Universal Measurement Apparatus Controlled by Quantum Software

Jaromír Fiurášek, Miloslav Dušek, and Radim Filip
Department of Optics, Palacký University, 17. listopadu 50, 772 00 Olomouc, Czech Republic
(Received 26 February 2002; published 16 October 2002)

We propose a quantum device that can approximate any projective measurement on a qubit—a quantum "multimeter." The desired measurement basis is selected by the quantum state of a "program register." Two different kinds of programs are considered and in both cases the device is optimized with respect to maximal average fidelity (assuming uniform distribution of measurement bases). Quantum multimeters exhibiting the covariance property are introduced and an optimal covariant multimeter with a single-qubit program register is found. Possible experimental realization of the simplest proposed device is presented.

DOI: 10.1103/PhysRevLett.89.190401 PACS numbers: 03.67.Lx, 03.65.Wj, 42.50.Ct

Programmable quantum "multimeters" are devices that can realize any desired generalized quantum measurement from a chosen set (either exactly or approximately) [1]. Their main feature is that the particular positive operator valued measure (POVM) is selected by the quantum state of a "program register"(quantum software). In this sense they are analogous to universal quantum processors [2–4]. Quantum multimeters could play an important role in quantum state estimation and quantum information processing.

In this Letter, we will describe a programmable quantum device that can approximately accomplish *any* projective von Neumann measurement on a single qubit. Since it is impossible to encode an arbitrary unitary operation (acting on a finite-dimensional Hilbert space) into a state of a finite-dimensional quantum system [2] it is also impossible to encode arbitrary projective measurement on a qubit into such a state [1]. However, it is still possible to encode POVMs that represent, in a certain sense, the best approximation of the required projective measurements.

Suppose we want to measure a qubit in the basis represented by two orthogonal vectors $|\psi\rangle$ and $|\psi_\perp\rangle$. We want this measurement basis to be controlled by the quantum state of a program register, $|\phi_p(\psi)\rangle$. An *ideal* multimeter would map the composite state of the measured system and the program register to two fixed orthogonal pure states $|0\rangle$ and $|1\rangle$ according to

$$|\psi\rangle \otimes |\phi_p(\psi)\rangle \rightarrow |0\rangle, \qquad |\psi_\perp\rangle \otimes |\phi_p(\psi)\rangle \rightarrow |1\rangle. \quad (1)$$

As mentioned above, such a transformation cannot be implemented exactly. Thus, our task is to find a realistic linear trace-preserving completely positive (CP) map that represents the closest approximation to this nonrealistic map. We focus on the scenario when we always obtain one of the two measurement results $|0\rangle$ or $|1\rangle$, but errors, i.e., deviations from the ideal map (1), may appear. Our aim is to minimize the probability of error; i.e., we will maximize the probability of the correct discrimination between states $|\psi\rangle$ and $|\psi_\perp\rangle$. In general we could optimize both the program and the fixed transformation so as to optimally approximate the map (1) for a given dimension of the program register. However, this is an extremely hard problem that we will not attempt to solve in its generality. Instead, we optimize the fixed transformation for two natural choices of the program.

First we assume that the program register contains N copies of the state $|\psi\rangle$, $|\phi_p(\psi)\rangle = |\psi\rangle^{\otimes N}$. Our second choice of the program—the two-qubit state $|\psi\rangle|\psi_\perp\rangle$—is motivated by recent results on optimum quantum state estimation. Gisin and Popescu showed that the state of two orthogonal qubits $|\psi\rangle|\psi_\perp\rangle$ encodes the information on the state $|\psi\rangle$ better than state of two identical qubits $|\psi\rangle|\psi\rangle$ [5]. If we possess one copy of the state $|\psi\rangle|\psi_\perp\rangle$, then we can estimate $|\psi\rangle$ with fidelity $\mathcal{F}_\perp = (1 + 1/\sqrt{3})/2 \approx 0.7887$ which is slightly higher than the fidelity of the optimal estimation on one copy of two identical qubits, $\mathcal{F}_{||} = 3/4$ [5,6]. One would thus expect that the state $|\psi\rangle|\psi_\perp\rangle$ should also give an advantage when used as a program of the multimeter. Rather surprisingly, this is not the case and we shall see that it is better to use two identical qubits $|\psi\rangle|\psi\rangle$.

In what follows we benefit from the isomorphism between CP maps and bipartite positive semidefinite operators [7,8]. Let $\mathcal{H}$ and $\mathcal{K}$ denote the Hilbert spaces of input and output states, respectively. Choose basis $|i\rangle$ in $\mathcal{H}$, define a maximally entangled state $\sum_i |i\rangle|i\rangle$ on $\mathcal{H}^{\otimes 2}$, and apply the CP map to one part of this state. The density matrix χ of the resulting state on Hilbert space $\mathcal{H} \otimes \mathcal{K}$ represents the CP map and the relation between input and output density matrices reads [8]

$$\rho_{\rm out} = {\rm Tr}_{\rm in}[\chi \rho_{\rm in}^T \otimes \mathbb{1}_{\rm out}], \quad (2)$$

where T stands for the transposition in the basis $|i\rangle$ and $\mathbb{1}$ denotes an identity operator. The CP map is trace preserving if the positive semidefinite operator χ satisfies the condition ${\rm Tr}_{\rm out}[\chi] = \mathbb{1}_{\rm in}$.

Let us define the fidelity $F(\psi)$ of our multimeter projecting onto states $|\psi\rangle$ and $|\psi_\perp\rangle$ as the probability that a correct measurement result will be obtained when we send the states $|\psi\rangle$ or $|\psi_\perp\rangle$ to the input randomly each with probability one-half. This fidelity can be interpreted as a success rate of the discrimination between two orthogonal states $|\psi\rangle$ and $|\psi_\perp\rangle$. Assuming the program state to be $|\psi\rangle^{\otimes N}$, the two relevant input states of the multimeter read

$$|\Psi\rangle = |\psi\rangle \otimes |\psi\rangle^{\otimes N}, \qquad |\Psi_\perp\rangle = |\psi_\perp\rangle \otimes |\psi\rangle^{\otimes N}. \tag{3}$$

The input Hilbert space of the multimeter is a tensor product of the Hilbert space of signal qubit $\mathcal{H}_s$ and symmetric (bosonic) subspace $\mathcal{H}_+^N$ of the Hilbert space of N qubits, $\mathcal{H} = \mathcal{H}_s \otimes \mathcal{H}_+^N$ and $\dim\mathcal{H} = 2(N+1)$. A trace-preserving completely positive map χ transforms the input states onto the states of a single output qubit that is subsequently measured in the computational basis. The outcome $|0\rangle$ corresponds to the projection onto $|\psi\rangle$ while $|1\rangle$ is associated with the projection onto $|\psi_\perp\rangle$. Making use of the input-output relation (2) we have

$$F(\psi) = \tfrac{1}{2}\mathrm{Tr}[\chi(|\Psi\rangle\langle\Psi|)^T \otimes |0\rangle\langle 0|] + \tfrac{1}{2}\mathrm{Tr}[\chi(|\Psi_\perp\rangle\langle\Psi_\perp|)^T \otimes |1\rangle\langle 1|]. \tag{4}$$

The figure of merit that we maximize is the mean fidelity obtained on averaging $F(\psi)$ over all pure qubit states $|\psi\rangle$, i.e., over the surface of the Bloch sphere,

$$F = \int_\psi d\psi F(\psi) = \mathrm{Tr}[\mathrm{R}\chi]. \tag{5}$$

The positive semidefinite operator R reads

$$R = R_0^T \otimes |0\rangle\langle 0| + R_1^T \otimes |1\rangle\langle 1|, \tag{6}$$

where the operators R_0 and R_1 acting on the input Hilbert space $\mathcal{H}$ are given by integrals

$$R_0 = \frac{1}{2}\int_\psi d\psi |\Psi\rangle\langle\Psi|, \qquad R_1 = \frac{1}{2}\int_\psi d\psi |\Psi_\perp\rangle\langle\Psi_\perp|. \tag{7}$$

A straightforward calculation reveals that R_0 is proportional to the projector onto symmetric subspace $\mathcal{H}_+^{N+1}$ of the Hilbert space of $N+1$ qubits,

$$R_0 = \frac{1}{2(N+2)}\Pi_+^{(N+1)}. \tag{8}$$

Furthermore, the sum of operators R_0 and R_1 is proportional to the identity operator on the input Hilbert space, $R_0 + R_1 = \mathbb{1}_{\mathrm{in}}/[2(N+1)]$. Thus we immediately have

$$R_1 = \frac{1}{2(N+1)}\mathbb{1}_{\mathrm{in}} - \frac{1}{2(N+2)}\Pi_+^{(N+1)}. \tag{9}$$

The determination of the optimum CP map amounts to the maximization of the linear function (5) under the constraints $\chi \geq 0$ and $\mathrm{Tr}_{\mathrm{out}}[\chi] = \mathbb{1}_{\mathrm{in}}$. The optimum CP map that maximizes the mean fidelity (5) must satisfy the extremal equations [8,9]

$$(\lambda \otimes \mathbb{1}_{\mathrm{out}} - R)\chi = 0, \tag{10}$$

$$\lambda \otimes \mathbb{1}_{\mathrm{out}} - R \geq 0, \tag{11}$$

where λ is a positive definite operator on the input Hilbert space. Notice that the extremal equations (10) and (11) resemble the Helstrom equations for optimal POVM that maximizes the success rate in ambiguous quantum state discrimination [10]. One can prove that if both Eqs. (10) and (11) are satisfied, then χ is indeed an optimal CP map and F attains its global maximum on the convex set of trace-preserving CP maps [9,11].

Because of the specific structure of the operator R we can, without any loss of generality, assume that the optimal CP map has the structure

$$\chi = \Pi_0^T \otimes |0\rangle\langle 0| + \Pi_1^T \otimes |1\rangle\langle 1| \tag{12}$$

and the optimal transformation on the signal and program states is a joint two-component generalized measurement described by the POVM whose two elements Π_0 and Π_1 are positive semidefinite operators summing up to the identity operator. The two outcomes Π_0 and Π_1 form the two possible outputs of the multimeter. On inserting the CP map (12) into Eq. (5) we find that we have to maximize the fidelity $F = \mathrm{Tr}[R_0\Pi_0 + R_1\Pi_1]$ under the constraints $\Pi_j \geq 0$, $\Pi_0 + \Pi_1 = \mathbb{1}_{\mathrm{in}}$. Formally, this is equivalent to the problem of optimum discrimination between two mixed quantum states R_0 and R_1 which was solved by Helstrom [10]. We must find the eigenstates and eigenvalues of the operator $\Delta R = R_0 - R_1$ and then Π_0 and Π_1 are the projectors onto subspaces spanned by the eigenstates with positive and negative eigenvalues, respectively.

It turns out that the optimum POVM is formed by the projector onto symmetric subspace of the $N+1$ qubits and its orthogonal counterpart,

$$\Pi_0 = \Pi_+^{(N+1)}, \qquad \Pi_1 = \mathbb{1}_{\mathrm{in}} - \Pi_+^{(N+1)} \equiv \Pi_-^{(N+1)}. \tag{13}$$

On inserting the expressions (6) and (12) into Eq. (5), we obtain the mean fidelity

$$F = \frac{2N+1}{2N+2}. \tag{14}$$

Taking into account the trace-preservation condition $\mathrm{Tr}_{\mathrm{out}}[\chi] = \mathbb{1}_{\mathrm{in}}$, we find from Eq. (10) that $\lambda = \mathrm{Tr}_{\mathrm{out}}[R\chi]$. After some algebra we arrive at

$$\lambda = \frac{1}{2(N+1)}\mathbb{1}_{\mathrm{in}} - \frac{1}{2(N+1)(N+2)}\Pi_+^{(N+1)}. \tag{15}$$

It is easy to check that the first extremal equation (10) is satisfied for the POVM (13). The inequality (11) splits into two independent inequalities for operators acting on input Hilbert space, $\lambda - R_0^T \geq 0$ and $\lambda - R_1^T \geq 0$. One can easily verify that these two inequalities are satisfied which proves the optimality of the POVM (13).

We can now determine the effective POVM carried out on the signal qubit,

$$\begin{aligned}\Pi_{||} &= \mathrm{Tr}_p[\mathbb{1}_s \otimes (|\psi\rangle\langle\psi|)^{\otimes N}\,\Pi_+^{(N+1)}],\\ \Pi_\perp &= \mathrm{Tr}_p[\mathbb{1}_s \otimes (|\psi\rangle\langle\psi|)^{\otimes N}\,\Pi_-^{(N+1)}],\end{aligned} \tag{16}$$

where Tr_p denotes trace over the program qubits. The outcome $\Pi_\perp$ cannot occur if the input state is $|\psi\rangle$ because the input state $|\Psi\rangle$ belongs to the symmetric subspace of $N+1$ qubits and $\Pi_{||}$ clicks with certainty. Hence the POVM element $\Pi_\perp$ must be proportional to the projector

$|\psi_\perp\rangle\langle\psi_\perp|$. Since the sum of POVM elements (16) is an identity operator, we have the following ansatz:

$$\Pi_{\|} = |\psi\rangle\langle\psi| + (1-p)|\psi_\perp\rangle\langle\psi_\perp|, \quad \Pi_\perp = p|\psi_\perp\rangle\langle\psi_\perp|. \tag{17}$$

The probability p that $\Pi_\perp$ clicks when the input state is $|\psi_\perp\rangle$ is given by $p = \langle\Psi_\perp|\Pi_-^{(N+1)}|\Psi_\perp\rangle$. After some algebra we get $p = N/(N+1)$ and the effective POVM representing our universal multimeter reads

$$\Pi_{\|} = \frac{1}{N+1}\mathbb{1} + \frac{N}{N+1}|\psi\rangle\langle\psi|, \quad \Pi_\perp = \frac{N}{N+1}|\psi_\perp\rangle\langle\psi_\perp|. \tag{18}$$

Notice that the POVM (18) is asymmetric, which reflects the asymmetry of the program register. Furthermore, the fidelity $F(\psi)$ is independent of ψ and equal to the mean fidelity (14). In the limit of an infinitely large program register ($N \to \infty$), the POVM (18) approaches the ideal projective measurement.

The program $|\psi\rangle^{\otimes N}$ exhibits an important covariance property. First note that the measurement basis $\{|\psi\rangle, |\psi_\perp\rangle\}$ can be obtained from the computational basis $\{|0\rangle, |1\rangle\}$ via a unitary transformation $U_2(\psi)$ that belongs to the two-parametric subset of the SU(2) group that is holomorphic to the surface of the Bloch sphere and parametrized by two Euler angles θ and φ: $U_2(\psi)|0\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$, $U_2(\psi)|1\rangle = -e^{-i\varphi}\sin\frac{\theta}{2}|0\rangle + \cos\frac{\theta}{2}|1\rangle$. Similarly, the program $|\psi\rangle^{\otimes N} \in \mathcal{H}_+^N$ is related to the program $|\phi_{\rm p}(0)\rangle \equiv |0\rangle^{\otimes N}$ via unitary transformation $U_{N+1}(\psi)$ that belongs to the two-parametric subset of the $N+1$ dimensional irreducible representation of SU(2).

We define *covariant multimeters* as all the multimeters that satisfy the property

$$|\phi_{\rm p}(\psi)\rangle = U_{N+1}(\psi)|\phi_{\rm p}(0)\rangle. \tag{19}$$

The restriction to the two-parametric subset of the SU(2) group guarantees that the program state $|\phi_{\rm p}(\psi)\rangle$ is uniquely defined for all $|\psi\rangle$ and $|\phi_{\rm p}(0)\rangle$. The covariance property means that we can change the measurement basis simply by properly rotating the quantum state of the program register. For covariant multimeters, the problem of simultaneous optimization of the program and the fixed measurement reduces to the determination of a *single* optimum program state $|\phi_{\rm p}(0)\rangle \equiv |\phi_0\rangle$. The operators R_0 and R_1 for the program (19) read

$$R_0(\phi_0) = \frac{1}{2}\int_\psi d\psi|\psi\rangle\langle\psi| \otimes U_{N+1}(\psi)|\phi_0\rangle\langle\phi_0|U_{N+1}^\dagger(\psi),$$
$$R_1(\phi_0) = \frac{1}{2}\int_\psi d\psi|\psi_\perp\rangle\langle\psi_\perp| \otimes U_{N+1}(\psi)|\phi_0\rangle \times \langle\phi_0|U_{N+1}^\dagger(\psi).$$

Once we determine $R_0(\phi_0)$ and $R_1(\phi_0)$ we can express the maximum achievable fidelity in terms of the sum of absolute values of the eigenvalues $\lambda_j(\phi_0)$ of the operator $\Delta R = R_0(\phi_0) - R_1(\phi_0)$ [10],

$$F(\phi_0) = \frac{1}{2} + \frac{1}{2}\sum_j |\lambda_j(\phi_0)|. \tag{20}$$

We must find the maximum of $F(\phi_0)$ over all possible programs $|\phi_0\rangle = \cos\frac{\theta_0}{2}|0\rangle + e^{i\varphi_0}\sin\frac{\theta_0}{2}|1\rangle$. We have performed explicit calculations for a single-qubit program register, $N = 1$, and found an analytic expression for the mean fidelity,

$$F = \frac{1}{2} + \frac{1}{24\sqrt{2}}\sqrt{25 + 7\cos(2\theta_0)} + \frac{1}{24}\left|\cos\theta_0 - \frac{1}{2}\sin\theta_0\right| + \frac{1}{24}\left|\cos\theta_0 + \frac{1}{2}\sin\theta_0\right|. \tag{21}$$

It is optimal to set $\theta_0 = 0$, i.e., to choose $|\phi_0\rangle = |0\rangle$ and the optimum program of the covariant multimeter with a single-qubit register is given by $|\phi_{\rm p}(\psi)\rangle = |\psi\rangle$. We can conjecture that the multimeters with the program $|\psi\rangle^{\otimes N}$ are optimum covariant multimeters also for $N > 1$.

We now turn our attention to the program $|\psi\rangle|\psi_\perp\rangle$. The optimum CP map for this program can be found following the same procedure as described above for the program $|\psi\rangle^{\otimes N}$. Briefly, one has to calculate the operator R and solve extremal equations (10) and (11). We will not give the details of calculations here and present only the results. Similarly as before, the optimum CP map is equivalent to a generalized measurement on the signal qubit and two program qubits. The two elements of this three-qubit POVM read

$$\Pi_0 = \tfrac{1}{2}\Pi_+^{(3)} + |\phi_1\rangle\langle\phi_1| + |\phi_2\rangle\langle\phi_2|, \quad \Pi_1 = \mathbb{1}_3 - \Pi_0, \tag{22}$$

where $\mathbb{1}_3$ is an identity operator on Hilbert space of three qubits and

$$|\phi_1\rangle = \frac{1}{2\sqrt{3}}[(\sqrt{3}+1)|0\rangle_{\rm s}|01\rangle_{\rm p} - (\sqrt{3}-1)|0\rangle_{\rm s}|10\rangle_{\rm p} - 2|1\rangle_{\rm s}|00\rangle_{\rm p}],$$
$$|\phi_2\rangle = \frac{1}{2\sqrt{3}}[(\sqrt{3}+1)|1\rangle_{\rm s}|10\rangle_{\rm p} - (\sqrt{3}-1)|1\rangle_{\rm s}|01\rangle_{\rm p} - 2|0\rangle_{\rm s}|11\rangle_{\rm p}]. \tag{23}$$

Here the subscripts "s" and "p" label the states of signal and program qubits, respectively. After some algebra, we find the effective POVM carried out on the signal qubit,

$$\Pi'_{\|} = \frac{3-\sqrt{3}}{6}\mathbb{1} + \frac{\sqrt{3}}{3}|\psi\rangle\langle\psi|, \quad \Pi'_\perp = \frac{3-\sqrt{3}}{6}\mathbb{1} + \frac{\sqrt{3}}{3}|\psi_\perp\rangle\langle\psi_\perp|. \tag{24}$$

This POVM is symmetric (reflecting the symmetry of the program $|\psi\rangle|\psi_\perp\rangle$). The fidelity $F(\psi)$ is state independent and equal to the mean fidelity

$$F' = \frac{1}{2}\left(1 + \frac{1}{\sqrt{3}}\right). \tag{25}$$

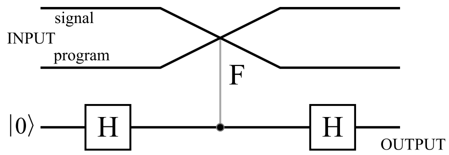

FIG. 1. Quantum circuit representing simple programmable multimeter: H—Hadamard gate; F—Fredkin gate.

Notice that $F' = \mathcal{F}_{\perp}$. This is not a mere coincidence; the optimum strategy for program $|\psi\rangle|\psi_{\perp}\rangle$ is to carry out an optimal estimation of $|\psi\rangle$ and then measure the signal qubit in the basis formed by estimated state $|\psi_{\rm est}\rangle$ and its orthogonal counterpart. The POVM (22) is an explicit implementation of this procedure. We emphasize here that F' is a maximum fidelity attainable with program $|\psi\rangle|\psi_{\perp}\rangle$, because the corresponding CP map solves the extremal equations (10) and (11). With the program $|\psi\rangle|\psi\rangle$ we achieve the fidelity $5/6 \approx 0.8333$ which is higher than $F' \approx 0.7886$; hence the program $|\psi\rangle|\psi\rangle$ exhibits better performance than $|\psi\rangle|\psi_{\perp}\rangle$.

It is important that the proposed universal measurement devices can be realized experimentally. As an example, we now describe a possible realization of the simplest one that uses a single-qubit program register. Such a device can be built up from one Fredkin (controlled swap) and two Hadamard gates [12] as shown in Fig. 1. Signal and program enter the Fredkin gate as "controlled" qubits; the result can be read out from the ancilla serving as a "control" qubit. There are several ways how to implement the presented scheme on real physical systems [13]. Very recently, an experimental implementation of the scheme shown in Fig. 1 on an NMR quantum computer has been reported [14].

There is also a very promising way of the quantum-optical implementation of the simplest multimeter. For a single-qubit program, the optimum measurement on the program and data qubits is the projection onto symmetric and antisymmetric subspaces spanned by singlet and triplet Bell states, respectively. The incomplete Bell state analysis distinguishing between triplet and singlet polarization states of two photons can be performed with just a single beam splitter and two photodetectors [15–17]. These kinds of measurements have already been successfully carried out in the experiments on dense coding [18] and quantum state teleportation [19]. This simple optical scheme has recently been implemented experimentally in our laboratory [20].

In summary, we have investigated the quantum multimeters that can approximate any projective measurement on a single qubit. The main feature of the multimeters is that the measurement basis is selected by the quantum state of the program register. We have considered two different kinds of programs encoding the basis $\{|\psi\rangle, |\psi_{\perp}\rangle\}$: first we have considered the program $|\psi\rangle^{\otimes N}$ and then the program $|\psi\rangle|\psi_{\perp}\rangle$. In both cases we have determined the optimal multimeter that maximizes the average fidelity. Remarkably, it turns out that the program $|\psi\rangle|\psi\rangle$ leads to higher average fidelity than $|\psi\rangle|\psi_{\perp}\rangle$.

Generally, one would like to optimize both the program and the joint measurement on the program and signal registers simultaneously in order to determine the truly optimal multimeter. This seems to be a very hard but interesting problem that deserves further investigation. Here we have made the first steps in this direction. We have introduced covariant multimeters whose programs are mutually related via unitary transformations and we have found the truly optimal covariant multimeter with a single-qubit program register. Finally, we have pointed out that there is a very simple quantum-optical implementation of the simplest multimeter with a single-qubit program.

In this Letter we have investigated the multimeters that always provide one of the two possible measurement outcomes, but errors may occur. Note that, alternatively, one can consider a probabilistic multimeter that performs the exact projective measurements but with the probability of success lower than 1. Such a multimeter would be conceptually analogous to the probabilistic programmable quantum gates [2–4].

This research was supported under Project No. LN00A015 of the Ministry of Education of the Czech Republic.

[1] M. Dušek and V. Bužek, Phys. Rev. A **66**, 022112 (2002).
[2] M. A. Nielsen and I. L. Chuang, Phys. Rev. Lett. **79**, 321 (1997).
[3] G. Vidal *et al.*, Phys. Rev. Lett. **88**, 047905 (2002).
[4] M. Hillery *et al.*, Phys. Rev. A **65**, 022301 (2002).
[5] N. Gisin and S. Popescu, Phys. Rev. Lett. **83**, 432 (1999).
[6] S. Massar, Phys. Rev. A **62**, 040101(R) (2000).
[7] A. Jamiołkowski, Rep. Math. Phys. **3**, 275 (1972).
[8] J. Fiurášek, Phys. Rev. A **64**, 062310 (2001).
[9] K. Audenaert and B. De Moor, Phys. Rev. A **65**, 030302(R) (2002).
[10] C.W. Helstrom, *Quantum Detection and Estimation Theory* (Academic Press, New York, 1976).
[11] J. Fiurášek *et al.*, Phys. Rev. A **65**, 040302(R) (2002).
[12] A. K. Ekert *et al.*, Phys. Rev. Lett. **88**, 217901 (2002).
[13] R. Filip, Phys. Rev. A **65**, 062320 (2002).
[14] X. Fei *et al.*, quant-ph/0204049.
[15] H. Weinfurter, Europhys. Lett. **25**, 559 (1994).
[16] S. L. Braunstein *et al.*, Phys. Rev. A **51**, R1727 (1995).
[17] S.M. Barnett, A. Chefles, and I. Jex, quant-ph/0202087.
[18] K. Mattle *et al.*, Phys. Rev. Lett. **76**, 4656 (1996).
[19] D. Bouwmeester *et al.*, Nature (London) **390**, 575 (1997).
[20] M. Hendrych *et al.*, quant-ph/0208091.