Nonlinear Theory of the Ablative Rayleigh-Taylor Instability

J. Sanz, J. Ramírez, and R. Ramis
E.T.S.I. Aeronáuticos, Universidad Politécnica de Madrid, Madrid 28040, Spain

R. Betti and R. P. J. Town
Laboratory for Laser Energetics, University of Rochester, Rochester, New York 14623
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A fully nonlinear sharp-boundary model of the ablative Rayleigh-Taylor instability is derived and closed in a similar way to the self-consistent closure of the linear theory. It contains the stabilizing effect of ablation and accurately reproduces the results of 2D DRACO simulations. The single-mode saturation amplitude, bubble and spike evolutions in the nonlinear regimes, and the seeding of long-wavelength modes via mode coupling are determined and compared with the classical theory without ablation. Nonlinear stability above the linear cutoff is also predicted.

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The Rayleigh-Taylor instability (RTI) has great relevance in inertial confinement fusion (ICF) and astrophysics [1]. In ICF, the imploding shell is accelerated inward by the low density ablating plasma and the outer shell surface is unstable to the RTI. Mass ablation is caused by the heat front propagating through the shell and driven by the laser energy absorbed at the critical surface.

It has long been known [2] that mass ablation reduces the growth rate of the RTI in the linear regime. However, only recently [3] has a self-consistent linear theory for subsonic ablation flows identified the physical mechanisms of stabilization. These are clearly revealed for ablation fronts with large Froude numbers typical of direct drive ICF capsules with cryogenic deuterium-tritium (DT) ablators. Here the Froude number is a dimensionless parameter $F_r = V_a^2/gL_a$, where g is the inward target acceleration, L_a is the characteristic thickness of the ablation front, and V_a is the ablation velocity of the material with density ρ_a. The average mass ablation rate is $\dot{m}_{\rm av} \equiv \rho_a V_a$. For perturbations with wave number $k = 2\pi/\lambda$, the ablative RTI growth rate for $\mathrm{Fr} \gg 1$ is $\gamma \approx \sqrt{(2kV_a)^2 + kg - k^2V_a^2/\mu_k} - 2kV_a$, where $\mu_k = (2kL_a/n)^{1/n}$, n being the exponent in the Spitzer thermal conductivity ($\kappa_{\rm Sp} = \overline{K}T^n$). The cutoff wave number is estimated from $k_c L_a \approx \mu_{kc}/\mathrm{Fr} \ll 1$, indicating that the unstable spectrum consists only of long-wavelength modes. The density defined as $\rho_k \equiv \mu_k \rho_a$ has the meaning of the hot blowoff density at a distance $(2k)^{-1}$ from the ablation front. Because of heat flux, the high blowoff velocity ($\sim V_a/\mu_k$) produces restoring forces, similar to the rocket effect. The ablation and the vorticity generated through the ablation front damp the growth, giving the $-2kV_a$ term. A simple linear sharp-boundary model (SBM) with two incompressible fluids, of densities ρ_a and $\rho_h \ll \rho_a$, was developed [4] (large n or dominant heat conduction region underlies such approximation), which reproduces the same results when the interface is approximated by an isotherm, and ρ_h is substituted for ρ_k, for each k-Fourier mode of the perturbation. The use of the SBM with this self-consistent closure (SCC) has proved to be very fruitful in linear theory [5].

Attempts have been made to infer the physics of the nonlinear RTI at ablation fronts [6]. These theories used heuristic approaches leading to inaccurate results. Here we present a fully nonlinear model based on the recently developed 2D-SBM. It is derived from first principles and closed with an approximation similar to the SCC of the linear theory. It is applicable for ablation fronts with large Fr, when the cutoff of the unstable spectrum occurs for long-wavelength perturbations, $kL_a \ll 1$. As the model correctly captures the physics of the ablative stabilization, it is a basic tool to study many aspects of the single-mode nonlinear ablative RTI and multimode interaction including, self-consistently, nonlinear ablation effects.

For simplicity, we consider a planar foil of thickness ℓ, subject to an acceleration g due to the ablation pressure P_a generated by the heat flux coming from the corona: P_a, g, V_a, and ℓ are assumed constant over the characteristic RTI time scale. Attention is restricted to a region of characteristic thickness k_c^{-1} about the ablation front, such that $L_a \ll k_c^{-1} \ll \ell$, and let $\varepsilon^2 \equiv k_c V_a^2/g$ ($\approx \mu_{kc} \ll 1$). In the frame (orthonormal vectors $\vec{e}_x$, $\vec{e}_y$, $\vec{e}_z$) moving with the unperturbed ($y = 0$) ablation front (acceleration $-g\vec{e}_y$), we use one-fluid equations for an ideal gas with heat conduction [3]: continuity, momentum, and an isobaric approximation (subsonic flow) for the energy

$$\begin{aligned} &\partial_t \rho + \nabla \cdot (\rho \vec{v}) = 0, \\ &\rho(\partial_t + \vec{v}\cdot\nabla)\vec{v} = -\nabla p + \rho g \vec{e}_y, \\ &\nabla \cdot (\tfrac{5}{2} P_a \vec{v} - \overline{K} T^n \nabla T) \approx 0, \\ &P = \rho T \approx P_a, \end{aligned} \tag{1}$$

where ρ, $\vec{v}$, P, and T are density, velocity, pressure, and temperature, respectively, $p \equiv P - P_a$ is the perturbed pressure ($|p| \ll P_a$), $\overline{K}$ is a constant, and no motion occurs in the z direction [$\vec{r} = (x, y)$]. The thickness of the ablation front is neglected and the ablation surface, arbitrarily parametrized, is at $\vec{r}_a = [\eta(\alpha, t), \xi(\alpha, t)]$. The solution in the two regions is matched using mass $\dot{m} \equiv \rho \vec{v}' \cdot \vec{n}$, momentum $p\vec{n} + \dot{m}\vec{v}'$, and energy $\frac{5}{2} P_a \vec{v}' \cdot \vec{n} - \overline{K} T^n \nabla T \cdot \vec{n}$ fluxes conservation; $\vec{t}$ and $\vec{n}$ are the tangent and normal (towards the expanding plasma) unit vectors at the interface, and $\vec{v}'$ is the fluid velocity relative to it ($\vec{v}' \cdot \vec{n} \equiv \vec{n} \cdot \vec{v} - \vec{n} \cdot \partial_t \vec{r}_a$). The momentum flux conservation yields the continuity of both the tangential velocity, $\vec{v} \cdot \vec{t}$, and $q \equiv p + \dot{m}^2/\rho$. The solution in the dense region (D_c) is simplified by neglecting the heat flux and assuming a potential flow with constant density ρ_a and velocity $\vec{v}_c = \nabla\phi + V_a \vec{e}_y$. The perturbed velocity and pressure variations are of order $\sqrt{g/k_c}$ and $\rho_a g/k_c$, respectively. Since $\dot{m} \sim O(\rho_a V_a)$, we have $p \simeq q$ on the cold side of the interface. Hence, the fluid motion is described by the Laplace equation for ϕ, a kinematic equation for the interface, and the Bernoulli equation

$$\Delta\phi = 0 \quad (x, y \in D_c), \qquad \nabla\phi = 0 \ \text{ at } y = -\infty, \tag{2}$$

$$\vec{n} \cdot \partial_t \vec{r}_a = \vec{v}_c \cdot \vec{n} - \dot{m}/\rho_a \ \text{ at } \vec{r} = \vec{r}_a, \tag{3}$$

$$\partial_t \phi \simeq g\xi - \tfrac{1}{2}(\nabla\phi)^2 - V_a \partial_y \phi - q/\rho_a \ \text{ at } \vec{r} = \vec{r}_a. \tag{4}$$

Equations (2)–(4) are the time evolution equations of the ablating surface; q and $\dot{m}$ are determined below. Setting $q = \dot{m} = V_a = 0$, (2)–(4) describe the classical RTI with Atwood number unity. Next, we have the hot blowoff region (D_h), with density $\rho_h \sim \varepsilon^2 \rho_a$, velocity $v_h \sim V_a/\varepsilon^2$, and perturbed pressure $p_h \sim \rho_h v_h^2$. Integrating the energy equation in (1), we get $\vec{v}_h \simeq \vec{v}_r + \dot{m}_{\text{av}} \nabla\theta/\rho_h$, $\vec{v}_r$ being its rotational part and $\theta \equiv 2\overline{K}T^n/5n\dot{m}_{\text{av}}$. The vorticity, $\omega \vec{e}_z = \nabla \times \vec{v}_r$, is generated at the ablation front and convected to the underdense region. Now, using $\vec{v}_h$ and taking the ablation surface as an isotherm, the continuity of the energy flux and of the tangential velocity leads to $\vec{v}_c = \vec{v}_r$ at the interface. This allows us to assume that $\vec{v}_r$ remains small [$\sim O(\varepsilon)$] compared with the potential part ($\propto \nabla\theta$), throughout the D_h region. Then, substituting for $\vec{v}_h$ the velocity in the continuity Eq. (1), neglecting $O(\varepsilon/n)$ terms, one finds the following eigenvalue problem:

$$\Delta\theta \simeq 0, \qquad (x, y \in D_h), \tag{5}$$

$$\theta(\vec{r} = \vec{r}_a, t) \simeq 0, \qquad \partial_y \theta \simeq 1 \ \text{ at } y = \infty. \tag{6}$$

The latter of Eqs. (6) corresponds to a uniform flow in the far blowoff region. The solution of (5) and (6), taking into account energy conservation through the interface, yields the mass ablation rate

$$\dot{m} \equiv \rho_h \vec{v}_h' \cdot \vec{n}|_a \approx \dot{m}_{\text{av}} \nabla\theta \cdot \vec{n}|_a; \tag{7}$$

because of the faster ablation at the peaks with respect to the bottom of bubbles, the instability growth is slowed down. Finally, the description of the hot region is completed by using $\nabla \cdot \vec{v}_r = 0$, and momentum equation. The latter is simplified by using the incompressible flow approximation which holds as long as the power index for thermal conduction n is well above unity [4,5]. Using $\vec{v}_h$, the momentum equation, up to $O(\varepsilon)$, is $\rho_h \vec{v}_h \times \omega \vec{e}_z \simeq \nabla(\frac{1}{2}\rho_h v_h^2 + p_h + \dot{m}_{\text{av}} \partial_t \theta)$. Its integration along the interface ($\vec{t} \cdot \nabla \equiv \partial_s$), inside the hot region, yields $p_h \approx -\dot{m}^2/2\rho_h + \int \dot{m}\omega\, ds$, where the second term is an indefinite integral along the interface and s is the arclength. Then, $q \simeq (\dot{m}^2 - \dot{m}_{\text{av}}^2)/2\rho_h + \int \dot{m}\omega\, ds$, with the constant $\dot{m}_{\text{av}}^2/2\rho_h$ introduced for convenience. The leading term of the so-called rocket effect, q, is the stabilizing restoring force. The vorticity equation derived from the above momentum equation (2D flow) simplifies to $\vec{v}_h \cdot \nabla\omega \simeq 0$. It follows, neglecting $O(\varepsilon)$ in $\vec{v}_h$, that $\nabla\theta \cdot \nabla\omega \approx 0$, indicating that ω depends only on χ(harmonic conjugate function of θ). Setting $\vec{v}_r = \partial_y \psi \vec{e}_x + (V_a - \partial_x \psi)\vec{e}_y$ leads to the following eigenvalue problem for the stream function ψ in the D_h region: $\Delta\psi = -\omega(\chi)$ with ψ bounded, and $\vec{v}_c = \vec{v}_r$ at $\vec{r} = \vec{r}_a$. One can get the solution by the conformal map $(x, y) \rightarrow (\chi, \theta)$ and the k-Fourier transform on the χ coordinate ($\dot{m}_{\text{av}} d\chi = \dot{m} ds$). Then, the vorticity is obtained from the following linear integral equation:

$$\int_{-\infty}^{\infty} \omega\, d\chi \int_0^{\infty} \frac{e^{-|k|\theta - ik\chi} d\theta}{|\nabla\theta|^2} = \int_{-\infty}^{\infty} \frac{\nabla\phi \cdot (ik\vec{n} + |k|\vec{t})}{|k|(\dot{m}/\dot{m}_{\text{av}})} \times e^{-ik\chi}\, d\chi, \tag{8}$$

which in linear theory ($\theta \approx y$, $\chi \approx x$, $\eta = \alpha = x$) reproduces the vorticity obtained in self-consistent theory, $\omega = 2\partial_{xy}^2 \phi|_{y=0} \approx 2\partial_{xt}^2 \xi$ [3,4]. The model would be closed if in the leading term $[q_L \equiv (\dot{m}^2 - \dot{m}_{\text{av}}^2)/2\rho_h]$ of q the density ρ_h were specified. In the SBM linear theory ($\chi \approx x$) the self-consistent closure is introduced in Fourier space [4,5]. That is, let $\dot{m} = \dot{m}_{\text{av}} + \delta\dot{m}$, $\delta\dot{m}$ being a small perturbation, and $\delta\dot{m}_k = F(\delta\dot{m}) \equiv \int_{-\infty}^{\infty} \delta\dot{m} e^{-ik\chi} d\chi$ its k-Fourier transform on the χ. Then, the linearized leading restoring force, $\dot{m}_{\text{av}} \delta\dot{m} \rho_h^{-1}$, is substituted in Fourier space for $\delta q_{Lk} = \dot{m}_{\text{av}} \delta\dot{m}_k \rho_k^{-1}$. It follows that in physical space $\delta q_L = \dot{m}_{\text{av}} F^{-1}(\delta\dot{m}_k \rho_k^{-1})$, where F^{-1} is the inverse Fourier transform operator. Having the unstable spectrum modes with $kL_a \ll 1$ only, the contribution to that integral of the short wavelength modes, $kL_a > 1$, is negligible. Hence, it is unmodified if ρ_k^{-1} is redefined as $\rho_k^{-1} = \rho_a^{-1} \mu_k^{-1} H(1 - |k|L_a)$, where H is the Heaviside function, and defining $L_a = 2\overline{K}T_a^n/5\dot{m}_{\text{av}}$ as usual [3–5]. In this way, δq_L is a convolution product, $\dot{m}_{\text{av}}(\delta\dot{m} * \rho_{bl}^{-1}) \equiv \dot{m}_{\text{av}} \int_{-\infty}^{\infty} \delta\dot{m}(\chi', t) \rho_{bl}^{-1}(\chi - \chi') d\chi'$, where $\rho_{bl}^{-1}(\chi) = F^{-1}(\rho_k^{-1})$. Its simpler extension to nonlinear theory, which fulfills the requirements of reproducing the linear case, is performed by estimating the blowoff density, ρ_*, with the local characteristic thickness of the front $[\equiv 2\overline{K}T_a^n/5\dot{m} \equiv (\dot{m}/\dot{m}_{av})L_a]$ instead of L_a. Thus $\rho_*^{-1} = \sigma^{1/n} \rho_k^{-1}$, where $\sigma \equiv \dot{m}/\dot{m}_{\text{av}}$. Such a density will be lowered in the vicinity of the tip of the spike, with respect to the density ρ_k in linear theory. Then, the rule to get q_L is applied in a similar way, and the total rocket effect is

$$q \simeq \dot{m}_{\rm av}^2[(\sigma^2 - 1)\sigma^{1/n}] * (2\rho_{bl})^{-1} + \dot{m}_{\rm av} \int \omega \, d\chi. \quad (9)$$

For a mode with wave number k, Eqs. (2)–(9) reproduce the linear results [4,5]: setting $\xi = \xi_{kL}(t)\cos kx$, we get $\dot{m} \simeq \dot{m}_{\rm av}(1 + k\xi_{kL}\cos kx)$, etc., and then $d_t^2 \xi_{kL} \simeq -4kV_a d_t \xi_{kL} + kg(1 - \tilde{k})\xi_{kL}$, where $\tilde{k} = (k/k_c)^{1-1/n}$.

Our model is supported by the agreement with the results of the 2D DRACO [7] simulations. Now, we investigate the nonlinear evolution of an initially small perturbation, $\xi_1(0)\cos kx$. Laplace equations (2) and (5) are solved by means of a Fredholm integral equation of first kind [8]. Then we get $\vec{n} \cdot \nabla\phi$ on the interface and σ. Equation (8) is solved by expanding ω in Fourier series on χ, $\sum \omega_j e^{ijk\chi}$. Then, Eqs. (3) and (4) are numerically integrated with the orthogonal gauge $\vec{t} \cdot \partial_t \vec{r}_a = 0$; for each time step, we need $\dot{m}$, ω, and $q \sim \sum[(\sigma^2 - 1)\sigma^{1/n}]_j \cos(jk\chi)/j^{1/n}$, where $[\cdots]_j$ means the j-Fourier coefficient. In Fig. 1 we compare the model with the simulations. Dimensionless bubble and spike amplitudes versus time are shown for two wavelengths. The target was a 100 μm cryogenic (DT) foil irradiated by 0.35 μm laser light with an intensity of 5×10^{13} W/cm^2. The initial front-surface amplitude is taken as 0.1 μm. The parameters $n \simeq 2.1$ and $L_a \simeq 0.04$ μm were determined as in Ref. [9]. The ablation front was defined as the isodensity contour at some fraction $f = 0.2$ of the maximum density. Notice that both isodensity and isothermal contours overlap at the ablation front as the pressure varies slowly through the front. The results are not sensitive to the choice of f because the density profile is very sharp at the ablation front. After a transient time of 1.6 ns in which Richtmyer-Meshkov oscillations [5] take place, the foil starts to accelerate with the average values of $g \simeq 60$ μm/ns^2 and $V_a \simeq 3.3$ μm/ns ($F_r \simeq 4.5$). The cutoff wavelength predicted by linear theory [3,4] and simulation is about 10 μm. Equations (3) and (4) were integrated with a small enough initial amplitude to have a wide time interval still in linear regime where $\xi_1(t) \propto e^{\gamma t}$. Its time origin was shifted in order to compare with the linear amplitude of the numerical simulation after a transient time, and once the RTI has taken place. Large amplitudes are well reproduced by the model as well as the shape of the ablation front ($t \approx 4.6$ ns, $\lambda = 50$ μm). The good agreement confirms the accuracy of the theory. The shapes obtained from the model for 4.1 and 4.9 ns are also shown. The tip of the spike is flattened and widened due to the dynamical overpressure and consequently the shape becomes multivalued. The spike-bubble asymmetry begins at bigger amplitude than that of the classical RTI due to the nonlinear stabilizing ablation effects. This is more noticeable for shorter wavelengths.

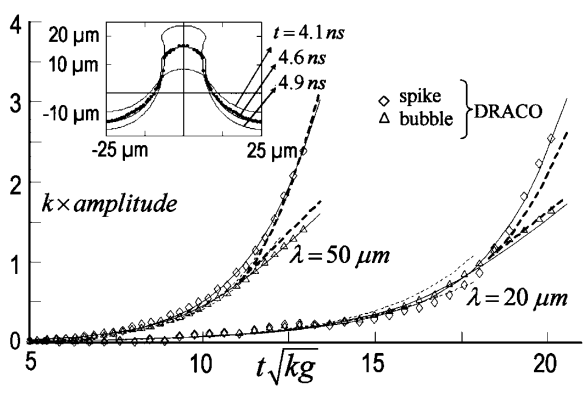

FIG. 1. Spike and bubble amplitudes versus time. Also the shapes of the ablation front (inset) are plotted ($\lambda = 50$ μm) and compared with that of the DRACO simulation (dots) at $t = 4.6$ ns. Thin and thick dashed lines correspond to the Eqs. (10) and (12), respectively.

Next we determine the essential nonlinear features of the ablative RTI: bubble-spike asymmetries, nonlinear coupling, bubble asymptotic velocity, and nonlinear single-mode saturation. In classical RTI theory [10], during the early nonlinear evolution, the higher harmonics are produced with phases enhancing the spike growth over the bubble growth and initiating the bubble-spike asymmetry. In the ablative RTI theory such a physical picture is somewhat different. In a weakly nonlinear analysis [$\xi = \sum \xi_j \cos(jkx)$] up to third order, the amplitudes of the fundamental mode ξ_1, second, and third harmonics are significantly modified respect to the classical RTI amplitudes (details will be published elsewhere). Here we report the final results for $k < k_c$

$$\begin{aligned} \xi_3 &\approx \tfrac{1}{8}(1 - 4\tilde{k})(3 - 4\tilde{k})k^2\xi_{kL}^3, \\ \xi_2 &\approx (1/2 - \tilde{k})k\xi_{kL}^2, \\ \xi_1 &\approx \xi_{kL} - \frac{(2 - \tilde{k})(1 - 2\tilde{k})}{8(1 - \tilde{k})}k^2\xi_{kL}^3, \end{aligned} \quad (10)$$

where ξ_{kL} is the linear amplitude. In the limit case $k/k_c \to 0$, the results of classical RTI [10] are recovered. It follows from (10) that the occurrence of negative or positive feedback to the harmonics depends, contrary to the classical RTI, on the wave number. The feedback to the fundamental mode is null when $\tilde{k} = 1/2$ leading to $k \simeq 0.27k_c$ for $n = 2.1$. Regarding the spectrum of generated modes, the following feature is roughly noticed: the portion of spectrum with j harmonic above k_c/k has the sign opposite to that of the first harmonic. Looking at the spike, $a_s \approx \xi_{kL} + \xi_2$, or bubble, $a_b \approx \xi_{kL} - \xi_2$, incipient amplitudes, the asymmetry of the shape is reduced or even inverted if $\tilde{k} > 1/2$. The thin dashed lines in Fig. 1 show the bubble and spike amplitudes obtained from (10). Observe that in the 20 μm case ($\tilde{k} > 1/2$), the highest one corresponds to the bubble. The same feature, though weaker, can be noticed in both the full nonlinear model and the simulation results in the interval $15 < t\sqrt{kg} < 17.5$. Another important aspect concerns the generation of long-wavelength modes via mode coupling. When two modes are initially present, nonlinear interaction leads to the appearance, in particular, of the beat mode $k_{12} \equiv k_1 - k_2$. The most relevant is the beating of modes with wave numbers about the wave number for maximum growth rate. At second order, Eqs. (2)–(9) yield their amplitude as

$$\xi_{k_{12}} \approx -\tfrac{1}{4}|k_{12}|(1 + \tfrac{3}{8}|\tilde{k}_{12}|^{-1/n})\xi_{k_1 L}\xi_{k_2 L}, \qquad (11)$$

indicating that the seeding of long-wavelength modes, $|k_{12}| \ll k_c$, is enhanced with respect to the classical case ($\propto |k_{12}|$) [6]. This is an important result as long-wavelength modes cause macroscopic distortion of the imploding shell leading to nonuniform compression of the hot spot.

A Layzer-type approach [11] yields the asymptotic bubble velocity $V_{b0} \approx \sqrt{g/3k} - V_a$ ($\sqrt{g/3k}$ is the Layzer asymptotic velocity for classical RTI). Observe that, contrary to the results of Oron [6], the bubbles with wavelength $\lambda < 6\pi F_r L_a$ are smoothed out by ablation. The same type of approach gives us the asymptotic spike acceleration $g_s \approx g[1 + V_a/V_{b0} + g\tilde{k}/(4kV_{b0}^2)]^{-1}$. A useful estimate of bubble-spike amplitudes in the nonlinear regime can be obtained by instantaneously switching the nonlinear evolution [11] from exponential growth to constant velocity bubble (or free fall), at a time t_0 in which the linear velocity equals the asymptotic bubble velocity

$$a_b \simeq V_{b0}(\gamma^{-1} + t - t_0), \qquad a_s \simeq a_b + \tfrac{1}{2}g_s(t - t_0)^2. \qquad (12)$$

In such expressions (thick dashed lines in Fig. 1), $S(k) \equiv \gamma^{-1}V_{b0}$ is usually defined as the saturation amplitude. The classical saturation amplitude of 0.1λ is recovered when the long-wavelength limit $k/k_c \to 0$ is considered. However, for shorter wavelength modes well into the ablative regime, the saturation amplitude is significantly different from the classical prediction. Values of 0.15λ and 0.2λ are obtained for 50 and 20 μm wavelength perturbations, respectively (in agreement with the simulations), indicating that mass ablation extends the linear growth phase with respect to the classical case. This result is particularly important in light of the fact that the output of one dimensional ICF implosion simulations are commonly postprocessed using Rayleigh-Taylor models based on the classical linear saturation amplitude of 0.1λ. For wavelengths shorter than 20 μm, our model predicts saturation amplitudes up to 0.3λ near the cutoff wavelength (~ 12 μm). Simulations near the cutoff are difficult to perform as small variations from steady state lead to oscillations in the cutoff wavelength. However, a 14 μm wavelength simulation has clearly indicated a saturation amplitude slightly exceeding 0.2λ (that is, somewhat less than the theoretical prediction of 0.25λ).

Finally, we show some analytical results for $k \geq k_c$. About the equation governing $\xi_{kL}(t)$, the periodic small perturbation $\xi = \xi_{kL}(0)\cos k_c x$ is an equilibrium shape. Then, a steady equilibrium, $\xi_e(x) = \sum \xi_{ej}\cos(jkx)$, with fundamental wave number k should have a bifurcation at $k = k_c$. Setting $\partial_t = 0$ in Eqs. (3) and (4) a weakly nonlinear analysis close to k_c yields ξ_{ej} for $\tilde{k} > 1$: $\xi_{e1} \approx \pm k^{-1}(\tilde{k} - 1)^{1/2}$, $\xi_{e2} \approx -k\xi_{e1}^2/4$, etc. Obviously such a bifurcated equilibrium is unstable. Hence, contrary to the common wisdom, for perturbations with $k > k_c$, the ablation surface may become nonlinearly unstable if the initial amplitude is outside an attraction basin, for instance, if $|k\xi_1(0)| > \sqrt{\tilde{k} - 1}$. In fact, at the cutoff k_c, any finite perturbation is indeed unstable, and its slow growth [approximately linear in time for $|k_c\xi_1(0)| \ll 1$], outside the scope of simulations, has a rate $\gamma_c \approx [k_c\xi_1(0)]^2 g/4V_a$.

In conclusion, a fully nonlinear SBM of the ablative RTI accounting ablation effects is developed. It reproduces the results of simulation codes and sheds new light on the nonlinear ablation physics. The model is a basic tool that is expected to cause further understanding of this problem.

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