Does the Reverse Brazil Nut Problem Exist?

In a recent Letter, Hong *et al.* [1] report the interesting numerical observation that a crossover from the Brazil nut problem (BNP) to the reverse BNP (RBNP) might be achieved under a condensation-driven segregation mechanism. The starting point of their theory is their previous observation [2] that there exists a critical temperature, T_c, below which a monodisperse system of hard spheres condenses in the presence of gravity. They obtain as T_c the expression $mgd\mu/\mu_0$, where m and d are, respectively, the mass and diameter of the elastic hard spheres considered, and μ and μ_0 are quantities related to the filling height of the column and the packing structure. Then, they consider a binary mixture of hard spheres with species A and B having mass m_A and m_B and diameters d_A and d_B, respectively. By altering the values of mass and diameters, they are able to invert the segregation of species from the BNP to the RBNP, not only in 2D but also in 3D.

In their molecular dynamics computational experiments, Hong *et al.* [1] test their theory by starting the binary system from a completely mixed state, at high temperature and no gravity. Thereafter, they turn on gravity and quench the system to a temperature T lower than $T_c(B)$ but higher than $T_c(A)$ [where the following relationship holds: $T_c(A)m_Bd_B = T_c(B)m_Ad_A$]. The system is then allowed to relax to a steady state. In a phase diagram, where in the vertical axis $y = d_A/d_B$ and in the horizontal axis $x = m_A/m_B$, the crossover line from the BNP to the RBNP found by the authors is a quadratic curve $y^2 = x$ (for 3D).

The BNP states that in a binary mixture of spheres, the ones with larger diameters segregate to the top when subjected to vibrations or shaking. This phenomenon has been corroborated not only by many computational experiments, but also in the laboratory. In several experiments carried out by our group we confirm indeed this observation, not only for a single particle as originally pointed out by Rosato *et al.* [3], but also for a bidisperse mixture.

However, after many experiments carried out to verify the RBNP observed by Hong *et al.* [1], we do not find evidence of this phenomenon. We prepare our experiments under the same conditions observed by the authors: at $\mu_A = \mu_B$, at different points in the proposed phase diagram ($x \geq 6$ and $y \leq 2$; see Fig. 4b in [1]), with different containers, and at different "temperatures" (frequencies and amplitudes of vertical vibrations), following their proposed prescription. Hong *et al.* define T_c as the temperature where the system becomes fully fluid [2]. With this definition, we experimentally obtain $T_c(A)$ and $T_c(B)$ for monodisperse species A and B. Thereafter, we run the experiment at different temperatures between $T_c(A)$ and $T_c(B)$. Invariably, regardless how much time we run the experiments, we get a mixed state. While in the counterpart problem, the normal BNP, the tendency of segregation is clearly observed as soon as the experiment begins; in the RBNP this tendency never shows up. An obvious criticism [4] is that molecular dynamics (MD) simulations and the experiment are difficult to compare since the former are carried out at a global temperature, but a temperature gradient exists along the vertical in our granular vibrating bed. However, even when this is true, all temperatures within this gradient are also contained between both condensation temperatures (as a matter of fact, as long as a fraction of a solid and liquid coexists, this requirement is fulfilled). During the experiments, since T is always greater than $T_c(B)$, the smaller particles behave as a fluid and the larger and heavier ones spread out around the bed compelled by the stream of this fluid. This is the crucial point, in the MD simulations the drag exerted by the fluid is not taken into account; rather, they consider this drag does not exist at all since the fluid is considered to be a nonconvective phantom phase in an ideal container. Hong *et al.* assume that the bidisperse granular system they study could be seen as a binary system where opposite particles do not feel one another and they refer to this assumption as a crude one. In fact, it is difficult to think of a binary system of particles behaving the way they propose.

The effect reported by Hong *et al.* is an interesting segregation effect in an ideal bidisperse mixture, where only similar particles see each other and have the same thermal energy given computationally. We do not aim to refute what they observe in their ideal conditions, but only point out that the RBNP was not observed in real conditions in the laboratory.

This work has been partially supported by Conacyt, México, under Grant No. 36256E.

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Received 14 December 2001; published 9 October 2002
DOI: 10.1103/PhysRevLett.89.189601
PACS numbers: 64.70.Dv, 05.20.Dd, 51.10.+y

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