Comment on "Ruling Out Chaos in Compact Binary Systems"

Intuitively, black hole binaries are obvious candidates for chaotic dynamics. First, they are highly nonlinear systems and chaos is an expression of extreme nonlinearity. Second, there are isolated unstable orbits around a Schwarzschild black hole, and unstable orbits are a red flag for the onset of chaos. Surely the orbits become only more unstable and more complex when there are two black holes.

There is no question that there is chaos in black hole binaries for some range of parameters [1,2]. Yet the authors of Ref. [3] claim to rule out chaos by finding no positive Lyapunov exponents along the fractal of Ref. [2]. Before carrying out a detailed calculation, there is reason to be suspicious of the absence of positive Lyapunov exponents. Even the Schwarzschild solution has positive Lyapunov exponents for unstable circular orbits [4]. Though completely integrable and so nonchaotic, the orbits are unstable. *As long as there are unstable circular orbits in the post-Newtonian (PN) equations, there are positive Lyapunov exponents for these orbits.* This is not to say that the orbits will definitely be chaotic, just that there will be instability and a positive Lyapunov exponent.

Chaos was detected in the PN expansion of the two-body problem for rapidly spinning bodies [2] using the method of fractal basin boundaries. One fully expects positive Lyapunov exponents at least very near the fractal basin boundaries, if not elsewhere in the phase space. We sampled some orbits near the boundary and found positive Lyapunov exponents as illustrated by the positive slope in Fig. 1. The reason for the discrepancy seems to be that the authors of Ref. [3] define the maximum exponent as "the Cartesian distance between the dimensionless 12-component coordinate vectors $[\vec{\mathbf{r}}, \dot{\vec{\mathbf{r}}}, \vec{\mathbf{S}}_1, \vec{\mathbf{S}}_2]$ and $[\vec{\mathbf{r}}', \dot{\vec{\mathbf{r}}}', \vec{\mathbf{S}}'_1, \vec{\mathbf{S}}'_2]$ of two nearby trajectories" [3]. However, *this is not a Lyapunov exponent.* The Lyapunov exponent is obtained by linearizing the equations of motion about a given trajectory. An *approximation* to the Lyapunov exponent can be made using the Cartesian distance between two trajectories by continually rescaling the shadow trajectory so that it is always infinitesimally close to the original trajectory as was done by [1]. However, the result does depend on the rescaling and can give false answers. A more thorough treatment of the Lyapunov exponents and their interpretation is given elsewhere [5].

Chaos has not been ruled out; rather it has been confirmed by positive Lyapunov exponents. In fact, an even stronger claim can be made. The two-body problem has not been solved in relativity and it would not be too daring to conjecture that it will never be solved. The system shows every indication of being fully nonintegrable.

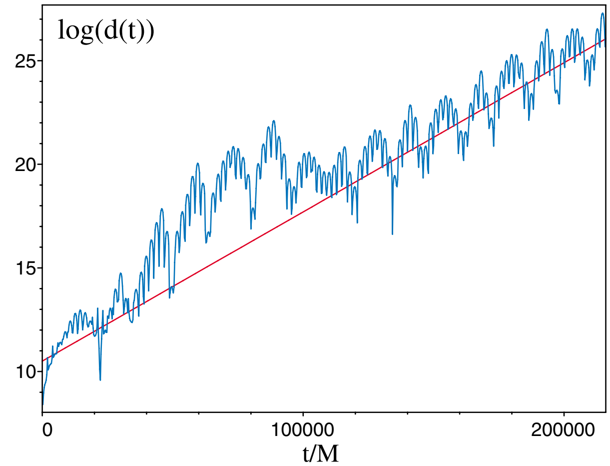

FIG. 1 (color online). Determining the Lyapunov exponent for an orbit taken from Fig. 3 of Ref. [2].

Regardless of how we would like Nature to behave to make our lives as observers easier, she may not be so accommodating. Understandably there is a sense of urgency for the removal of any obstacles to the detection of gravitational waves. However, chaos need not be a terrible obstacle. There are ways that chaos can enhance signals or amplify salient features in the sky [6] to aid our quest to observe gravitational waves.

Neil J. Cornish[1] and Janna Levin[2]
[1]Department of Physics,
Montana State University,
Bozeman, Montana 59717
[2]DAMTP,
Cambridge University,
Wilberforce Road,
Cambridge CB3 0WA, United Kingdom

Received 2 July 2002; published 8 October 2002
DOI: 10.1103/PhysRevLett.89.179001
PACS numbers: 04.30.Db, 04.25.Nx, 05.45.Jn, 95.30.Sf

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