Spin-Zero Sound in One- and Quasi-One-Dimensional ^{3}He

E. S. Hernández

Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, 1428 Buenos Aires and Consejo Nacional de Investigaciones Científicas y Técnicas, Argentina

(Received 12 December 2001; published 11 October 2002)

The zero sound spectrum of fluid ^{3}He confined to a cylindrical shell is examined for configurations characterizing strictly one-dimensional and quasi-one-dimensional regimes. It is shown that the restricted dimensionality makes room to the possibility of spin-zero sound for the attractive particle-hole interaction of liquid helium. This fact can be related to the suppression of phase instabilities and thermodynamic phase transitions in one dimension.

DOI: 10.1103/PhysRevLett.89.185301 PACS numbers: 67.20.+k, 67.70.+n, 67.55.-s, 67.90.+z

It is common wisdom that bulk liquid ^{3}He does not exhibit longitudinal spin-zero sound. This is an experimental fact whose physical ground, according to Landau's theory of Fermi liquids, is the weak attractiveness of the effective particle-hole (ph) interaction in the spin channel of bulk helium. Theoretical descriptions of spin-density fluctuations of finite wavelength, carried within polarization potential (PP) theory [1], the random-phase approximation (RPA) in the monopole model [2] and finite range density functional (FRDF) theory [3] do support the experimental data. The restriction does not hold for transverse spin-zero sound (Silin waves), whose prediction in the late 1950s [4] was experimentally confirmed after several years [5]. It has been also shown that in partially polarized ^{3}He, for any finite value of the magnetization, the effective ph interaction derived by double functional differentiation of the FRDF acquires sufficient repulsive intensity to build up two collective oscillations of the spin density, both with substantial strength [6]. This picture is further enriched with the prediction of a dispersion relation for transverse spin-zero sound, in the same density functional frame [7].

The purpose of this Letter is to examine a dimensionality effect which may show up in one- and quasi-one-dimensional (1D and Q1D) confinements, as those taking place in the adsorbing field of a curved surface such as a graphite nanotube [8] or another cylindrical pore. It will be shown that stable spin oscillations of the zero sound type may take place in 1D and Q1D ^{3}He, and that this possibility is related to a feature of the free Lindhard function as spatial dimensions are suppressed. In the cylindrical environment, the wave functions of free ^{3}He quasiparticles (qp's) take the form

$$\Phi_{kl}(\mathbf{r}) = \frac{1}{\sqrt{2\pi R}} f_0(r) e^{i(kz+l\varphi)}. \tag{1}$$

Here R, L, ($R \ll L$) are the radius and length of the cylinder, and $f_0(r)$ is the ground state for the transverse radial motion. For strong adsorbing potentials, the latter is narrow enough to be represented as a delta function restricting the atoms to the cylindrical surface, and the first radial excited state lies much higher than the threshold for angular motion, $\varepsilon_1 = \hbar^2/(2mR^2)$ (see, e.g., Ref. [9]). The wave function (1) corresponds to a band spectrum $\varepsilon_{kl} = \hbar^2 k^2/2m + \varepsilon_l$, being $\varepsilon_l = \varepsilon_1 l^2$, which according to Fermi statistics, at zero temperature is populated up to a Fermi energy ε_F related to the linear atom density by

$$\rho_1 = \frac{N}{L} = \frac{2}{\pi} \sum_l k_{F_l} \Theta(\varepsilon_F - \varepsilon_l), \tag{2}$$

where $k_{F_l} = \sqrt{2m(\varepsilon_F - \varepsilon_l)/\hbar^2}$ is the Fermi momentum for each occupied angular momentum band (hereafter, a Fermi segment), and $\Theta(x)$ is the step function. Thus, as linear density increases, the gas of qp's experiences a crossover between the strictly 1D and the 2D geometries, through a Q1D regime which takes place when a few Fermi segments of length $2k_{Fl}$ are populated.

For this noninteracting gas, the 1D Lindhard function at zero temperature reads [10]

$$\chi_0^{1\mathrm{D}}(q,\omega) = \nu_F^{1\mathrm{D}} \frac{k_F}{q} \left\{ \ln\left[\frac{\omega^2 - \omega_-^2}{\omega_+^2 - \omega^2}\right] - i\pi[\Theta(\omega_+ - \omega) - \Theta(\omega_- - \omega)] \right\}, \tag{3}$$

where $\nu_F^{1\mathrm{D}} = m/(2\pi\hbar^2 k_F)$ is the 1D density of states at the Fermi level per spin degree of freedom, and

$$\omega_\pm = \pm \frac{\hbar}{2m} q^2 + q v_F. \tag{4}$$

The real part of this response is regular within the ph band $\omega_- \leq \omega \leq \omega_+$ and diverges at the edges, while the strength on the 1D-ph continuum $S_0 = -\mathrm{Im}\chi_0/\pi$ is a constant equal to $\nu_F^{1\mathrm{D}} k_F/q$ for energies belonging to the ph band. For very small q, S_0 becomes

$$S_0 \approx \nu_F^{1\mathrm{D}} k_F \delta(\omega - q v_F), \tag{5}$$

which is the characteristic Luttinger liquid behavior [11].

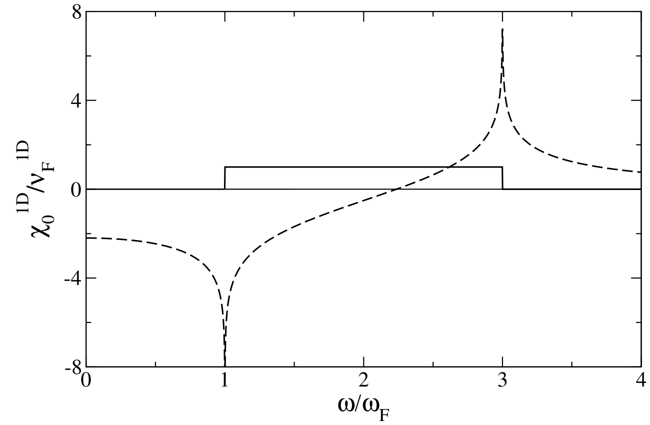

FIG. 1. Dynamical susceptibility of a 1D Fermi gas as a function of reduced energy for finite transferred momentum $q = k_F$. Full and dashed lines, respectively, correspond to $-\mathrm{Im}\chi_0^{1D}/\pi\nu_F^{1D}$ and $\mathrm{Re}\chi_0^{1D}/\nu_F^{1D}$.

This susceptibility is displayed in Fig. 1 for finite transferred momentum q. The real part presents opposite signs on either side of the nonvanishing continuum strength, which opens the possibility, for *attractive* effective interactions $V_{ph}(q, \omega)$, of generating low energy collective oscillations, i.e., to satisfy the simple RPA dispersion relation in the monopolar approximation

$$\varepsilon(q, \omega) = 1 - V_{ph}(q, \omega)\chi_0^{1D}(q, \omega) = 0. \quad (6)$$

In fact, such a possibility may occur in higher dimensional, homogeneous ^{3}He. The concentration of continuum ph strength between two finite, positive energy edges is a well-known property of 2D and 3D Fermi gases for transferred momentum larger than $2k_F$. This is depicted in Fig. 2, where the real and imaginary part of the 2D and 3D dimensionless Lindhard functions are plotted for $q = 2.3k_F$. We realize that in the low energy region $\omega < \omega_-$, the dielectric function $\varepsilon(q, \omega)$ in Eq. (6) may vanish for a negative value of V_{ph} such that $\nu_F^D|V_{ph}| > 1$. However, this is an undesirable situation, since the fluid would be unstable against such oscillations.

The evolution of the Lindhard function with decreasing dimension is more clearly visualized in the Landau limit; i.e., both q and ω approach zero with finite reduced phase velocity $s = \omega/qv_F$, $|s| \leq 1$. This is shown in Fig. 3, where the real and imaginary parts of the dimensionless Landau susceptibilities $\Omega_0^D(s) = -\lim_{q,\omega\to 0} \chi_0^D/\nu_F^D$ are displayed as functions of s for $D = 1$ to 3, with $D = 1$ corresponding to the Luttinger liquid [11].

Two facts are visible in this picture. First, we appreciate the progressive expulsion of low energy continuum strength towards the Fermi surface, due to the reduction in ph momentum space associated to the suppression of one or both angular degrees of freedom. Second, we realize that in the 1D geometry, while a repulsive interaction always gives rise to a collective oscillation above

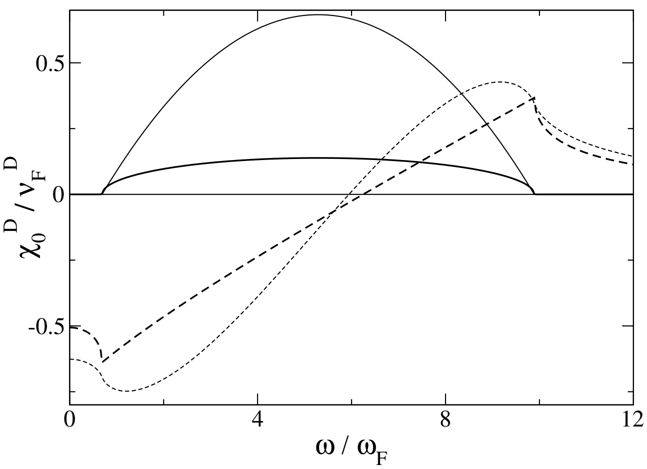

FIG. 2. Same as Fig. 1 for a 2D and a 3D Fermi gas (thick and thin lines, respectively) at transferred momentum $q = 2.3k_F$.

the ph continuum, stable low energy collective poles may also appear for sufficiently weak attraction, $\nu_F^D|V_{ph}| < 1$. This is possible due to the change in curvature of $\mathrm{Re}\Omega_0^D$ at the origin.

To fix ideas, hereafter I focus on a specific, although largely simplified, description of ^{3}He bound to a cylindrical surface of radius R at given linear density ρ_1. I have chosen to take into account the gross features of ph

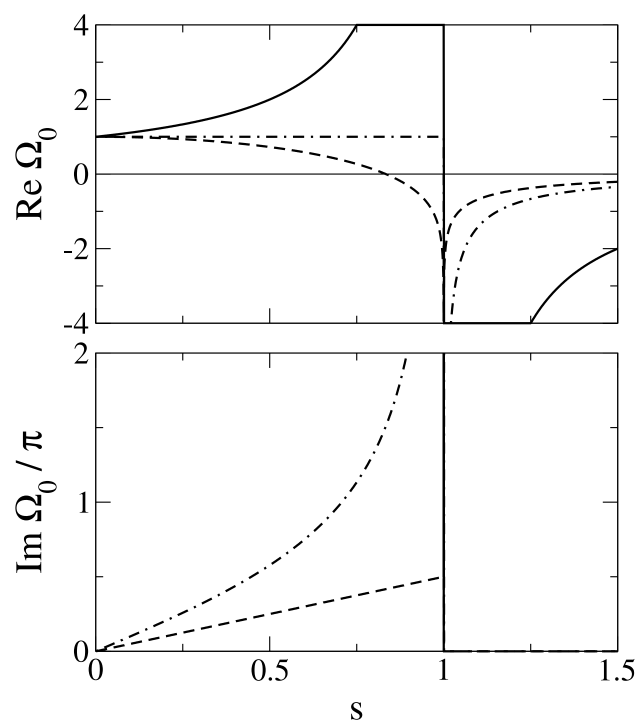

FIG. 3. The real (upper frame) and imaginary (lower frame) parts of the dynamical response of a Fermi gas in the Landau limit as a function of the reduced phase velocity. Full, dot-dashed, and dashed lines, respectively, correspond to the 1D, 2D, and 3D cases.

propagation in either the symmetric (s) or the antisymmetric (a) spin channel by means of polarization potential theory [1]. In other words, the spectrum of collective excitations is given by Eq. (6) with an effective qp interaction

$$V_{ph}(q,\omega) = f_\sigma(q) + (\tfrac{\omega}{q})^2 h_\sigma(q), \tag{7}$$

where σ stands for s or a, $f_\sigma(q)$ is the monopolar self-consistent field, which at infinite wavelength provides the monopole Landau parameter F_0^σ, and $h_\sigma(q)$ incorporates backflow effects so as to account for the momentum-dependent effective mass and for the magnetic Landau parameter F_1^a.

In the present model, the restricted geometry has been incorporated as follows. The self-consistent fields are the projections of the spatial pseudopotentials [1] onto the components $e^{i(kz+l\varphi)}$ of the excitation operator. In Ref. [12] the backflow field h_s is parametrized as a function of transferred momentum and h_a is a constant in momentum space. These fields have been Fourier antitransformed in 3D to obtain their spatial representation. Note that the particles see each other in three-dimensional space, i.e., the distance between interacting qp's is $r = [4R^2\sin^2(\varphi/2) + z^2]^{1/2}$. The parameters and strengths are taken at bulk saturation [12]. In fact, although meaningful linear densities for the fluid under consideration correspond to 3D figures well below saturation, PP's are not known for such dilute systems. Several questions may be raised concerning the validity of the picture itself; one could reasonably expect that core suppression—an essential ingredient in pseudopotential theory—is sensitive to dimensionality. Indeed, a reduction in the number of nearest neighbors of an atom would remove short-range screening and enhance the strength of the repulsion, thus of the attraction in the spin channel. Although apparently this issue has not been explored yet, it is plausible that the qualitative features to be discussed below are robust against fine improvements in the effective ph interaction, for the particular restricted geometry here adopted.

As a first illustration, a cylindrical shell of ^{3}He atoms with radius $R = 4$ Å and linear coverage $\rho_1 = 0.1$ Å^{-1} is investigated. Taking into account the backflow field $h_s(q)$, the momentum-dependent effective mass is, in 1D,

$$m_q^* = m + \rho_1 h_s(q). \tag{8}$$

This formula generalizes in a straightforward manner the expression derived in 3D PP theory [1]. One finds $m_0^* = 1.29m$, m_q^* approaching unity monotonically with increasing momentum. For this cylinder, the threshold ε_1 for angular momentum excitations is 0.5 K if the bare ^{3}He mass is considered. The momentum dependence of the effective mass permits to push the first bandhead downwards to 0.38 K; however, within a sizeable range of wavelengths, this band never becomes populated for linear densities below 0.1 Å^{-1}, and the system remains 1D. The dispersion relations numerically obtained solving Eq. (6) are displayed in Fig. 4, where we can appreciate that in addition to the density-zero sound mode, which departs from the ph continuum with phase velocity around $3.4v_F$, a low energy spin-density oscillation appears, which evolves into a damped rotonlike minimum at a transferred momentum near 0.45 Å^{-1}. The phase velocity of this second mode, which in addition, carries collective strength around 10 times that of the density-density mode, is about one half the Fermi velocity.

For higher coverages, one cannot expect a single shell monolayer for the adsorbed fluid. However, in a wider pore the angular spectrum may become sufficiently compressed, so that the fluid shell covers more than one Fermi segment and can be regarded as a Q1D system. Even for a nanotube of standard size, say, around 7 Å, the gas adsorbed on the outer surface would lie at a radius near 10 Å, which makes room to significant Q1D structure and thermodynamics even at coverages slightly above 0.1 Å^{-1}. The Lindhard function for Q1D systems has been anticipated in Ref. [13] and a complete survey of results is presented in Ref. [14].

A situation with two populated Fermi segments, corresponding to $R = 10$ Å and $\rho_1 = 0.11$ Å^{-1}, permits a qualitative illustration of the Q1D response. Figure 5 shows the longitudinal Q1D Lindhard function, for $l = 0$ and transferred momentum $q = 0.05$ and 0.12 Å^{-1}; it permits a previous guess of the multiplicity of the collective spectrum, which is portrayed in Fig. 6, together with the edges of the two ph continua. The largely reduced scales, as compared to those in Fig. 4, are chosen to display the two undamped modes which lie between the two neighboring 1D continua, the lower (higher) energy one corresponding to a density (spin-density) fluctuation. This kind of spectrum has been already found in the case of adsorbed quasi-two-dimensional ^{3}He [15], for

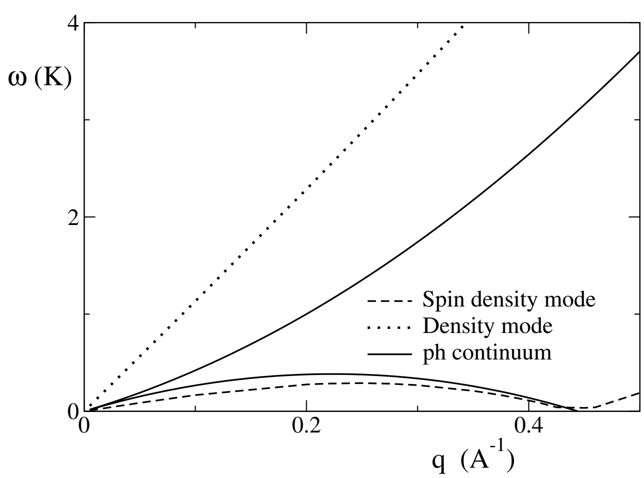

FIG. 4. The dispersion relation for the low and high energy collective modes of 1D ^{3}He quasiparticles at $\rho 1 = 0.1$ Å^{-1}.

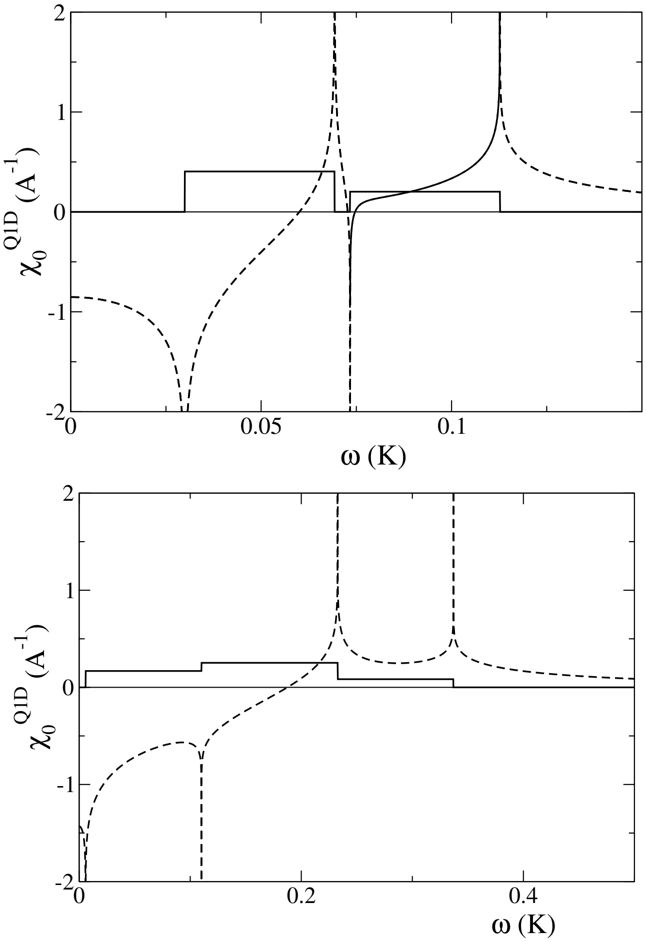

FIG. 5. Free susceptibility of Q1D ^{3}He quasiparticles for transferred momentum $q = 0.05$ Å^{-1} (upper panel) and 0.12 Å^{-1} (lower panel).

density-zero sound modes. It can be verified that the density oscillation of higher energy persists up to sizeable momentum; near 1 Å^{-1}, this mode departs from the linear dispersion relation and approaches the continuum edge so as to mimic the 2D trend [10].

The purpose of this work is to call attention to a dynamical manifestation of reduced dimensionality which may be linked to the suppression of thermodynamic phase transitions in one dimension; in this sense, experimental verification remains the key to further understanding and completion of physical insight. It should be kept in mind that the calculations here presented are model dependent; the choice of the single particle spectrum, of the effective *ph* interaction, and of the values of the force parameters may modify the shape of the dispersion relations, the values of the phase velocities of the density and spin modes, and the phonon-maxon-roton appearance of the 1D spin dispersion. Further research investigating the properties of ^{3}He adsorbed in cylindrical environments would shed light on the robustness of these quantitative results.

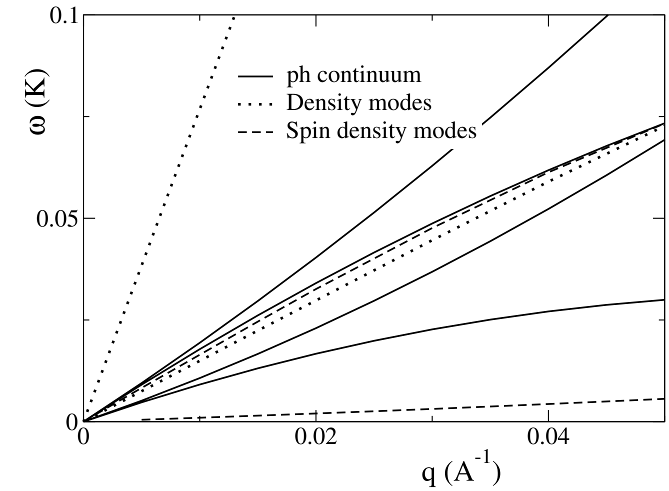

FIG. 6. Dispersion relations for Q1D ^{3}He quasiparticles.

The author is grateful to Silvina Gatica for helpful comments. This work was supported by Grant No. EX103 from Universidad de Buenos Aires, Argentina.

[1] C. H. Aldrich III and D. Pines, J. Low Temp. Phys. **25**, 677 (1976); **25**, 691 (1976); C. H. Aldrich III, C. J. Pethick, and D. Pines, Phys. Rev. Lett. **37**, 845 (1976); C. H. Aldrich III and D. Pines, J. Low Temp. Phys. **32**, 689 (1978).

[2] H. R. Glyde, *Excitations in Liquid and Solid Helium* (Clarendon Press, Oxford, 1994).

[3] M. Barranco, E. S. Hernández, and J. Navarro, Phys. Rev. B **54**, 7394 (1996).

[4] V. P. Silin, Sov. Phys. JETP **6**, 945 (1958); **8**, 870 (1958).

[5] L. R. Corruccini, D. S. Osheroff, D. M. Lee, and R. C. Richardson, J. Low Temp. Phys. **8**, 229 (1972); J. R. Owers-Bradley, H. Chocolacs, R. M. Mueller, Ch. Buchal, M. Kubota, and F. Pobell, Phys. Rev. Lett. **51**, 2120 (1983); L. J. Wei, N. Kalenchofsky, and D. Candela, Phys. Rev. Lett. **71**, 879 (1993).

[6] S. M. Gatica, E. S. Hernández, and J. Navarro, Phys. Rev. B **60**, 15 302 (1999).

[7] E. S. Hernández, J. Low Temp. Phys. **121**, 755 (2000).

[8] M. M. Calbi, M.W. Cole, S. M. Gatica, M. J. Bojan, and G. Stan, Rev. Mod. Phys. **73**, 857 (2001).

[9] G. Stan and M.W. Cole, Surf. Sci. **395**, 280 (1998).

[10] S. Das Sarma and E. H. Hwang, Phys. Rev. B **54**, 1936 (1996); D.W. Wang and S. Das Sarma, cond-mat/0101061.

[11] See, e.g., the review article by J. Voit, Rep. Prog. Phys. **57**, 977 (1994).

[12] D.W. Hess and D. Pines, J. Low Temp. Phys. **72**, 24 (1988).

[13] E. S. Hernández and J. Navarro, "Microscopic Approaches to Quantum Liquids in Confined Geometries," edited by E. Krotscheck and J. Navarro (World Scientific, Singapore, to be published).

[14] E. S. Hernández, J. Low Temp. Phys. **127**, 153 (2002).

[15] M. M. Calbi and E. S. Hernández, J. Low Temp. Phys. **120**, 1 (2000).