Benchmarks for the Forward Observables at RHIC, the Tevatron-Run II, and the LHC

J. R. Cudell,[1] V. V. Ezhela,[2] P. Gauron,[3] K. Kang,[4] Yu. V. Kuyanov,[2] S. B. Lugovsky,[2] E. Martynov,[1,5] B. Nicolescu,[3]
E. A. Razuvaev,[2] and N. P. Tkachenko[2]

(COMPETE Collaboration)*

[1]*Institut de Physique, Bâtiment B5, Université de Liège, Sart Tilman, B4000 Liège, Belgium*
[2]*COMPAS group, IHEP, Protvino, Russia*
[3]*LPNHE † Theory Group, Université Pierre et Marie Curie, Tour 12 E3, 4 Place Jussieu, 75252 Paris Cedex 05, France*
[4]*Physics Department, Brown University, Providence, Rhode Island 02912*
[5]*Bogolyubov Institute for Theoretical Physics, 03143 Kiev, Ukraine*

(Received 19 June 2002; published 28 October 2002)

We present predictions on the total cross sections and on the ρ parameter for present and future pp and $\bar{p}p$ colliders, and on total cross sections for $\gamma p \rightarrow$ hadrons at cosmic-ray energies and for $\gamma\gamma \rightarrow$ hadrons up to $\sqrt{s} = 1$ TeV. These predictions are based on an extensive study of possible analytic parametrizations invoking the largest hadronic dataset available at $t = 0$. The uncertainties on total cross sections reach 1.9% at the Relativistic Heavy Ion Collider, 3.1% at the Tevatron, and 4.8% at the Large Hadron Collider, whereas those on the ρ parameter are, respectively, 5.4%, 5.2%, and 5.4%.

DOI: 10.1103/PhysRevLett.89.201801 PACS numbers: 13.85.–t, 11.55.-m, 12.40.Nn, 13.60.Hb

In recent works [1,2], we have performed an exhaustive study of the analytic parametrizations of soft forward data. For this purpose, we gathered the largest available set of data at $t = 0$, which includes all measured total cross sections and ratios of the real part to the imaginary part of the elastic amplitude (ρ parameter) for the scattering of pp, $\overline{p}p$, $\pi^{\pm}p$, $K^{\pm}p$, and total cross sections for γp, $\gamma\gamma$, and $\Sigma^{-}p$ [3,4].

Several experiments are under way [6], or being planned, to measure the hadronic amplitudes at $t = 0$. Some authors [7–9] also presented what they feel are reference values for the total γp and $\gamma\gamma \rightarrow$ hadrons cross sections. Thus it is timely and appropriate to present independently our predictions for the forward observables at the Relativistic Heavy Ion Collider (RHIC), the Tevatron-run II, and the Large Hadron Collider (LHC) as well as for γp total cross section at cosmic-ray energies and for $\gamma\gamma$ total cross sections up to 1 TeV.

We can summarize [10] the general form of the parametrizations by quoting the total cross sections for the scattering of m on n, from which the ρ parameter is obtained via analyticity. The ingredients are the contribution $y^{mn}(s)$ of the highest meson trajectories (ρ, ω, a, and f) and the rising $C = +1$ term $h^{mn}(s)$ from the pomeron contribution to the total cross section:

$$\sigma_{\text{tot}}^{mn} = (y^{mn}(s) + h^{mn}(s))/s. \tag{1}$$

The first term is parametrized via Regge theory, and we allow the lower trajectories to be partially degenerate, i.e., our experience shows that it is enough to introduce one intercept for the $C = +1$ trajectories, and another one for the $C = -1$ [11]. A further lifting of the degeneracy is certainly possible, but does not seem to modify significantly the results [12]. Hence we use

$$y^{mn}(s) = Y_{+}^{mn}(s/s_1)^{\alpha_+} \pm Y_{-}^{mn}(s/s_1)^{\alpha_-}, \tag{2}$$

with $s_1 = 1\ \text{GeV}^2$. The contribution of these trajectories is represented by RR in the model abbreviations.

As for the part rising with energy, we consider here two main options: it can rise as a $\ln s$, or as a $\ln^2 s$, with in each case the possibility to add a constant term. We shall not consider the simple-pole parametrization [13], because, after refitting, it is excluded at the the 98% C.L. if one lowers the energy cutoff to 5 GeV [14]. In the following, we shall refer explicitly only to our preferred parametrization of h^{mn}, noted PL2:

$$h^{mn}(s) = s(B^{mn} \ln^2(s/s_0) + P^{mn}), \tag{3}$$

where s_0 is a universal scale parameter (to be determined by the fits) identical for all collisions.

We consider several possible constraints on the parameters of Eqs. (2) and (3): degeneracy of the reggeon trajectories ($\alpha_+ = \alpha_-$); universality of rising terms (B^{mn} independent of the hadrons) [15–17]; factorization for the residues in the case of the $\gamma\gamma$ and γp cross sections ($h_{\gamma\gamma} = \delta h_{\gamma p} = \delta^2 h_{pp}$); quark counting rules [18] (predicting the Σp cross section from pp, Kp, and πp); and finally the Johnson-Treiman-Freund [19] relation for the cross section differences.

Out of the 256 possible variants, we keep only models which have an overall $\chi^2/\text{dof} \leq 1.0$ and a non-negative pomeron contribution at all energies. This leaves us 24 possible models for fits to σ_{tot} for $\sqrt{s} \geq 10$ GeV, and 5 for $\sqrt{s} \geq 4$ GeV(see Table XI from [1]). If we include the ρ data, we are left with 20 (respectively 4) variants for $\sqrt{s} \geq 10$ (respectively 5) GeV (see Table XIV from [1]).

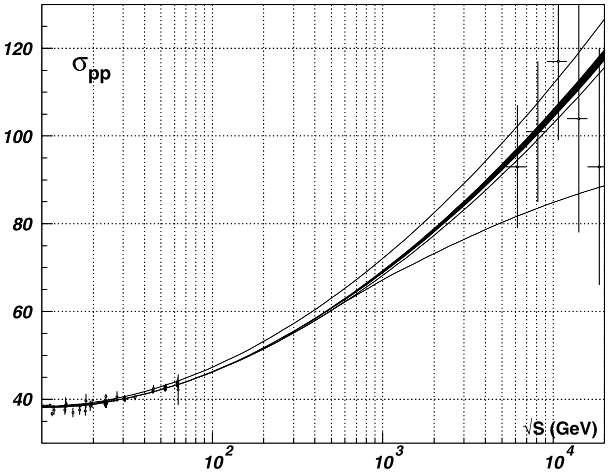

FIG. 1. Predictions for total cross sections. The black error band shows the statistical errors to the best fit, the closest curves near it give the sum of statistical and systematic errors to the best fit due to the ambiguity in Tevatron data, and the highest and lowest curves show the total errors bands from all models considered in this Letter (note that the upper curve showing the systematic error is indistinguishable from the highest curve in this case).

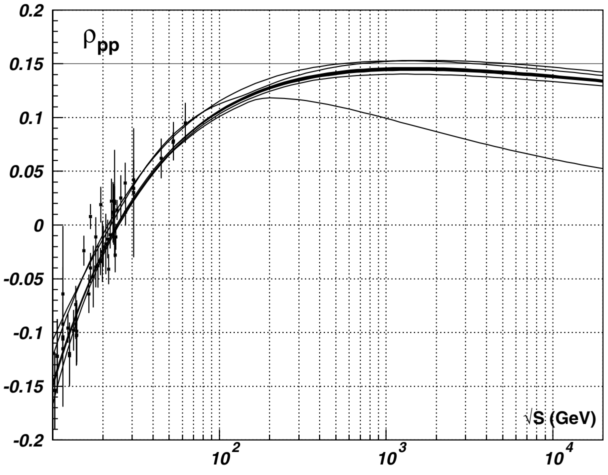

FIG. 2. Predictions for the ρ parameter. The curves and band are as in Fig. 1.

Here we shall neither give the list of models nor spell out ranking criteria based on new indicators that quantify certain qualities of the fits, but simply mention that the triple-pole parametrization $\mathrm{RRP_{nf}L2_u}$ [15,16] was determined to be the highest-ranking model leading to the most satisfactory description of the data. This parametrization has a universal (u) $B\ln^2(s/s_0)$ term, a nonfactorizing (nf) constant term and nondegenerate lower trajectories.

We start by giving the predictions of this model, adjusted for ($\sqrt{s} \geq 5$ GeV), with updated data points from ZEUS [5]. The quality of the fit is shown in Table I, column 2. For pp and $\bar{p}p$, the central value of the fit gives

$$\sigma_{\mathrm{tot}}^{\bar{p}p,pp} = 42.6\ s^{-0.46} \pm 33.4\ s^{-0.545} + 35.5 + 0.307 \ln^2\left(\frac{s}{29.1}\right), \tag{4}$$

with all coefficients in mb and s in GeV^2.

The following predictions include statistical errors calculated from the full error matrix E_{ij}. We define

$$\Delta Q = \sum_{ij} E_{ij} \frac{\partial^2 Q}{\partial x_i \partial x_j}, \tag{5}$$

with $Q = \sigma_{\mathrm{tot}}$ or ρ and x_i the parameters of the model. These errors are shown in Figs. 1 and 2 by a filled band, and in Tables II and III.

In these figures and tables, we also give our estimate of the systematic uncertainty coming from the discrepancy between different FNAL measurements of σ_{tot}: we fit $\mathrm{RRP_{nf}L2_u}$ either to the high data (CDF) or to the low ones (E710/E811), and get two error bands. The distances from the central value of the combined fit to the upper (respectively lower) border of these bands give us the positive (respectively negative) systematic errors. We

TABLE I. Summary of the quality of the fits at different stages of the Review of Particle Physics (RPP) database (DB): DB02, the 2002 RPP DB; DB02Z, the 2002 RPP DB with new ZEUS data; DB02Z-CDF, with the CDF point removed; DB02Z-E710/E811, with E710/E811 points removed. The first line gives the overall χ^2/dof for the global fits; the other lines give the χ^2/nop for data subsamples; the last line gives in each case the parameter controlling the asymptotic form of cross sections.

Sample	DB02	DB02Z	DB02Z −CDF	DB02Z −E710/E811
Total	0.968	0.966	0.964	0.951
Total cross sections				
$\bar{p}p$	1.15	1.15	1.12	1.05
pp	0.84	0.84	0.84	0.84
$\pi^- p$	0.96	0.96	0.96	0.96
$\pi^+ p$	0.71	0.71	0.71	0.71
$K^- p$	0.62	0.62	0.62	0.61
$K^+ p$	0.71	0.71	0.71	0.71
$\Sigma^- p$	0.38	0.38	0.38	0.38
γp	0.61	0.58	0.58	0.58
$\gamma\gamma$	0.65	0.64	0.64	0.63
Elastic forward Re/Im				
$\bar{p}p$	0.52	0.52	0.52	0.53
pp	1.83	1.83	1.83	1.80
$\pi^- p$	1.10	1.10	1.09	1.14
$\pi^+ p$	1.50	1.50	1.52	1.46
$K^- p$	1.00	0.99	1.01	0.96
$K^+ p$	1.07	1.07	1.10	0.98
Values of the parameter B				
	0.307(10)	0.307(10)	0.301(10)	0.327(10)

TABLE II. Predictions for $\sigma_{\rm tot}$ and ρ, for $\bar{p}p$ (at $\sqrt{s} = 1960$ GeV), and for pp (all other energies). The central values and statistical errors correspond to the preferred model RRP$_{\rm nf}$L2$_{\rm u}$, and the systematic errors come from the consideration of two choices between CDF and E710/E811 $\bar{p}p$ data in the simultaneous global fits.

$\sqrt{s}$ (GeV)	σ (mb)	ρ
100	$46.37 \pm 0.06 ^{+0.17}_{-0.09}$	$0.1058 \pm 0.0012 ^{+0.0040}_{-0.0021}$
200	$51.76 \pm 0.12 ^{+0.39}_{-0.21}$	$0.1275 \pm 0.0015 ^{+0.0051}_{-0.0026}$
300	$55.50 \pm 0.17 ^{+0.57}_{-0.30}$	$0.1352 \pm 0.0016 ^{+0.0055}_{-0.0028}$
400	$58.41 \pm 0.21 ^{+0.71}_{-0.36}$	$0.1391 \pm 0.0017 ^{+0.0056}_{-0.0030}$
500	$60.82 \pm 0.25 ^{+0.82}_{-0.45}$	$0.1413 \pm 0.0017 ^{+0.0057}_{-0.0030}$
600	$62.87 \pm 0.28 ^{+0.94}_{-0.48}$	$0.1416 \pm 0.0018 ^{+0.0058}_{-0.0031}$
1960	$78.27 \pm 0.55 ^{+1.85}_{-0.96}$	$0.1450 \pm 0.0018 ^{+0.0057}_{-0.0030}$
10000	$105.1 \pm 1.1 ^{+3.6}_{-1.9}$	$0.1382 \pm 0.0016 ^{+0.0047}_{-0.0027}$
12000	$108.5 \pm 1.2 ^{+3.8}_{-2.0}$	$0.1371 \pm 0.0015 ^{+0.0046}_{-0.0026}$
14000	$111.5 \pm 1.2 ^{+4.1}_{-2.1}$	$0.1361 \pm 0.0015 ^{+0.0058}_{-0.0025}$

estimate the total errors as the sum of the systematic and of the statistical uncertainties [20].

One can see that the total errors on total cross sections are of the order of 1.9% at RHIC, of the order of 3.1% at the Tevatron, and as large as 4.8% at the LHC and dominated by the systematic errors. The errors on the ρ parameter are much larger, reaching 5.4% at RHIC, 5.2% at the Tevatron, and 5.4% at the LHC. This comes from the fact that experimental errors are bigger, hence less constraining, and also from the incompatibility of some low-energy determinations of ρ [1]. This means that the systematic error is always bigger than the statistical one.

Concerning the contradictory data, we are forced to use them in our fits until the discrepancy is resolved by further experiments. In the case of the Tevatron data, the discrepancy results in a big shift (of more than 1σ) in the central value of the coefficient B of the $\ln^2 s$ term, which controls the asymptotic behavior, and hence that asymptotic predictions are appreciably weakened by the present situation. The opportunities to measure $\sigma_{\rm tot}$ and ρ will be scarce in the future, thus any new measurement at RHIC, the Tevatron-run II and the LHC should not be missed. Unfortunately, the recent publication of E811 [21] does not clear the problem as their value for ρ is fully compatible with our preferred model, whereas their number for $\sigma_{\rm tot}$ (which is highly correlated with ρ) has hardly changed.

TABLE III. Predictions for $\sigma_{\rm tot}$ for $\gamma p \rightarrow$ hadrons and for $\gamma\gamma \rightarrow$ hadrons for cosmic ray energies. The central values, the statistical errors, and the systematic errors are as in Table II.

$p_{\rm lab}^{\gamma}$ (GeV)	$\sigma_{\gamma p}$ (mb)	$\sqrt{s}$ (GeV)	$\sigma_{\gamma\gamma}$ (μ b)
0.5×10^6	$0.24 \pm 0.01 \pm 0.01$	200	$0.55 \pm 0.03 \pm 0.03$
1.0×10^6	$0.26 \pm 0.01 \pm 0.01$	300	$0.61 \pm 0.04 \pm 0.04$
1.0×10^7	$0.33 \pm 0.02 \pm 0.02$	400	$0.66 \pm 0.04 \pm 0.04$
1.0×10^8	$0.42 \pm 0.02 ^{+0.03}_{-0.02}$	500	$0.70 \pm 0.05 \pm 0.05$
1.0×10^9	$0.52 \pm 0.03 ^{+0.04}_{-0.03}$	1000	$0.84 \pm 0.07 \pm 0.07$

It is interesting to note that the choice of one FNAL result or the other leads to a variation of the overall fit quality, as shown in Table I (last two columns) [22]: the variant with CDF data has slightly better overall χ^2/dof and better χ^2/nop distribution over subsamples. We can consider this as an indication that the global picture emerging from fits to all data on forward observables supports the CDF data and disfavors the E710/E811 data at $\sqrt{s} = 1.8$ TeV (see also [23]).

Finally, we also present in Figs. 1 and 2 our estimate of the region where new physics would be discovered. For each of the 20 parametrizations which satisfy our criteria for applicability [1] for $\sqrt{s} \geq 10$ GeV, and which obey the Froissart-Martin bound [24], we construct error bands according to Eq. (5). This gives us 20 1σ-error bands. Their union represents the "allowed region" where analytic models built according to (1) can reproduce the data. A measurement outside this region would imply that new physics ingredients are needed.

To conclude, we believe that we have given here the best possible estimates for present and future pp and $\bar{p}p$ facilities. Although one might be tempted to use only data in an energy range close to the one measured, one must realize that analytic parametrizations are constrained both by lower-energy data, and by their asymptotic regime. Because the pomeron mixes (physically and numerically) with the f trajectory, fits to all data help to disentangle the two contributions.

A sharpening of our error bars would enable one to decide if the unitarization plays an essential role and what form it takes. This in turn can have an impact on the determination of the survival of probability gaps in hard scattering, and on the usefulness of pomeron exchange as a detection tool.

Any significant deviation from the predictions based on model RRP$_{\rm nf}$L2$_{\rm u}$ will lead to a reevaluation of the hierarchy of models and presumably change the preferred parametrization to another one. A deviation from the "allowed region" would be an indication that strong interactions demand a generalization of the analytic models discussed so far, e.g., by adding odderon terms, or new pomeron terms, as suggested by pQCD.

The COMPAS group was supported in part by the Russian Foundation for Basic Research Grants No. RFBR-98-07-90381 and No. RFBR-01-07-90392. K. K. is in part supported by the U.S. D.o.E. Contract No. DE-FG-02-91ER40688-Task A. We thank Professor Jean-Eudes Augustin for the hospitality at LPNHE-Université Paris 6, where part of this work was done. We thank O. Selyugin for his corrections to the database, A. Sobol for discussions on the soft physics program of the TOTEM &CMS, and Yu. Kharlov for discussions on possibilities for $\gamma\gamma$ physics at RHIC and at the LHC.

Note added.—After this study was completed, Menon [25] pointed out to us that one of the Akeno points (at $p_{\rm lab} = 1.85 \times 10^8$ GeV) should be changed to 100 mb instead of the 93 mb we used. We have checked that this does not significantly modify our results—the main effect being to reduce the lower systematic errors [10].

---

*COmputerized Models, Parameter Evaluation for Theory and Experiment.

†Unité de Recherche des Universités Paris 6 et Paris 7, Associée au CNRS.

[1] J. R. Cudell *et al.*, Phys. Rev. D **65**, 074024 (2002).

[2] J. R. Cudell *et al.*, Phys. Rev. D **61**, 034019 (2000); **63**, 059901(E) (2001).

[3] K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002), available on the PDG WWW pages (http://pdg.lbl.gov/).

[4] The latest ZEUS point [5] on $\sigma_{\rm tot}(\gamma p \rightarrow \text{hadrons})$, that supersedes the previous two ZEUS points, is included in the present study for $\text{RRP}_{\text{nf}}\text{L2}_{\text{u}}$. Also the whole database was changed slightly since the last session of the cross assessment [1,3] due to misprints and errata corrections. This improves slightly the global fit and central values are shifted noticeably within the error bars when one compares with results calculated from the parametrizations of [1,2].

[5] ZEUS Collaboration, S. Chekanov *et al.*, Nucl. Phys. **B627**, 3 (2002).

[6] See, e.g., http://www.rhic.bnl.gov/pp2pp/ and http://totem.web.cern.ch/Totem/

[7] S. I. Dutta, M. H. Reno, I. Sarcevic, and D. Seckel, Phys. Rev. D **63**, 094020 (2001).

[8] G. Baur, K. Hencken, D. Trautmann, S. Sadovsky, and Y. Kharlov, Phys. Rep. **364**, 359 (2002).

[9] M. M. Block and A. B. Kaidalov, Phys. Rev. D **64**, 076002 (2001); M. M. Block and G. Pancheri, hep-ph/0206166.

[10] The details of the parametrizations used in this paper can be found at http://nuclth02.phys.ulg.ac.be/compete/publications/benchmarks_details/

[11] J. R. Cudell, K. Kang, and S. K. Kim, Phys. Lett. B **395**, 311 (1997).

[12] P. Desgrolard, M. Giffon, E. Martynov, and E. Predazzi, Eur. Phys. J. C **18**, 555 (2001).

[13] A. Donnachie and P.V. Landshoff, Phys. Lett. B **296**, 227 (1992).

[14] The original Donnachie-Landshoff parametrization [13] provides an adequate fit to total cross sections only for $\sqrt{s} \geq 10$ GeV. The inclusion of the ρ parameter forces one to lift the degeneracy of the leading meson trajectories, and leads to an intercept value of 1.095 [2,11]. Keeping all data down to $\sqrt{s} = 5$ GeV leads to the exclusion of such a simple form. It is, however, remarkable that the refitted parametrization of [2] is almost identical with our preferred model in pp and $\bar{p}p$ collisions. The original parametrization [13] leads to somewhat smaller values of the cross section. Both fall into the theoretical error bands shown in Figs. 1 and 2.

[15] L. D. Soloviev, Pis'ma Zh. Eksp. Teor. Fiz. **18**, 455 (1973); **19**, 185 (1974) [JETP Lett. **19**, 116 (1974)].

[16] P. Gauron and B. Nicolescu, Phys. Lett. B **486**, 71 (2000).

[17] E. Ferreiro, E. Iancu, K. Itakura, and L. McLerran, hep-ph/0206241.

[18] E. M. Levin and L. L. Frankfurt, Pis'ma Zh. Eksp. Teor. Fiz. **3**, 652 (1965) [JETP Lett. **2**, 65 (1965)].

[19] K. Johnson and S. B. Treiman, Phys. Rev. Lett. **14**, 189 (1965); P. G. O. Freund, Phys. Rev. Lett. **15**, 929 (1965).

[20] Apart from the tiny statistical error, none of the errors or bands given in this work has a probabilistic interpretation.

[21] E811 Collaboration, C. Avila *et al.*, Phys. Lett. B **537**, 41 (2002).

[22] We have constructed a preliminary version of a Web predictor, which gives the central values and fit errors without systematics from our highest rank models at any collision energies $\sqrt{s} \geq 5$ GeV; see http://nuclth02.phys.ulg.ac.be/compete/predictor.html

[23] P. Gauron and B. Nicolescu, Phys. Lett. B **333**, 142 (1994); M. G. Albrow, A. Beretvas, L. Nodulman, and P. Giromini, Report No. FERMILAB-TM-2071, 1999.

[24] M. Froissart, Phys. Rev. **123**, 1053 (1961); A. Martin, Nuovo Cimento A **42**, 930 (1966); see also W. Heisenberg, Z. Phys. **133**, 65 (1952).

[25] M. J. Menon (private communication).