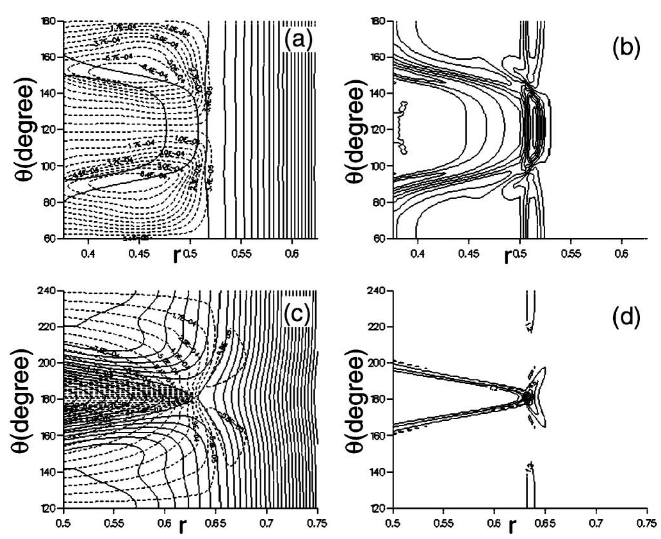

FIG. 4. Contours of the helical flux function, ψ^*, solid curves in (a) and (c); the flow potential, ϕ, dotted curves in (a) and (c); and the toroidal current, $j_{l>0}$, solid curves in (b) and (d): (a) contours of ψ^* and ϕ at $t = 130$ for the standard DTM ($\Delta r = 0.115$), (b) contours of $j_{l>0}$ at $t = 130$ for the standard DTM ($\Delta r = 0.115$), (c) contours of ψ^* and ϕ at $t = 330$ for the nonlinearly destabilized DTM ($\Delta r = 0.285$), (d) contours of $j_{l>0}$ at $t = 330$ for the nonlinearly destabilized DTM ($\Delta r = 0.285$).

from the simulation that the critical value is related to the critical harmonics number (Fig. 1) and also to the distance between two rational surfaces. From these features, the phenomenon is referred as "structure-driven nonlinear destabilization." A remarkable feature of this structure-driven mode is the dependence of the spontaneous growth rate, $\gamma_{\rm temp}$, on the resistivity, η; that is, the dependence of $\gamma_{\rm temp}$ on η in the explosive growth phase is very weak, $\gamma_{\rm temp} \sim \eta^{\alpha}$, $\alpha \simeq 0$, as shown in Fig. 5. The features of the current point formation and the weak dependence of the growth rate on the resistivity are similar to the Petschek-type reconnection model. The structure-driven destabilization, however, is caused by the interaction of two island chains of the weakly coupled DTM, and the process is explosive with accelerated spontaneous growth rate and is terminated by expelling the magnetic islands. Hence, it does not fit to any model of the steady state driven reconnection such as the Petschek-type one.

We note that the linear stability of this mode follows the tearing mode scaling at the initial equilibrium. Hence, the mode experiences three time scales in its time evolution; $\gamma \propto \eta^{3/5}$ in the linearly unstable phase, $\gamma \propto \eta$ in the Rutherford-type nonlinear glowing phase, and $\gamma \propto \eta^{\alpha}$, $\alpha \simeq 0$ in the nonlinearly destabilized phase. In the recent large tokamak plasmas, the resistivity, η, becomes about $\eta \simeq 10^{-8}$. The result suggests that the nonlinear destabilization of DTM could be triggered in the fast time scale, after long term evolution of DTM in the Rutherford-type regime.

In summary, we have shown the new process of the nonlinear destabilization of DTM caused in the reversed shear profile in a tokamak. It was found that the slowly growing DTM can be nonlinearly destabilized and changes to the explosively growing DTM. The explosive growth of DTM was shown to be originated not from both any type of the quasilinear destabilization and the turbulence driven instability, where the increase of the transport coefficients driven by the higher harmonics accelerates the growth of the mode. As discussed in this Letter, the slow growth such as in the Rutherford regime and the sufficient interaction of the inner and outer islands are necessary for the nonlinear destabilization of DTM. We think that the key physics of the explosive growth is the formation of the sharp triangularity in the magnetic structure and the resultant intense current point, which enhances the magnetic reconnection. These are special features of this nonlinear process. This is the reason we call it the structure-driven one.

In the case of the low β disruption in a negative shear plasma, perturbations growing with a resistive time scale are sometimes observed around each rational surfaces [11]. After the growth in the resistive time scale, the perturbation shows the explosive growth. These features

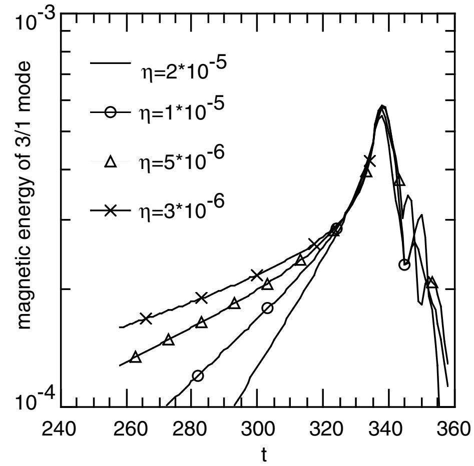

FIG. 5. Time evolutions of the magnetic energies of 3/1 harmonics at the nonlinear destabilization phase of DTM for the different resistivity.

are roughly consistent with our observation of the nonlinear destabilized DTM, although, at the present stage, the relationship between the precursor with the resistive time scale and the fast time scale phenomenon is not clear in experiments and the shear flow evolution seems to have the effect on the mode stability, which is not included in the present simulation.

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