Direct Measurement of the Wigner Function of a One-Photon Fock State in a Cavity

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We have measured the complete Wigner function W of the vacuum and of a single-photon state for a field stored in a high-Q cavity. This experiment implements the direct Lutterbach and Davidovich method [L. G. Lutterbach and L. Davidovich, Phys. Rev. Lett. **78**, 2547 (1997)] and is based on the dispersive interaction of a single circular Rydberg atom with the cavity field. The nonclassical nature of the single-photon field is exhibited by a region of negative W values. Extensions to other nonclassical cavity field states are discussed.

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The statistical properties of a single mode light field (or of a one-dimensional atomic motion) are described by the Wigner function W [1], which is a quasiprobability distribution in phase space. Whereas, classically, this quantity is always strictly positive, W may take negative values for a quantum state. The existence of negative quasiprobabilities is generally considered as a signature of nonclassical states. Coherent fields, thermal fields, and even squeezed states have completely positive Wigner functions and can thus, in this respect, be considered as "classical." Fock states with a defined nonzero photon or phonon number and superpositions of coherent states with different phases or amplitudes ("Schrödinger cat" states [2–4]) exhibit negative values of W and are, according to this criterium, "nonclassical."

Several reconstruction methods have been proposed and experimentally demonstrated to obtain W. A procedure similar to medical tomography amounts to integrating W along many directions in phase space. The Radon inversion [5] is then used to extract W. This method has been used for various field states [6,7], leading recently to a measurement of W for a single-photon running field [7,8]. The Radon inversion has also been applied to the atomic motion in an interferometer, leading to the observation of a negative W value [9].

Other methods, which directly yield the Wigner function, based on the measurement of the photon number distribution after a displacement of the field state in phase space, have also been proposed [10] and realized for "classical" states [11]. The photon [12] or phonon [13] number distribution, and, hence, W, can also be obtained indirectly, by analyzing the quantum Rabi oscillation of a two-level atom coupled to the displaced oscillator. This method has been applied to a trapped ion single-phonon state [13]. In practice, both these methods require truncation of the photon (phonon) number distribution at some finite number.

An elegant method proposed by Lutterbach and Davidovich (LD procedure) [14,15] yields directly W at a given point α in phase space. Without determining explicitly (or truncating) the photon number distribution, it directly measures the photon number parity of the field displaced by $-\alpha$. This measurement is performed by atomic interferometry, in a situation where an atomic state superposition coupled to the field is phase shifted by π each time the number of photons increases by one. We have already used this method to measure the negative value of W for a one-photon state at the origin of phase space [16]. This experiment was, however, unable to determine W at other points, because the photon-induced phase shift, produced in a resonant Rabi oscillation [17], applied only to the zero- or one-photon subspace.

In this Letter, we report the complete determination of W for a cavity field, using the LD procedure. The π-phase shift per photon is realized by a nonresonant atom-field interaction [18]. We have applied the method to a small thermal field, with a Gaussian-shaped positive Wigner distribution, and to an approximate one-photon Fock state, giving a "Mexican hat"-shaped distribution, with a well-marked negative feature at the origin.

The LD method is based on a simple expression of $W(\alpha)$, at point α in phase space, for a field with density matrix ρ [19]:

$$W(\alpha) = 2\mathrm{Tr}[D(-\alpha)\rho D(\alpha)P], \tag{1}$$

where $P = \exp(i\pi a^\dagger a)$ is the field parity operator [its action on Fock state $|n\rangle$ being $P|n\rangle = (-1)^n|n\rangle$] and $D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)$ is the displacement operator. With this normalization, $-2 \leq W \leq 2$ and $\int W(\alpha)d^2\alpha = \pi$. For an n-photon Fock state, $W(0) = 2(-1)^n$.

W is twice the expectation value of P in the field state displaced by an amplitude $-\alpha$, whose density matrix is $\rho(\alpha) = D(-\alpha)\rho D(\alpha)$. If the initial field is stored in a cavity C, we merely have to "shift" it by injecting in C a coherent field with amplitude $-\alpha$, then to measure the parity operator on the resulting field. Repeating this experiment many times for each α value, we get $W(\alpha)/2 = \mathrm{Tr}[\rho(\alpha)P] = \langle P\rangle = \sum_n(-1)^n\rho_{n,n}(\alpha)$.

In order to measure $\langle P \rangle$, we send repeatedly across C a single atom with a known velocity. This atom has two levels, e and g, the $e \rightarrow g$ transition being slightly off resonant with the field (frequency mismatch Δ). In an n-photon field, the atomic transition frequency is light shifted at cavity center by $\Omega^2 n/2\Delta$ [18], where Ω is the resonant vacuum Rabi frequency [20]. We adjust Δ so that a single-photon produces a π-phase shift on an e/g coherence during the atom-cavity interaction time.

This phase shift is revealed by Ramsey interferometry [21]. We subject the atom to two resonant $\pi/2$ pulses mixing e and g before and after the interaction with C. The probability p_e (respectively, p_g) for detecting the atom in e (respectively, g) exhibits modulations versus the phase ϕ of the interferometer. For an empty cavity and a proper choice of phase reference, $p_e = (1 + \cos\phi)/2$. The phase shift induced by an n-photon field being $n\pi$, p_e becomes, in the presence of a displaced field, $p_e(\phi, \alpha) = [1 + \sum_n (-1)^n \rho_{n,n}(\alpha) \cos\phi]/2 = [1 + \langle P \rangle \cos\phi]/2$. Hence, W is directly related to the fringes contrast $c(\alpha)$:

$$W = 2\langle P \rangle = 2c(\alpha) = 2[p_e(0, \alpha) - p_e(\pi, \alpha)]. \qquad (2)$$

$W(\alpha)$ is thus determined from the expectation value of a measurement performed on the atom, after its interaction with the displaced field, without inversion procedure.

The realization of the LD method is challenging. We need to generate a large dispersive phase shift per photon. We must also perform the measurement in a time short compared to the field damping time—the time scale for the washing out of nonclassical features for fields with photon number of the order of unity. We have performed the experiment with our cavity QED setup, described in detail elsewhere [22], which provides the tools required to realize these difficult conditions.

A sketch of the setup is shown in Fig. 1(a). Its central part is a superconducting cavity C made of two niobium mirrors in a Fabry-Perot configuration, cooled down to 1.3 K. It sustains two Gaussian field modes M_a and M_b (waist $w = 6$ mm) separated by a frequency interval $\delta = 128$ kHz around 51.1 GHz. The field damping times are $T_a = 800$ μs for the upper frequency mode M_a and $T_b = 730$ μs for M_b. The two modes initially contain a thermal field with about one photon. This field is erased, before the experimental sequence starts, by sending through the cavity a train of absorbing atoms [22]. This reduces the background photon number in both modes. The residual photon number in mode M_a is about 0.1.

Let us describe now the sequence of operations required to measure W for a one-photon Fock state stored in M_a. This field is produced by a velocity-selected circular Rydberg atom (velocity $v = 150$ m/s). The atom is prepared in the circular level with principal quantum number 51 (level e) and undergoes in M_a a π-Rabi pulse on the transition from e to the circular state with principal quantum number 50 (level g) [23]. The single-photon

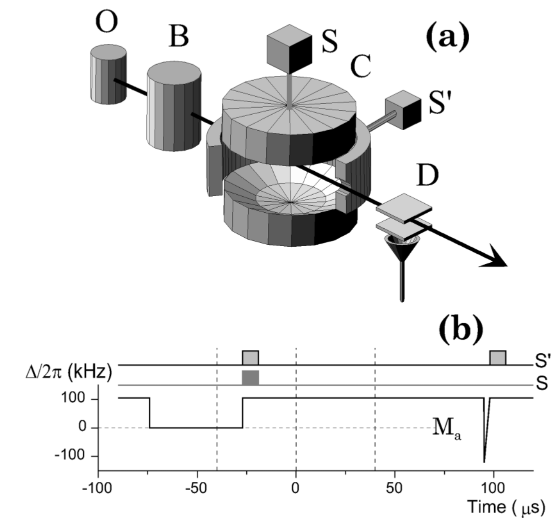

FIG. 1. (a) Scheme of the experimental apparatus. The rubidium atomic beam effuses from oven O. Circular atoms are prepared one at a time in box B. They cross the cavity C before being counted in the field-ionization detector D. The source S is used for the cavity field displacement. The source S' is used for the Ramsey interferometer. (b) Experimental timing for the one-photon Wigner function measurement. The time origin corresponds to the atom crossing the cavity axis (central vertical dotted line). The two other dotted lines indicate the cavity mode waist limits. The lower curve presents, versus time, the detuning Δ between the atomic transition and mode M_a. The horizontal dotted line corresponds to the atom-M_a resonance condition. The resonant period between $t = -74$ μs and $t = -27$ μs corresponds to the π-Rabi pulse preparing a single photon in M_a. The narrow feature around 95 μs is the field pulse used to tune the phase ϕ of the Ramsey interferometer. The two upper curves present the "on" periods for sources S and S'.

Rabi frequency is $\Omega/2\pi = 49$ kHz [24]. To adjust the Rabi pulse duration, a time varying electric field is applied across the cavity mirrors. It switches by Stark effect the atomic transition, initially out of resonance, into resonance with M_a for the required time interval, then out of resonance again. The corresponding timing is depicted on Fig. 1(b) (lower trace). The final $e \rightarrow g$ transition frequency is set to be $\Delta/2\pi = 105$ kHz above the frequency of M_a. The atomic transition being from then on nonresonant with M_a and M_b, the atom-cavity interaction becomes dispersive. It is used to measure, with the same atom, the Wigner function of the field in C.

The field displacement is achieved by injecting in M_a, in a pulsed process, a coherent field generated by the source S. The injected amplitude is calibrated, with a $\pm 3\%$ precision, in an auxiliary experiment, by measuring the Ramsey fringes phase shift, proportional to the average photon number for large detunings Δ [18]. The injection occurs immediately after the one-photon preparation. Since there is no phase information in the cavity field preparation, we deal only with phase-independent Wigner functions. The phase of the injected amplitude, well controlled in our setup [22], is thus not varied here. We assume it, without loss of generality, to be zero.

We start, simultaneously with the displacement, the Ramsey interferometer operation. Two pulses generated by the source S' in a low-Q transverse mode, resonant on the $e \rightarrow g$ transition (whose frequency is detuned from M_a and M_b), are applied on the atom [Fig. 1(b)], the second being fired after the atom has exited C. The atom-cavity detuning and the timing of the first pulse are adjusted to achieve the required π-phase shift per photon. The duration of each experimental sequence is shorter than the cavity mode damping time.

The duration of the Ramsey pulses (7.8 μs) is so short that Fourier components would coincide, if no precautions were taken, with the frequencies of M_a and M_b, leading to a field leakage in these modes. In order to avoid it, we adjust the square shape Ramsey pulses central frequencies and durations so that their sine-cardinal Fourier transform has zeros at the frequencies of M_a and M_b. We have checked by an independent experiment that the Ramsey pulses do not feed more than 0.03 photon on the average in M_a.

The final atomic state is detected by the state-selective field-ionization detector D. The phase ϕ of the Ramsey interferometer is swept by applying, before the second Ramsey pulse (when the atom is already out of the mode), a short pulse of electric field of variable amplitude [see Fig. 1(b)]. Ramsey fringes are recorded by accumulating data on many runs of the experiment. For comparison, a similar experiment is carried out without the atomic emission in C, with a slightly different timing. The Stark field, realizing the photon generating Rabi pulse, is not applied in this case.

We start by discussing the results when no photon is injected in C, realizing an approximation of the vacuum field, modified by the presence of a small thermal field. Figures 2(a)–2(c) show the Ramsey fringes obtained for three increasing values (top to bottom) of α. The experimental fringes are fitted with sine curves [solid lines in Figs. 2(a)–2(c)] providing the contrast $c(\alpha)$ (with a precision ± 0.02). The phase invariance versus α indicates that the shift per photon is close to π (a precise fit of the data yields 3.3 ± 0.15 rad). Note that the imprecision on this shift affects only quadratically $c(\alpha)$ ($\pm$ 5% errors for coherent fields containing up to four photons).

Figure 2(d) presents the experimental W values. The measurements are affected by the finite contrast of the Ramsey interferometer due to various imperfections, among which the phase shift induced by the residual thermal field in M_b (about 0.4 photon on average). Since the atom-M_b detuning is $\Delta + \delta \approx 2\Delta$, a photon in M_b produces a $\pi/2$ phase shift, reducing $c(\alpha)$. We thus multiply the raw data by a normalization factor 2.44, so that the integral of the W values over the phase space is equal to π, as required. This procedure assumes that the contrast reduction is independent of α, a reasonable assumption for the low photon numbers involved here. The

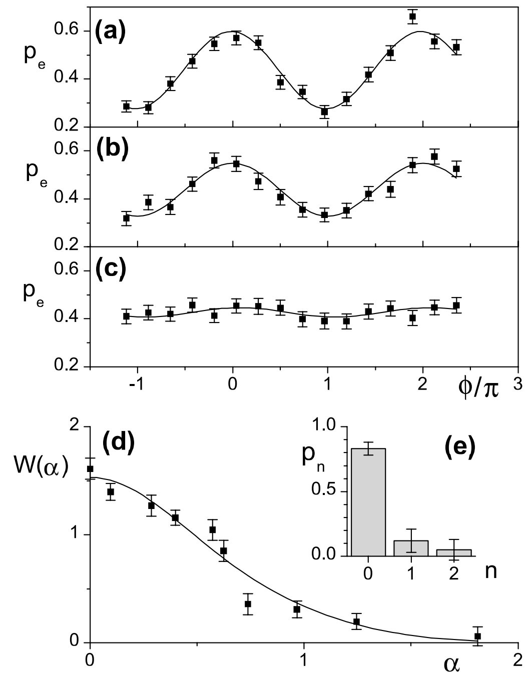

FIG. 2. Determination of the "vacuum state" Wigner function. (a) Ramsey fringes for an injected amplitude $\alpha = 0$. Probability p_e for detecting the atom in state e as a function of the Ramsey interferometer phase ϕ/π. Dots are experimental with error bars reflecting the variance of the binomial detection statistics. The solid curve is a sine fit. (b) and (c) Ramsey fringes for $\alpha = 0.57$ and $\alpha = 1.25$, respectively. (d) Dots: vacuum state Wigner function versus α with error bars reflecting the uncertainty on the Ramsey fringes fit. The solid line is a theoretical fit (see text). (e) Corresponding photon number distribution p_n.

Wigner function is a Gaussianlike curve whose variation when α increases reflects the decrease of $c(\alpha)$. The solid line in Fig. 2(d) is a fit on the Wigner function of a mixture of the zero-, one-, and two-photon states. The adjustable parameters are the photon number probabilities p_n of the initial cavity field, before displacement. The agreement between the fit and the experiment is very good. The corresponding photon number distribution is shown in Fig. 2(e). It is in good agreement with a thermal field (average photon number $\overline{n} = 0.2$), consistent with the return to thermal equilibrium during the duration of the experiment.

We now turn to the study of the "one-photon" Wigner function. Figures 3(a) and 3(b) show the Ramsey fringes for $\alpha = 0$ and $\alpha = 0.81$, respectively. Contrary to the "vacuum" field case, we see that the phase of the fringes is shifted by π between these two values. This phase reversal indicates a change of the sign of W from negative (small α) to positive (large α). The Wigner function $W(\alpha)$ is shown in Fig. 3(c). The normalization factor is now 4.16. The reduction of the contrast, compared to the vacuum case, is mainly due to the imperfections of the π-Rabi pulse preparing the one-photon state in C. A

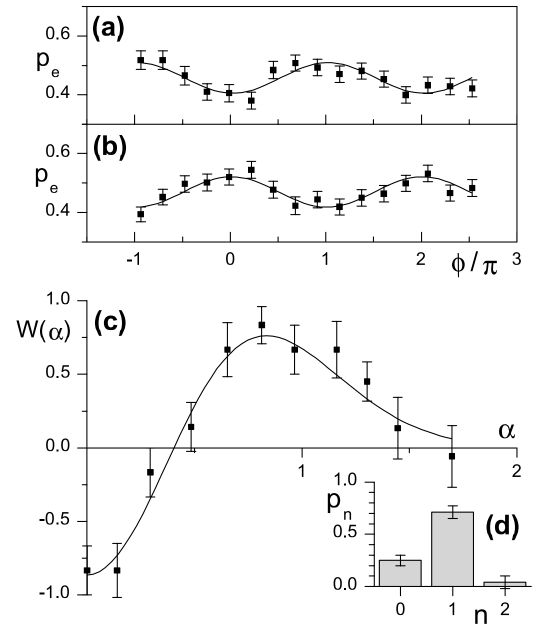

FIG. 3. Determination of the "one-photon" Wigner function. (a) Ramsey fringes for an injected amplitude $\alpha = 0$. (b) Ramsey fringes for $\alpha = 0.81$ (c) Dots: experimental Wigner function. The solid line is a theoretical fit. (d) Inferred photon number distribution.

fraction (16%) of the atoms stay in e. Since they experience the Ramsey pulses, they contribute to out of phase signals which reduce the observed contrast. The measured Wigner function exhibits a strongly nonclassical feature around $\alpha = 0$. The solid line is a fit on a mixture of Fock states with adjustable photon number probabilities. The photon distribution p_n before injection, inferred from the fit, is shown in Fig. 3(d). It exhibits a 71% probability for the one-photon state. This value is again explained by field relaxation during the experiment.

We have realized a complete measurement of the Wigner function in phase space for an approximation of a zero- and one-photon field using the LD method. This opens interesting perspectives for the study of nonclassical fields. An immediate extension is the determination of W for a superposition of zero- and one-photon states. The atom is initially prepared, by a classical microwave pulse, in a superposition of e and g. A π-Rabi pulse copies this superposition onto the cavity state [23]. The same atom is then used to determine W. A two-photon Fock state Wigner function could also be measured easily. An extra source atom is then used to prepare a first photon in the cavity before the above-described sequence starts. The study of the Wigner function of a decaying "Schrödinger cat" state [4] is also within reach. The long interaction times required to prepare the "cat" state and to probe the Wigner function make it necessary to use different atoms for the field preparation and detection. This puts more severe limitations on the cavity damping time. The corresponding conditions will be met with a slightly improved cavity.

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