Preavalanche Instabilities in a Granular Pile

Lydie Staron,[1] Jean-Pierre Vilotte,[1] and Farhang Radjai[2]
[1]*IPGP, 4 Place Jussieu, F-75252 Paris cedex 05, France*
[2]*LMGC, CNRS-Université Montpellier II, Place Eugène Bataillon. F-34095 Montpellier cedex, France*
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We investigate numerically the transition between static equilibrium and dynamic surface flow of a 2D cohesionless granular system driven by a continuous gravity loading. This transition is characterized by intermittent local dynamic rearrangements and can be described by an order parameter defined as the density of critical contacts, i.e., contacts where the friction is fully mobilized. Analysis of the spatial correlations of critical contacts shows the occurrence of "fluidized" clusters which exhibit a power-law divergence in size at the approach of the stability limit. The results are compatible with recent models that describe the granular system during the static/dynamic transition as a multiphase system.

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Avalanches and debris flows are of special interest both for industrial and natural processes. Although many advances have been made in the physical understanding of granular flow both from a microscopic and a continuum point of view (see, for example, [1–6]), the mechanisms leading to the transition of a granular material from a static equilibrium to a dynamic state are still unclear, and difficult to analyze experimentally. A recent hydrodynamic model based on phase transition theory describes the granular material as a two-phase material with a "solid" (static) and a "liquid" (flowing) phase [7]. This transition, also advocated in others conditions [8,9], is important for the understanding of avalanche instability in the context of risk assessment.

The purpose of this paper is to investigate numerically this static to dynamic transition in a minimal model configuration, e.g., a 2D cohesionless granular medium driven by continuously loading body force, here a continuous tilt under gravity. We find that the evolution is chacterized by intermittent local instabilities and can be described by the fraction of contacts where friction force is completely mobilized. A more detailed description involves "fluidized" areas which percolate when the system reaches the stability limit. These results would be in favor of a description in terms of a multiphase system undergoing a phase transition.

The contact friction is described by the classical Coulomb's law. Relative slip between two particles in contact can occur only when the friction force is fully activitated, i.e., $f_t = \pm\mu f_n$, where f_t and f_n are, respectively, the tangential and normal forces at the contact and μ is the contact friction coefficient. This defines the Coulomb threshold. Otherwise, no slip can occur and $f_t \in [-\mu f_n, \mu f_n]$. The numerical simulations were performed using the contact dynamics method [10,11], based on a fully implicit resolution of the contact forces. This allows for an accurate determination of the ratio f_t/f_n independently of any numerical regularization parameter. In all the simulations, the contact friction coefficient is $\mu = 0.5$ and collisions are perfectly inelastic.

The initial configuration of the pile is generated by random deposition of $N_p = 4000$ disks in a rectangular box; the disks have a uniform distribution of diameters within the range $[D_{\min}, D_{\max}]$. We used the ratio $D_{\max}/D_{\min} = 1.5$, but we checked that for a polydispersity as large as $D_{\max}/D_{\min} = 10$ the results remain essentially unchanged. The granular samples prepared by random deposition have a rectangular shape with a nearly flat surface, a thickness of $30D$, and a width of $120D$, where D is the mean disk diameter. The initial coordination number, i.e., the mean number of contacts of a particle, is $z \simeq 3.6$. It shows only weak fluctuations (up to 5%) in the course of tilting. The results presented below were obtained from 15 independent runs in which the random processing of grain sizes is the only source of noise.

The granular bed is slowly tilted with a constant rotation rate (0.001° per time step). The slope increases from $\theta = 0°$ to the maximum angle of repose $\theta_c \simeq 20°$ at which a surface avalanche occurs. Within the picture of ideal Coulomb's material, θ_c is related to the internal coefficient of friction of the pile $\mu_{\rm eff}$ through the relation $\mu_{\rm eff} = \tan(\theta_c) \simeq 0.35$ [12]. However, the evolution of the pile towards θ_c is not monotonous. Indeed, we observe the occurrence of local instabilities owing to the mobilization of friction between particles. Initially f_t at the contacts is only partially activated, namely $|f_t| < \mu f_n$. But upon tilting, a number of contacts reach the Coulomb threshold, i.e., $|f_t| = \mu f_n$. These contacts cannot sustain further shear force increment and can lead eventually to a slip instability or the disappearance of the contact. We call them *critical contacts*. We describe the evolution of the pile in terms of the fraction ν of critical contacts over a volume V and as a function of θ:

$$\nu(\theta, V) = \left\langle \frac{N_c}{N} \right\rangle_V, \tag{1}$$

where N_c and N are, respectively, the number of critical contacts and the total number of contacts in V at slope angle θ. Since the density of contacts remains almost constant due to close-packing ($N \propto V$), ν also represents the density of critical contacts. It characterizes the plastic state of the pile.

The evolution of $\nu(\theta, V_{\text{pile}})$, where V_{pile} is the volume of the whole packings, is reproducible from run to run even though there are rapid fluctuations of ν within the system [see Fig. 1(a)]. When averaging the evolution of ν over 15 independant realizations [inset graph in Fig. 1(a)], we observe a regular increase of $\nu(\theta)$ from an intial value $\nu = 0$ to a maximum value $\nu \simeq 0.08$ at the maximum angle of repose θ_c. This indicates that a partial plastification occurs well before the stability limit of the pile. Such a transition is, however, characterized by rapid fluctuations of ν. They are the signature of intermittent instabilities during which loss of critical contacts occurs through local dynamical rearrangements, as can be seen in the fluctuations of the mean kinetic energy in Fig. 1(b). This suggests that even though the density of critical contacts increases in the mean during the mobilization stage of the system, the population of critical contacts is renewed by these intermittent instabilities and a single critical contact is only of short lifetime due to its metastable state. During the intermittent evolution, the system can explore extreme ν states as seen in Fig. 1(a). When averaging over small θ size windows the extremal states at each θ and for all independant realizations, their evolution exhibits a well-defined limit envelope [see Fig. 1(c)]. It shows a roughly exponential convergence towards an asymptotic limit characterized by a critical state $\nu_c \simeq 0.08$, corresponding to the stability limit. The granular system can therefore be described in terms of an order parameter $\Phi = \nu/\nu_c$ which varies from zero to one and characterizes the partially fluidized transition.

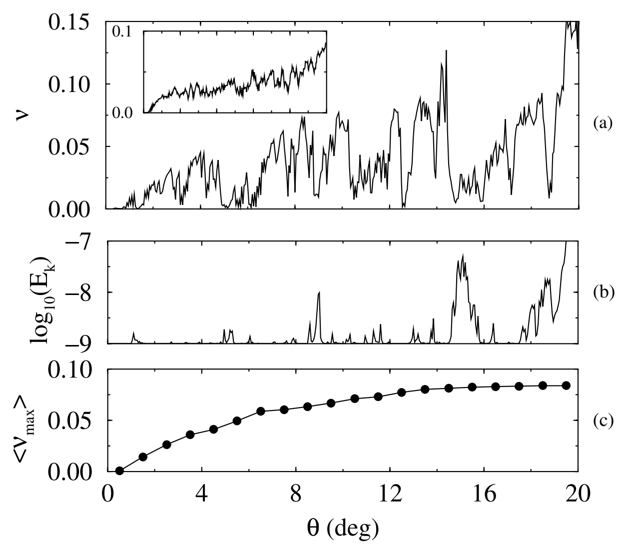

FIG. 1. (a) Density ν of critical contacts in the pile as a function of the tilt angle θ for a single run (main curve) and averaged over 15 independant runs (inset curve); (b) Mean translational kinetic energy E_k ($\text{kg}{\cdot}\text{m}^2 \cdot \text{s}^{-2}$) of particles as a function of θ for the same single run; (c) mean envelope of the maximum values of ν evaluated over the 15 runs.

Owing to the intrinsic geometrical and stress fluctuations of each individual realization, each pile can explore rare extremal states beyond the critical limit ν_c in the course of its evolution. These states are metastable and followed eventually by local dynamical rearrangements, the size of which can be characterized by the fraction $\Delta N/N$ of contacts lost in that event. When analyzing the size of the local rearrangements as a function of the ν-state explored by the system prior to the event (see Fig. 2), we found that the pile evolution is characterized by frequent weak rearrangements, less than 1%, that correspond to ν states of the system below ν_c, and rare and strong rearrangements corresponding to extremal states $\nu > \nu_c$. The intermittent rearrangements during the mobilization occur both in the bulk and close to the free surface.

The evolution of ν may be also interpreted in terms of the mean separation distance $\lambda(\theta)$ between two critical contacts

$$\lambda(\theta) = \sqrt{\frac{2}{z(\theta)\nu(\theta)}}D, \tag{2}$$

where $z(\theta)$ is the coordination number. During the mobilization, λ decreases from $+\infty$ when $\nu = 0$ to $\lambda_c = 2.5D$ when $\nu = \nu_c$. For $\lambda < \lambda_c$, i.e., $\nu > \nu_c$, the packing undergoes large-scale instabilities due to spatial correlations among critical contacts (see Fig. 2).

To investigate the distribution of critical contacts, we introduce the probability density function (pdf) $P(\nu)$ of finding a local state ν in the neighborhood of a contact. The size of this neighborhood must not be smaller than the minimum length separating two critical contacts, of the order of D, nor larger than the typical size of spatial heterogeneities of ν (to be analyzed later). In particular, we introduce the pdf $P_c(\nu)$ at critical contacts and the pdf $P_{nc}(\nu)$ at noncritical contacts. The distributions P_c and P_{nc} time averaged over all the loading phase from $\theta = 0$ to θ_c are displayed in Fig. 3. The range of local states ν is

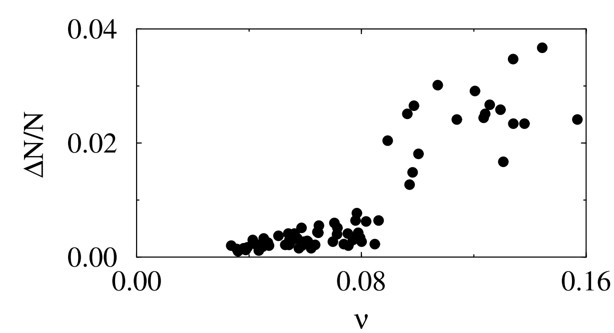

FIG. 2. Proportion $\Delta N/N$ of contacts lost in the packing during rearrangements as a function of ν.

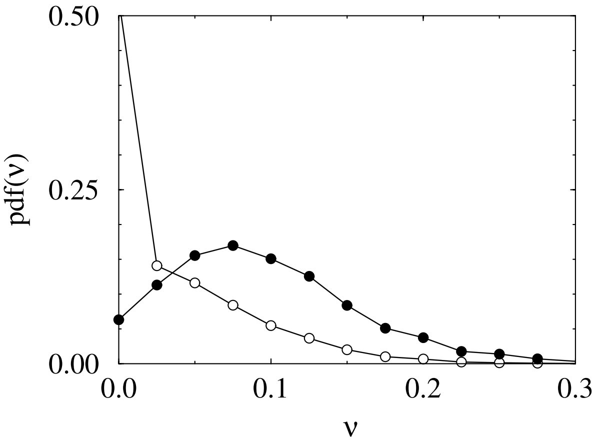

FIG. 3. Probability density function (pdf) of local states ν in the neighborhood of critical contacts (filled circles) and non-critical contacts (opaque circles), for a neighborhood radius $r = 6D$.

quite wide for both distributions: P_c has a well-defined peak at $\nu \simeq \nu_c$, whereas P_{nc} is a decreasing function with a high peak at $\nu = 0$. This indicates that critical contacts tend to appear preferentially in the vicinity of other critical contacts, and are strongly correlated spatially. It is worth noting here that the most probable local state is found to be the critical state ν_c that characterizes the whole pile at the stability limit.

We study the spatial correlations by defining the mean state $\bar{\nu}$ in a neighborhood of critical contacts as a function of the neighborhood's size r and of the inclination angle θ of the pile. In practice, we explore $r \in [1D, 20D]$. Denoting ν_i the local state in the vicinity of a contact i, $\bar{\nu}(\theta, r)$ can be expressed as follows:

$$\bar{\nu}(\theta, r) = \frac{1}{N_c} \sum_{i \in \{N_c\}} \nu_i(\theta, r), \quad (3)$$

where N_c is the number of critical contacts in the pile. The evolution of $\bar{\nu}(\theta, r)$ is displayed in Fig. 4(a) for three values of θ. Regardless of the inclination θ, $\bar{\nu}$ decreases rapidly as a function of r as long as $r \lesssim 10D$. Beyond this distance, $\bar{\nu}$ remains almost constant and equal to the state of the pile $\nu(\theta, V_{\text{pile}})$. Such an evolution of $\bar{\nu}(\theta, r)$ can be approximated by [see Fig. 4(b)]:

$$\overline{\nu}(\theta, r) = \begin{cases} \nu_\infty(\theta)[1 + A(\theta) r^{\alpha(\theta)}] & \text{if } r \leq 10D, \\ \nu_\infty(\theta) & \text{if } r > 10D, \end{cases} \quad (4)$$

where $\nu_\infty(\theta)$ is the state of the pile at θ, and $A(\theta)$ and $\sigma(\theta)$ are affine functions of θ. The distance $L \simeq 10D$ arises here as the characteristic length of the spatial correlations between critical contacts.

The spatial correlations can be illustrated when looking at two snapshots of the pile for different values of the inclination angle θ (see Fig. 5). In particular, the system exhibits clusters of critical contacts where $\nu \geq \nu_c$. These

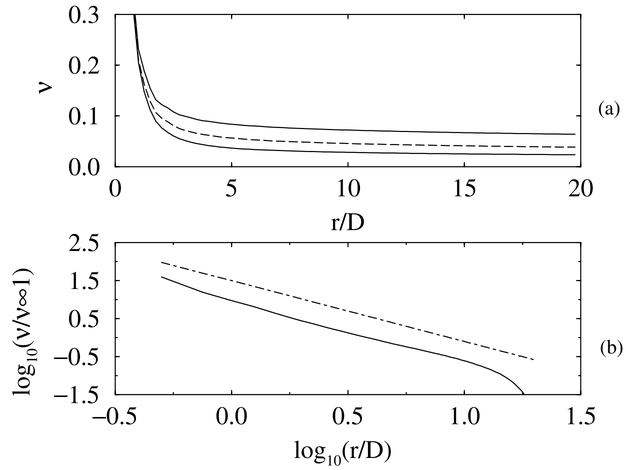

FIG. 4. Mean state $\bar{\nu}$ in the neighborhood of a critical contact as a function of the size r of the neighborhood (a) in linear scale for three values of the tilt angle $\theta = 6°$ (solid line), $\theta = 8°$ (dashed line), and $\theta = 12°$ (dotted line); and (b) in logarithmic scale for $\theta = 6°$ (solid line) shown with a line with a slope $\alpha = -1.5$ (dot-dashed line).

clusters can no longer sustain shear load increment and eventually lead to local rearrangements. They can be analyzed as "fluidized" zones with θ increasing size.

We define r_c the mean size of the clusters of critical contacts characterized by a state $\nu = \nu_c$. Equation (4) yields

$$r_c(\theta) = \left[\frac{1}{A(\theta)}\left(\frac{\nu_v}{\nu_\infty} - 1\right)\right]^{1/\alpha(\theta)}, \quad (5)$$

where ν_∞ is defined as the state of the pile $\nu(\theta, V_{\text{pile}})$ averaged over all the simulations (see inset graph in Fig. 1). The evolution of $r_c(\theta)$, displayed in Fig. 6, shows a slow increase from around $2D$ to $4D$ as long as $\theta \lesssim 15°$. For larger θ, the size increases quickly with a power-law divergence $\propto (\theta_c - \theta)^{-\beta}$. That is reminiscent of a percolationlike process. The value $\theta = 15°$ may be related to

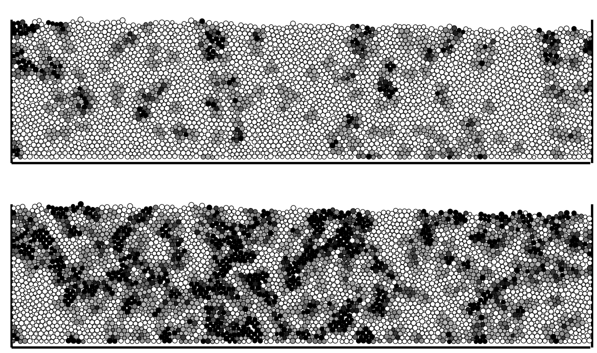

FIG. 5. Density of critical contacts in the pile at $\theta \simeq 5°$ (upper picture) and at $\theta \simeq 16°$ (bottom picture). Areas where $\nu \geq \nu_c$ are in black, whereas the state $\nu = 0$ are in white. Intermediate values of ν ($\nu \in |0, \nu_c|$) are represented in gray.

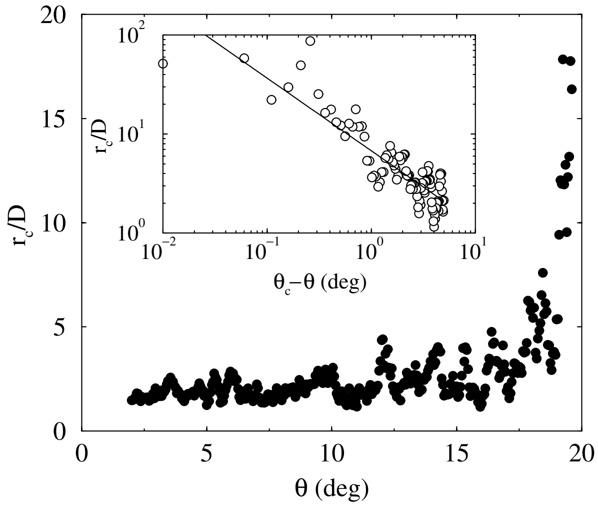

FIG. 6. Evolution of the mean size r_c of clusters of critical contacts (see text) as a function of the tilt angle θ in linear scale (main curve) and in logarithmic scale (inset curve), where $\theta_c = 19.96°$ and the line is drawn with a slope $-\beta \simeq 0.5$.

the dynamical angle of repose of the pile, but this would require further investigation.

The mobilization of the granular pile is related to the occurrence of "fluidized" clusters of critical contacts. The size of these clusters increases with θ, with a power-law divergence as $\theta \rightarrow \theta_c$. This result is in support of a description of the granular system during mobilization as a multiphase system. The transition from a static equilibrium to a dynamical flow is controlled by the order parameter $\Phi = \nu/\nu_c$, and the stability threshold would result from a multiphase instability. Detailed analysis of the phases interaction during transition still requires further analysis.

Forces in granular media have been shown to obey a broad distribution and to define a strong and a weak contact network. Forces transmitted along the strong contact network exceed the mean force in the media and are responsible for the mechanical strength of the medium, with a typical correlation length $\xi \simeq 10D$ [13,14]. By contrast, forces transmitted along the weak contact network mainly contribute to an average pressure. In an extended forthcoming paper, we show that critical contacts appear mostly within the weak contact network. They coincide with areas of lower pressure, screened by the strong contact network which controls the evolution of static stresses. Critical contacts tend to be organized as "fluidized" clusters that eventually destabilize the pile when their size becomes comparable with the correlation length of strong forces ξ. The avalanche instability appears as a two phase instability triggered by the interaction between these correlated clusters and a solid skeleton made of the strong contact network. The influence of material parameters such as the coefficient of friction and the degree of configurational disorder in the pile are the purpose of further work. The extension of the problem to three dimensions has to be investigated.

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