Flavor Ordering of Elliptic Flows at High Transverse Momentum

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Based on the quark coalescence model for the parton-to-hadron phase transition in relativistic heavy ion collisions, we relate the elliptic flow (v_2) of high p_T hadrons to that of high p_T quarks. For high p_T hadrons produced from an isospin-symmetric and quark-antiquark–symmetric partonic matter, magnitudes of their elliptic flows follow a flavor ordering as $(v_{2,\pi} = v_{2,N}) > (v_{2,\Lambda} = v_{2,\Sigma}) > v_{2,K} > v_{2,\Xi} > (v_{2,\phi} = v_{2,\Omega})$ if strange quarks have a smaller elliptic flow than light quarks. The elliptic flows of high p_T hadrons further follow a simple quark counting rule if strange quarks and light quarks have the same high p_T spectrum and coalescence probability.

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Elliptic flow in heavy ion collisions is a measure of the azimuthal asymmetry of particle momentum distributions in the plane perpendicular to the beam direction. It results from the initial spatial asymmetry in the transverse plane in noncentral collisions and is thus sensitive to the properties of the dense matter formed during the initial stage of heavy ion collisions [1–8]. There have been extensive experimental [9–14] and theoretical [1–8,15] studies of elliptic flow in heavy ion collisions at various energies. At the Relativistic Heavy Ion Collider (RHIC), the elliptic flow has been measured as functions of the centrality of collisions [11–14], as well as the particle transverse momentum [11,12,14] and pseudorapidity [13]. Theoretical studies indicate that these experimental results provide not only information on the equation of state of nuclear matter at high density and temperature [4–7] but also on the scattering cross section of partons produced in the collisions [16–19].

The elliptic flow has also been measured for different hadron species, such as pions, kaons, nucleons, and Λ, up to 3 GeV/c [20,21]. The experimental data show that at low p_T the elliptic flow of heavier particles is smaller than that of lighter particles. In the hydrodynamical model, this mass ordering of elliptic flow at low p_T is attributed to the mass dependence of radial flow [6]. For high p_T hadrons, we expect the flavor dependence to be different from that at low p_T, since high p_T hadrons originate from hard processes while low p_T particles are mostly produced from soft nonperturbative processes and are much closer to thermal equilibrium. Indeed, the observed saturation of hadron elliptic flow at $p_T >$ 2 GeV/c [21,22] contradicts the predictions from the hydrodynamical model [5], but is roughly consistent with the results expected from a large parton transport opacity [17] or energy loss [23]. Furthermore, fast moving heavy quarks have been shown to suffer less energy loss in a thermalized parton plasma than fast moving light quarks [24–26]. This would imply that high p_T strange and heavier quarks may have smaller scattering cross sections than light quarks in a partonic matter, leading to a smaller elliptic flow for strange quarks than for light quarks. It is then of great interest to study how different strengths of anisotropic flow in the deconfined stage of the collision are reflected in the elliptic flow of hadrons.

In this Letter, we shall study the flavor dependence of the elliptic flow of high p_T hadrons in ultrarelativistic heavy ion collisions, using a quark coalescence model to describe the phase transition from the partonic matter to the hadronic matter. As we shall show, the elliptic flow of high p_T hadrons derived from this model can be expressed in terms of the elliptic flow of high p_T quarks. As a result, several relations between the elliptic flow of hadrons of different flavors are obtained. We further discuss some special cases where these relations become more transparent, e.g., the elliptic flows of high p_T hadrons are simply given by the arithmetic average over the elliptic flows of the valence quarks that build up the hadrons if strange quarks and light quarks have the same high p_T spectrum and coalescence probability.

In the quark coalescence model, one assumes that quarks and antiquarks are the effective degrees of freedom in the parton phase near the phase transition, and they combine to form hadrons according to the valence quark structure of hadrons. A meson is thus formed from the coalescence of a quark and an antiquark, while a baryon is due to the coalescence of three quarks. The idea of quark coalescence has been used in models such as the ALCOR [27] or MICOR model [28] to describe hadron abundance and the AMPT model with string melting [19] to describe the elliptic flow at RHIC.

In ultrarelativistic heavy ion collisions, high p_T partons are produced from initial hard scatterings between nucleons for which the perturbative QCD is applicable. The high p_T parton spectrum thus roughly follows an inverse power law. On the other hand, low p_T partons, which are produced from initial soft processes, typically have an exponential spectrum close to a thermal distribution. The parton p_T spectrum can thus be represented by an exponential function below a certain momentum scale p_0 and an inverse power law above p_0.

Let us consider via the quark coalescence model the formation of a high p_T meson with transverse momentum $\vec{p}_H$ from one parton with $\vec{p}_H$ and one parton with zero p_T, or from two partons with equal high p_T of $\vec{p}_H/2$. The ratio of the probabilities for forming a high p_T meson in these two cases is then proportional to $(ep_T/4p_0)^n$, where n represents the exponent of the inverse power law for final high p_T partons. Since this ratio is much greater than one for $p_T \gg p_0$, a high p_T meson is dominantly formed from the coalescence of one high p_T parton and one soft parton. Similarly, a high p_T baryon is mainly formed from the coalescence of one high p_T parton and two soft partons.

The transverse momentum distribution $F(\vec{p}_T) = dN/(dp_x dp_y)$ of initial high p_T mesons formed after the phase transition can thus be expressed in terms of that of final high p_T partons, i.e.,

$$F_H(\vec{p}_T) = F_i(\vec{p}_T)c_j + F_j(\vec{p}_T)c_i, \tag{1}$$

where i and j denote the flavor of the valence quark and antiquark of meson H. The coefficient c_i represents the capture probability for a soft parton i by a high p_T parton to form a high p_T meson; it is thus related to the density of soft quarks near the phase transition. For high p_T baryons or antibaryons, one can write down a similar expression, involving the product of two c_i, for their transverse momentum distribution.

The elliptic flow is generated during the early stage of heavy ion collisions when the pressure gradient and the spatial azimuthal asymmetry are the largest [2,4,16,19]. In transport model studies, it has been found that the elliptic flow in heavy ion collisions at RHIC develops mostly in the initial partonic phase, with later hadronic interactions having negligible effects on its final value [4,19]. We expect that the elliptic flow of high p_T hadrons are even less affected by hadronic interactions, as the proper formation time from a high p_T parton to a hadron is increased by a large Lorentz boost factor in the laboratory frame, leading to a much lower hadronic density when high p_T hadrons are formed. We can thus use Eq. (1) to relate the *final* elliptic flow of high p_T hadrons to that of high p_T partons. For mesons, we have

$$\begin{aligned} v_{2,H}(p_T) &= \frac{\int \cos(2\phi')F_H(\vec{p}_T)d\phi'}{\int F_H(\vec{p}_T)d\phi'} \\ &= \frac{v_{2,i}(p_T)f_i(p_T)c_j + v_{2,j}(p_T)f_j(p_T)c_i}{f_i(p_T)c_j + f_j(p_T)c_i}, \end{aligned} \tag{2}$$

where ϕ' is the azimuthal angle with respect to the reaction plane, and $f(p_T) = dN/(2\pi p_T dp_T)$ denotes the transverse momentum distribution after averaging over the azimuthal angle. In the following, we omit the label p_T in the variables $v_2(p_T)$ and $f(p_T)$ but keep in mind that they are evaluated at a given high p_T.

For SU(3) hadrons consisting of u, d, s quarks and antiquarks, their v_2 values at high p_T are then given by

$$\begin{gathered} v_{2,\pi^+} = \frac{v_{2,u}f_u c_{\bar{d}} + v_{2,\bar{d}}f_{\bar{d}}c_u}{f_u c_{\bar{d}} + f_{\bar{d}}c_u}, \qquad v_{2,K^+} = \frac{v_{2,u}f_u c_{\bar{s}} + v_{2,\bar{s}}f_{\bar{s}}c_u}{f_u c_{\bar{s}} + f_{\bar{s}}c_u}, \qquad v_{2,\phi} = \frac{v_{2,s}f_s c_{\bar{s}} + v_{2,\bar{s}}f_{\bar{s}}c_s}{f_s c_{\bar{s}} + f_{\bar{s}}c_s}, \\ v_{2,p} = \frac{v_{2,u}f_u c_d + v_{2,d}f_d c_u/2}{f_u c_d + f_d c_u/2}, \qquad v_{2,\Lambda} = v_{2,\Sigma^0} = \frac{v_{2,u}f_u c_d c_s + v_{2,d}f_d c_u c_s + v_{2,s}f_s c_u c_d}{f_u c_d c_s + f_d c_u c_s + f_s c_u c_d}, \\ v_{2,\Xi^0} = \frac{v_{2,u}f_u c_s/2 + v_{2,s}f_s c_u}{f_u c_s/2 + f_s c_u}, \qquad v_{2,\Omega} = v_{2,s}, \end{gathered} \tag{3}$$

with similar expressions for isospin partners and antiparticles.

The above relations become simpler if the quantities $v_{2,i}$, f_i, and c_i are independent of isospin and are also the same for strange and antistrange quarks; i.e., $u = d \equiv q$, $\bar{u} = \bar{d} \equiv \bar{q}$, and $s = \bar{s}$. These conditions are approximately satisfied in heavy ion collisions at RHIC as the π^+/π^- ratio is almost 1 around central rapidity [29–31]. In this isospin-symmetric and strange-antistrange symmetric limit, the v_2 values for hadrons at a given high p_T are given by

$$\begin{gathered} v_{2,N} = v_{2,q}, \qquad v_{2,\bar{N}} = v_{2,\bar{q}}, \qquad v_{2,\phi} = v_{2,\Omega} = v_{2,s}, \qquad v_{2,\pi^+} = v_{2,\pi^0} = v_{2,\pi^-} = \frac{v_{2,q} + r_{\bar{q}}v_{2,\bar{q}}}{1 + r_{\bar{q}}}, \\ v_{2,K^+} = \frac{v_{2,q} + r_s v_{2,s}}{1 + r_s}, \qquad v_{2,K^-} = \frac{v_{2,\bar{q}} + r_s v_{2,s}/r_{\bar{q}}}{1 + r_s/r_{\bar{q}}}, \qquad v_{2,\Lambda} = v_{2,\Sigma} = \frac{2v_{2,q} + r_s v_{2,s}}{2 + r_s}, \\ v_{2,\bar{\Lambda}} = v_{2,\bar{\Sigma}} = \frac{2v_{2,\bar{q}} + r_s v_{2,s}/r_{\bar{q}}}{2 + r_s/r_{\bar{q}}}, \qquad v_{2,\Xi} = \frac{v_{2,q}/2 + v_{2,s}r_s}{1/2 + r_s}, \qquad v_{2,\bar{\Xi}} = \frac{v_{2,\bar{q}}/2 + v_{2,s}r_s/r_{\bar{q}}}{1/2 + r_s/r_{\bar{q}}}, \end{gathered} \tag{4}$$

with N denoting a nucleon. In the above, the p_T-dependent variables $r_{\bar{q}}$ and r_s are defined as

$$r_{\bar{q}} = \frac{f_{\bar{q}}c_q}{f_q c_{\bar{q}}}, \qquad r_s = \frac{f_s c_q}{f_q c_s}. \tag{5}$$

Since the K^+/K^- ratio is close to 1 and the $\bar{p}/p$ ratio is about 0.7 in heavy ion collisions at RHIC [29–31], and they should be closer to 1 in heavy ion collisions at the Large Hadron Collider, we consider the case where $v_{2,i}$, f_i, and c_i are the same for quarks and antiquarks, i.e., $q = \bar{q}$ and thus $r_{\bar{q}} = 1$. For such a quark-antiquark–symmetric partonic matter, Eq. (4) simplifies to

$$v_{2,\pi} = v_{2,N} = v_{2,q}, \qquad v_{2,\phi} = v_{2,\Omega} = v_{2,s}, \tag{6}$$

$$v_{2,K} = \frac{v_{2,q} + r_s v_{2,s}}{1 + r_s}, \qquad v_{2,\Lambda} = v_{2,\Sigma} = \frac{2v_{2,q} + r_s v_{2,s}}{2 + r_s},$$
$$v_{2,\Xi} = \frac{v_{2,q} + 2r_s v_{2,s}}{1 + 2r_s}. \tag{7}$$

Eliminating the variable r_s in Eq. (7), we obtain two relations involving the v_2 of four different hadron species, and they can be any two of the following three relations:

$$(v_{2,\pi} - v_{2,K})(v_{2,\Lambda} - v_{2,\phi}) = 2(v_{2,\pi} - v_{2,\Lambda})(v_{2,K} - v_{2,\phi}),$$
$$2(v_{2,\pi} - v_{2,K})(v_{2,\Xi} - v_{2,\phi}) = (v_{2,\pi} - v_{2,\Xi})(v_{2,K} - v_{2,\phi}),$$
$$(v_{2,\pi} - v_{2,\Xi})(v_{2,\Lambda} - v_{2,\phi}) = 4(v_{2,\pi} - v_{2,\Lambda})(v_{2,\Xi} - v_{2,\phi}). \tag{8}$$

These relations on the elliptic flow of hadrons of different flavors become even simpler in several limits for the value of r_s. In the limit of $r_s \to 0$ due to $f_s/f_q \to 0$, i.e., if there are very few high p_T strange quarks relative to light quarks, we have $v_{2,\pi} = v_{2,K} = v_{2,N} = v_{2,\Lambda} = v_{2,\Sigma} = v_{2,\Xi} = v_{2,q}$, $v_{2,\phi} = v_{2,\Omega} = v_{2,s}$. This is simply due to the fact that all strange hadrons with light valence quarks consist of leading light quarks. In the opposite limit of $r_s \to \infty$, i.e., if the number of high p_T strange quarks is much larger than that of light quarks or the capture probability of a soft strange quark is much smaller than that of a soft light quark, all strange hadrons with light valence quarks consist of leading strange quarks. In this case, we have $v_{2,\pi} = v_{2,N} = v_{2,q}$, $v_{2,K} = v_{2,\phi} = v_{2,\Lambda} = v_{2,\Sigma} = v_{2,\Xi} = v_{2,\Omega} = v_{2,s}$.

Another interesting limit is $r_s = 1$, which would be the case if the spectrum of high p_T strange quarks is the same as that of light quarks, and the capture probability of a soft strange quark is the same as that of a soft light quark, or even though the above two factors are different but they cancel each other. In this limit, Eq. (7) gives

$$v_{2,K} = \frac{v_{2,q} + v_{2,s}}{2}, \qquad v_{2,\Lambda} = \frac{2v_{2,q} + v_{2,s}}{3},$$
$$v_{2,\Xi} = \frac{v_{2,q} + 2v_{2,s}}{3}. \tag{9}$$

These relations, together with Eq. (6), show that the v_2 values of hadrons at high p_T follow a simple quark flavor counting rule when $r_s = 1$; i.e., they are the arithmetic average over the elliptic flows of the valence quarks that build up the hadrons.

Equations (6) and (7) show that the dependence of the elliptic flow of high p_T hadrons on their flavor composition is determined by the relative magnitude of the elliptic flow of high p_T strange quarks to that of high p_T light quarks. If strange quarks have the same elliptic flow as light quarks at high p_T, i.e., $v_{2,s} = v_{2,q}$, then $v_{2,H} = v_{2,q}$ for all SU(3) hadrons regardless of the value of r_s. This is true even if the p_T spectrum for strange quarks is different from that for light quarks.

On the other hand, if fast moving heavy quarks suffer less energy loss in a thermalized parton plasma than fast moving light quarks as predicted in many theoretical studies [24–26], then it is possible that the elliptic flow of high p_T strange quarks will be smaller than that of light quarks; i.e., $v_{2,s} < v_{2,q}$. In this case, we obtain from Eqs. (6) and (7) the following flavor ordering of the v_2 values for hadrons at a given high p_T:

$$(v_{2,\pi} = v_{2,N}) > (v_{2,\Lambda} = v_{2,\Sigma}) > v_{2,K} > v_{2,\Xi} > (v_{2,\phi} = v_{2,\Omega}). \tag{11}$$

In Fig. 1, we illustrate the flavor ordering of hadron elliptic flows at high p_T for the case of $v_{2,s} < v_{2,q}$. The spacings between different curves correspond to the case of $r_s = 1$ and thus follow the quark counting relations of Eqs. (6) and (9). We note that the vertical scale for v_2 is in arbitrary units, and the shape of v_2 as a function of p_T is also arbitrary. The scale p_0 denotes the typical transverse momentum above which the p_T spectra of final partons changes from soft to hard, and its value should probably be a few GeV/c. All curves are shown well above p_0, reflecting the fact that the relations derived in the present study apply only to hadron elliptic flows at high p_T. In general, r_s can take any finite positive value, but the flavor ordering of hadron elliptic flows remains similar to that shown in Fig. 1 as long as $v_{2,s} < v_{2,q}$. However, the spacings between different curves can be different, while still being constrained by the two relations given in Eq. (8). Since the v_2 of hadrons follow the mass ordering at low p_T and the flavor ordering at high p_T, the curve for kaon $v_2(p_T)$, which is above those for proton and Λ at low p_T, will cross and become lower than the latter as p_T increases. A similar relation exists between the curve for ϕ meson $v_2(p_T)$ and those for Λ and Ξ.

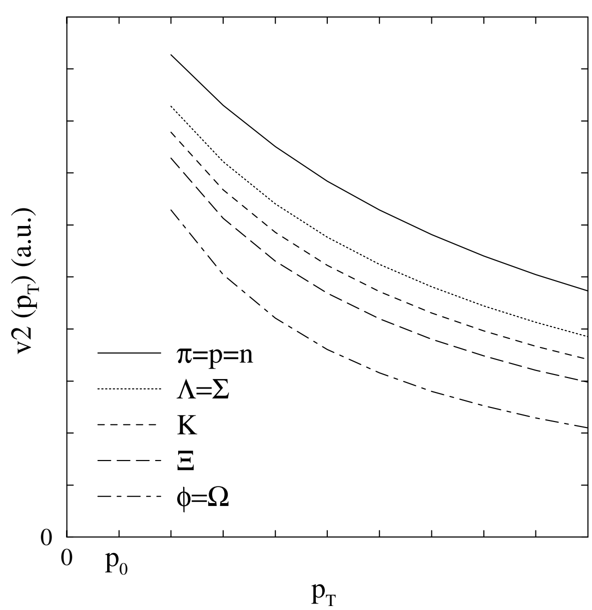

FIG. 1. Schematic plot for the flavor ordering of the elliptic flow of hadrons at high p_T. Details are given in the text.

We have not included the effects of resonance decays in this study. The relations shown in Eq. (3) can be extended to resonances such as η, ρ, ω, K^*, and Δ. These resonances at high p_T will decay to stable hadrons at different transverse momenta, thus complicating the relations we have thus derived for hadrons which are directly formed from the quark coalescence. Since the transverse momentum of a decay product is usually small compared to that of the parent hadron, and the inverse power law spectrum shows a rapid decrease with p_T, we expect that the resonance contribution to hadron elliptic flow at high p_T is small compared to the contribution from directly formed hadrons.

In summary, using a parton coalescence model to describe the formation of hadrons from the initial partonic matter in ultrarelativistic heavy ion collisions, we have studied the dependence of the elliptic flow of hadrons at high p_T on their flavor composition. Since the elliptic flow is generated mostly in the early partonic phase, and high p_T hadrons are mainly formed from the coalescence of a high p_T quark or antiquark produced from the initial hard processes and low p_T quarks or antiquarks from the soft processes, the magnitudes of the hadron elliptic flow at high p_T are determined by that of high p_T quarks. The relations between hadron and parton elliptic flows at high p_T also depend on the final quark spectrum at high p_T [$f_i(p_T)$] and the capture probability of a soft quark (c_i) by a high p_T quark to form a high p_T hadron. If strange quarks have a smaller elliptic flow than the light quarks, then the quark coalescence model leads to the flavor ordering in the elliptic flows of the hadrons formed from an isospin-symmetric and quark-antiquark–symmetric partonic matter; i.e., $(v_{2,\pi} = v_{2,N}) > (v_{2,\Lambda} = v_{2,\Sigma}) > v_{2,K} > v_{2,\Xi} > (v_{2,\phi} = v_{2,\Omega})$. We have also obtained two relations which are independent of $f_i(p_T)$ and c_i and involve the elliptic flows of four hadron species at high p_T. In the special case that $f_i(p_T)$ and c_i are the same for strange and light quarks, values of the elliptic flows of high p_T hadrons of different flavors are found to follow the quark counting rule. It will be very interesting to test these predictions in current and future heavy ion collisions. Such studies will provide valuable information on whether a partonic matter is formed in the collisions and the subsequent formation of hadrons can be described by the quark coalescence model.

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[31] We note that both the isospin symmetry and quark-antiquark symmetry will break down for quarks with p_T above the scale of the order of $\sqrt{s}/6$, with $\sqrt{s}$ being the center-of-mass energy, since the valence quark distribution function of nucleons starts to dominate the flavor of the produced jets.