Efficiency of Mesoscopic Detectors

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We consider a mesoscopic measuring device whose conductance is sensitive to the state of a two-level system. The detector is described with the help of its scattering matrix. Its elements can be used to calculate the relaxation and decoherence times of the system, and determine the characteristic time for a reliable measurement. We derive conditions needed for an efficient ratio of decoherence and measurement times. To illustrate the theory we discuss the distribution function of the efficiency of an ensemble of open chaotic cavities.

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Mesoscopic physics is evolving toward a stage where the understanding of the measurement process becomes of increasing importance. Of interest are detectors which allow a fast determination of the state of the system but at the same time leave the coherence of the measured system as unaffected as possible. These are conflicting requirements: For instance, a tunnel contact is an efficient but slow detector. Therefore, the question arises whether it is possible to develop detectors which are both fast and efficient. To answer this question we investigate mesoscopic multichannel conductors and analyze their speed and efficiency.

The effect of a detector on a phase-coherent mesoscopic system has been elegantly demonstrated in recent experiments [1–4]. Theoretical discussions addressed different aspects of weak measurement in mesoscopic systems: the relation to scattering theory [1,5,6] and screening [6], the measurement time and interactions [7], and the relation between detector noise and decoherence rate [8,9]. The time evolution of the system and detector has been studied in a master equation approach [10,11]. Refined calculations consider the conditional evolution of the system depending on the outcome of the measurement [12,13]. Tunnel contacts and single electron transistors have been identified as candidates for efficient measurement devices [14].

Both the measurement time and the decoherence rate depend on the scattering matrix of the detector. Consequently, both of these quantities depend on the sample specific geometry and impurity distribution of the detector. It is therefore necessary to investigate the distribution of the quantities of interest (measurement time, decoherence rate, and efficiency) of ensembles of macroscopically identical detectors. Here we focus on ballistic detectors for which ensemble members differ only in their geometry.

The model we consider is shown in Fig. 1. It consists of a double dot (DD) that plays the role of the system. It is an effective two-level system: The topmost electron in the DD can occupy either the upper or the lower dot. The detector is a mesoscopic two-terminal conductor (MC): Its conductance is sensitive to the charge on the upper dot. The system and detector are coupled by a set of capacitances C_1, C_2, C_i that link the charge Q on the MC to the charges on the dots Q_1 and $-Q_1$ (we abbreviate $C^{-1} = C_1^{-1} + C_2^{-1} + C_i^{-1}$). Single electron movement in such a setup was recently measured [4].

The interaction of DD and MC can be investigated from two different viewpoints. From the system side we are interested in the question of how fast a pure state prepared in the two-level system decays into a statistical mixture. We distinguish the thermal *relaxation* to an equilibrium distribution (described by a rate Γ_{rel}) and the often much faster *decoherence* of superpositions of states in the upper and lower dots (described by a rate Γ_{dec}). This decoherence depends on the temperature kT as well as on the potential difference eV applied at the MC. From the detector side we may ask how long it takes to *measure* the state of the two-level system (described by a rate Γ_m). The decoherence rate at zero temperature Γ_v is intimately related to the measurement rate Γ_m and satisfies the inequality $\Gamma_v \geq \Gamma_m$ [11,15].

We are interested in the conditions under which a MC turns out to be an efficient detector, i.e., fulfills $\Gamma_v \gtrsim \Gamma_m$. In order to be able to describe a wide class of detectors we

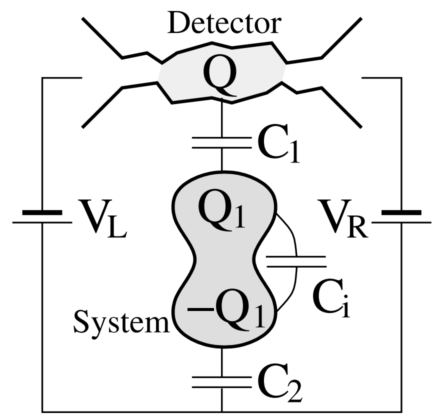

FIG. 1. A mesoscopic detector is capacitively coupled to one side of a double dot.

represent the MC by a scattering matrix $s_{\alpha\beta}$ that connects in- and outgoing states (α, β label left and right reservoirs). This enables us to treat multichannel MCs with arbitrary transmission probabilities T_n and to include screening effects between different channels. On the other hand, a minimum effort is put into the description of the coupling between the system and detector. We use a standard master equation (Bloch-Redfield approach [16]) in lowest order perturbation theory to study the evolution of the reduced density matrix of the DD. On this level of approximation the dynamics of the DD is influenced by the charge fluctuation spectrum S_{QQ} of the MC. A crucial role is therefore attributed to the Wigner-Smith time delay matrix ($\beta\gamma$ label the reservoirs)

$$N_{\beta\gamma} = \frac{1}{2\pi i} \sum_{\alpha} s^{\dagger}_{\beta\alpha} \frac{ds_{\gamma\alpha}}{dE} \tag{1}$$

that characterizes fully the low-frequency charge fluctuations [17]. We introduce the following four constants (e denotes the electron charge):

$$D = e^2 \mathrm{Tr} N, \qquad C_{\mu}^{-1} = C^{-1} + D^{-1},$$
$$R_q = \frac{1}{2}\frac{(\mathrm{Tr}N^2)}{(\mathrm{Tr}N)^2}, \qquad R_v = \frac{(\mathrm{Tr}N_{12}N_{21})}{(\mathrm{Tr}N)^2}. \tag{2}$$

These constants have been applied in many different contexts such as ac transport and noise [18,19]. D corresponds to the density of states at Fermi energy in the scattering region, C_{μ} is an effective electrochemical capacitance that characterizes the strength of interaction, R_q expresses the equilibrium contribution to the charge fluctuation spectrum S_{QQ}, and R_v is the nonequilibrium contribution.

The two-level system is conventionally represented by the Hamiltonian $\hat{H}_{DD} = \frac{\epsilon}{2}\hat{\sigma}_z + \frac{\Delta}{2}\hat{\sigma}_x$ where $\hat{\sigma}_i$ denote Pauli matrices. The energy difference between upper and lower dots is ϵ, and Δ accounts for tunneling between the dots. The full level splitting is thus $\Omega = \sqrt{\epsilon^2 + \Delta^2}$.

For the relaxation and decoherence rate in the DD we find the following expressions:

$$\Gamma_{\mathrm{rel}} = 2\pi \frac{\Delta^2}{\Omega^2}\left(\frac{C_{\mu}}{C_i}\right)^2 R_q \frac{\Omega}{2} \coth\frac{\Omega}{2kT}, \tag{3}$$

$$\Gamma_{\mathrm{dec}} = 2\pi \frac{\epsilon^2}{\Omega^2}\left(\frac{C_{\mu}}{C_i}\right)^2 (R_q kT + R_v e|V|) + \Gamma_{\mathrm{rel}}/2. \tag{4}$$

Equations (3) and (4) are the central result of this paper. It has formally the same appearance as the rates given in [11]. Its big virtue lies in the fact that the structure of the detector is condensed into the four parameters given in (2). An analysis of its properties reduces hence to a discussion of a few parameters. We postpone this discussion and explain first the derivation of Eqs. (3) and (4) in order to clarify the approximations made.

The Coulomb energy of our model can be found by circuit analysis,

$$\hat{H}_C = \frac{(\hat{Q}_1 - \bar{Q}_0)^2}{2C_i} + \frac{\hat{Q}_1\hat{Q}}{C_i} + \frac{\hat{Q}^2}{2C}. \tag{5}$$

Its first term contributes to the level splitting of the DD [$\bar{Q}_0$ is a background charge depending on the applied voltage $(V_L + V_R)/2$]. The charging energy $e^2/2C_i$ must be large compared to kT, $e|V|$ to allow us to consider only two levels of the DD. The second term $\hat{Q}_1\hat{Q}/C_i$ couples the system and detector. In the derivation of the master equation for the reduced density matrix of the system, we assume weak coupling and treat this term perturbatively. We apply a Markov approximation which is strictly speaking valid only at long time scales (compared to the correlation time of the detector). The third term influences the fluctuation spectrum of the charge operator $\hat{Q}$. In contrast to earlier work (with the exception of Ref. [6]) we do not completely disregard this term, but include it on the level of RPA. This Gaussian approximation restricts us to geometries, where Coulomb blockade effects are weak.

We will now discuss the meaning of the parameters given in Eq. (2) which describe the relation between detector geometry and relaxation or decoherence on the DD.

The parameter R_q lies always in the range $1/2 > R_q > 1/2N$ where N is the dimension of the scattering matrix. This observation indicates already that the relaxation and decoherence rates $\Gamma_{\mathrm{rel}}, \Gamma_{\mathrm{dec}}$ do not simply scale with the number of channels through the system. It is important to note that the multichannel result for the relaxation and decoherence rates cannot be obtained as a sum of rates due to each channel. For a large number N of open channels R_q and R_v behave as $1/N$, whereas the electrochemical capacitance $C_{\mu} \to C$ tends to a constant. We find therefore the somewhat surprising result that relaxation and decoherence decrease in the large channel limit. This result is a consequence of screening in the MC which reduces the charge fluctuations with increasing channel number N.

The charge response $D \propto \sum \tau_n$ and the relaxation parameter $R_q \propto \sum \tau_n^2/(\sum \tau_n)^2$ can be expressed entirely by the dwell times τ_n which are eigenvalues of the matrix (1). For the thermal charge fluctuations it is unimportant whether a scattering state is connected to the left or the right reservoir. This is reflected by the fact that the trace in the definition of R_q [see (2)] has to be taken over the entire matrix N. On the contrary, R_v does not show this symmetry. An applied voltage distinguishes the two reservoirs from one another. Thus, the trace in R_v [see (2)] cannot be expressed by dwell times only. To clarify the origin of R_v we note that the measurement is described by a measurement time [7,15] $\tau_m = 4S_{II}/(\Delta I)^2$ which is needed for a signal to noise ratio of 1. Here S_{II} denotes the low-frequency shot noise spectrum and $\Delta I = I_1 - I_2$

is the difference of current flowing through the MC depending on the state of the two-level system. This difference is evaluated by use of the Landauer formula

$$\Delta I = \Delta G|V| = \frac{e^2}{2\pi}|V| \sum \frac{dT_n}{dE}(e\Delta U), \tag{6}$$

where ΔG is the change of conductance between the two states of the double dot and $\Delta U = eC_\mu/D(C_i - C_\mu)$ the potential change on the MC. The shot noise is as usual $S_{II} = e|V|(e^2/2\pi)\sum R_n T_n$. Using weak coupling $C_1, C_2 \ll C_i$ one gets for the inverse measurement time

$$\tau_m^{-1} = \Gamma_m = 2\pi\left(\frac{C_\mu}{C_i}\right)^2 R_m e|V| \tag{7}$$

with the dimensionless constant

$$R_m = \frac{1}{16\pi^2}\frac{1}{(\mathrm{Tr}N)^2}\frac{(\sum \frac{dT_n}{dE})^2}{(\sum R_n T_n)}. \tag{8}$$

The energy derivatives dT_n/dE express the sensitivity of the conductance to a potential variation ΔU on the MC $[d/dE = -\partial/\partial(eU)]$. If this sensitivity is high, the MC is a fast measuring device. This fact is reflected by an important inequality between measurement rate and decoherence rate $\Gamma_{\rm dec} \geq \Gamma_m$ and $R_v \geq R_m$, respectively. To prove this inequality we introduce the matrices $A = s_{11}^\dagger s_{12}$ and $B = 2\pi i N_{12}$. Through this transformation we obtain a Cauchy-Schwarz inequality between R_v and R_m,

$$\mathrm{Tr}(AA^\dagger)\mathrm{Tr}(BB^\dagger) \geq |\mathrm{Tr}(AB^\dagger)|^2 \geq (\mathrm{Re}\mathrm{Tr}(AB^\dagger))^2. \tag{9}$$

Which conditions are needed to get the equality $\tau_m = \Gamma_{\rm dec}^{-1}$? The tunneling between the two double dots during the measurement must be negligible, $\Delta \simeq 0$, and the temperature must be much smaller than the applied voltage $kT \ll e|V|$. More interesting are constraints imposed on the symmetry of the scattering matrix by inequality (9): the matrices A and B must be linearly dependent. Using the polar decomposition of the scattering matrix [20], we find that this excludes channel mixing except for some singular cases. It implies that the scattering matrix can be written in block form,

$$s^{(n)} = \begin{pmatrix} -i\sqrt{R_n}e^{i(\phi_n+\phi_{A,n})} & \sqrt{T_n}e^{i(\phi_n-\phi_{B,n})} \\ \sqrt{T_n}e^{i(\phi_n+\phi_{B,n})} & -i\sqrt{R_n}e^{i(\phi_n-\phi_{A,n})} \end{pmatrix}. \tag{10}$$

Each block is defined by its transmission probability $T_n = 1 - R_n$ and three scattering phases ϕ_n, $\phi_{A,n}$, $\phi_{B,n}$. Using the definition of R_v [Eq. (2)] we arrive at [17]

$$R_v = \frac{\sum_n (\frac{1}{4T_nR_n}(\frac{dT_n}{dE})^2 + T_nR_n(\frac{d\phi_{A,n}}{dE} + \frac{d\phi_{B,n}}{dE})^2)}{(\sum_n \frac{d\phi_n}{dE})^2}. \tag{11}$$

Equation (11) can be connected to earlier results [1,7,8] in the infinite capacitance limit where $C_\mu^2 R_v$ in Eq. (4) can be replaced by $D^2 R_v$. Demanding equality of Eqs. (8) and (11) leads to additional constraints [let the MC be defined by an equilibrium electrostatic potential $V(x, y, z)$ open in z direction and confined in the xy plane].

(i) In order to have $d\phi_{B,n}/dE = 0$ in Eq. (11) the scattering Hamiltonian must obey time-reversal symmetry.

(ii) Furthermore, the derivatives $d\phi_{A,n}/dE$ have to vanish. This can be the case accidentally but is always fulfilled for symmetric detectors that obey an inversion symmetry $V(x, y, z) = V(x, y, -z)$. This condition is well known; see, for instance, [15]. In a spacially asymmetric detector, part of the information about the state of the DD is transferred to the phase of the scattered electrons. This phase does not influence a conductance measurement.

(iii) In the multichannel case $N > 1$ another condition is needed. The equality $R_m = R_v$ then implies that

$$\frac{dT_n/dE}{R_nT_n} = C(E). \tag{12}$$

The function $C(E) > 0$ does not depend on the index n. This restriction is of statistical origin: The total conductance of the detector is a sum of one channel conductances that have independent uncertainties. Under condition (12) the statistical uncertainty of their sum is minimized.

We now discuss some examples. The simplest case is that of a tunnel contact without channel mixing: In this case the probabilities T_n are dominated by the action in the forbidden region and Eq. (12) is independent of the channel number. Such a tunnel barrier is an efficient detector but has the drawback that its measurement time is long.

Detectors with shorter measurement times can be achieved in structures with higher transparencies. For such structures the condition Eq. (12) is now important. It can be interpreted as a differential equation for the transmission probabilities T_n. Their solutions are all of the form $T_n = [1 + e^{-[F(E)-F(E_n)]}]^{-1}$ with $dF/dE = C$. The only difference allowed between the different probabilities T_n is the offset energy E_n. Transmission probabilities of the type (12) thus occur automatically if the scattering problem is separable due to a potential of shape

$$V(x, y, z) = Z(z) + W(x, y). \tag{13}$$

This occurs, e.g., for the case $F = 2\pi E/\omega_z$ with a symmetric harmonic scattering potential $Z(z) = V_0 - m\omega_z^2 z^2/2$.

We demonstrate the importance of our findings with a generic model that includes channel mixing and violates condition (13). This condition states that a geometry with a separable potential is favorable to obtain an efficient detector in the case of more than one open channel. It is clear that a chaotic potential violates condition (13) by definition. We expect therefore that chaos reduces drastically the efficiency $\Gamma_m/\Gamma_v = R_m/R_v$ of a detector with two open channels, but has little impact on a detector with only one open channel.

To check this expectation we use a common model of a chaotic cavity coupled to a left lead with N_1 channels and

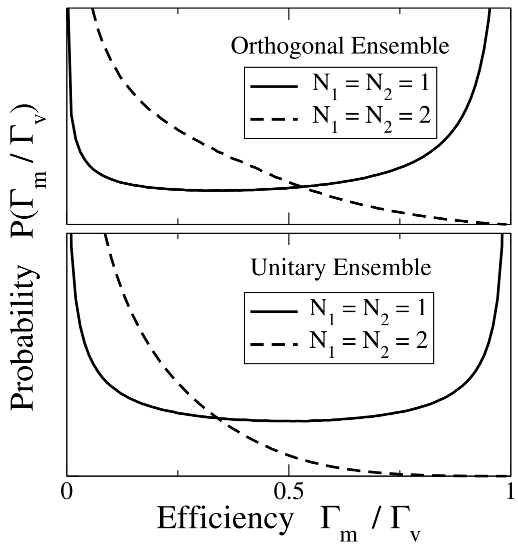

FIG. 2. Efficiency distribution of an ensemble of chaotic quantum cavity detectors: orthogonal ensemble (top panel) and unitary ensemble (lower panel) for single channel ($N_1 = N_2 = 1$) and double channel ($N_1 = N_2 = 2$) point contacts.

a right lead with N_2 channels: it can be described by a scattering matrix belonging to the circular ensemble of random matrix theory [20]. The distribution of the density of states matrix elements (1) is also known [21]. Using these distributions we obtain the probability distribution of the measurement efficiency R_m/R_v,

$$P\left(\frac{R_m}{R_v}\right) = \int ds dN_E \delta\left[\frac{R_m}{R_v} - \frac{R_m(s, N_E)}{R_v(s, N_E)}\right]. \qquad (14)$$

In Eq. (14) ds is a measure for the circular ensemble of scattering matrices and dN_E a measure for the symmetrized density of states matrix $N_E = s^{-1/2}(ds/dE)s^{-1/2}/2\pi i$. It turns out that the ratio R_m/R_v depends only on the eigenvectors of N_E and the scattering matrix, but not on the eigenvalues of N_E, the inverse dwell times τ_n^{-1}. The distribution of R_m/R_v is therefore the same in the canonical ($C \ll D$) and grand-canonical ensemble ($C \gg D$); see [22]. Figure 2 shows the distribution of the measurement efficiency R_m/R_v in the orthogonal (time-reversal symmetry) and unitary ensemble (broken time-reversal symmetry). The distributions were obtained by numerical integration. The distribution for $N_1 = N_2 = 1$ in the unitary ensemble can also be calculated analytically to be $P(R_m/R_v) = [R_m/R_v(1 - R_m/R_v)]^{-1/2}$. Surprisingly, despite the absence of inversion symmetry, a chaotic dot with open single channel contacts is with high probability an efficient detector. It is clearly visible that chaos reduces strongly the efficiency of the measurement device as soon as more than one channel contributes to the electric transport. The reduction due to a broken time-reversal symmetry is much less pronounced.

In this work we have analyzed coherent multichannel mesoscopic conductors with the aim to find both fast and efficient detectors. We find a new statistical condition necessary to carry out a quantum-limited measurement. This condition relates sensitivities and shot noises of different conductance channels. It leads us to a class of detectors (defined by separable potentials) that are both fast and efficient. We have assumed that a change in the state of the system causes only a small change in the potential landscape of the detector. Only under this condition is it possible to describe the detector response with the help of small differential changes of the scattering matrix and linear screening. We leave it as a future challenging problem to develop a theory of phase-coherent, nonlinear mesoscopic detectors.

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