Resonant Cooper-Pair Tunneling: Quantum Noise and Measurement Characteristics

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We study the quantum charge noise and measurement properties of the *double* Cooper-pair resonance point in a superconducting single-electron transistor (SSET) coupled to a Josephson charge qubit. Using a density-matrix approach for the coupled system, we obtain a full description of the measurement backaction; for weak coupling, this is used to extract the quantum charge noise. Unlike the case of a nonsuperconducting SET, the backaction here can induce population inversion in the qubit. We find that the Cooper-pair resonance process allows for a much better measurement than a similar nonsuperconducting SET, and can approach the quantum limit of efficiency.

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Among the many open issues related to solid state quantum computation, the question of how best to *measure* a solid state qubit remains a particularly interesting one. In the case where the qubit is a Cooper-pair box (i.e., a Josephson-junction single charge box), the standard choice for a readout device is the single-electron transistor (SET) [1–6]. An alternate and potentially more powerful approach is to use a *superconducting* single-electron transistor (SSET) biased at a point where the cyclic resonant tunneling of Cooper pairs dominates transport [7–12]. Such processes, known as Josephson quasiparticle (JQP) resonances, would appear to be an attractive choice for use in a measurement as their resonance structure implies an extremely high sensitivity. However, precisely because of their large gain, these processes may be expected to strongly alter the state of the qubit in a measurement. To assess the balance between these two opposing tendencies, a close examination of the physics of JQP tunneling is required.

In this paper, we focus on a *double* JQP process (DJQP) (see Fig. 1), which occurs at a lower SSET source-drain voltage than single JQP processes, and which has been used in a recent experiment [13]. We assess the potential of DJQP to act as a one-shot measurement of the state of a Cooper-pair box qubit. This involves characterizing both τ_{meas}, the time needed to discriminate the two qubit states in the measurement, and the backaction of the measurement on the qubit, which is described by a mixing rate Γ_{mix} and a dephasing rate $1/\tau_\varphi$. These quantities are intimately related to the noise properties of the SSET, which are of interest in themselves, given the novel nature of the DJQP process. τ_{meas} is determined by the shot noise of the process, while Γ_{mix} and τ_φ are related to the charge noise on the SSET island. While the shot noise of a *single* JQP process has been analyzed recently [14], the quantum charge noise has not been addressed.

To describe the measurement process in our system, we employ a density-matrix description of the *fully coupled* SSET plus qubit system; this is similar to the approach taken by Makhlin *et al.* [4] for a SET, but extended to deal with Josephson tunneling. This approach is not limited by a requirement of weak coupling, as are standard approaches which perturbatively link Γ_{mix} to the transistor charge noise [5,6]; nonetheless, in the limit of weak coupling it can be used to calculate the quantum charge noise of the SSET. We find that the quantum (i.e., asymmetric in frequency) nature of the noise is particularly pronounced for the DJQP feature, leading to regimes where the SSET can strongly relax the qubit. Moreover, due to the resonant nature of Cooper-pair tunneling, there exist regimes where the SSET can cause a pronounced *population inversion* in the Cooper-pair box. For typical device parameters, we find that a far better single-shot measurement is possible using the DJQP process than with a comparable SET. Significantly, one can also approach the quantum limit of measurement efficiency [3,4], where $\tau_\varphi/\tau_{meas} \uparrow 1$, in a regime which is both theoretically tractable *and* experimentally relevant.

Model.—The Hamiltonian of the coupled qubit plus SSET system is written as $\mathcal{H} = \mathcal{H}_S + \mathcal{H}_Q + \mathcal{H}_{int}$. The qubit itself (or "box"), described by $\mathcal{H}_Q$, consists of a superconducting metal island in the Coulomb blockade regime where only two charge states are relevant. These can be regarded as the σ_z eigenstates of a fictitious spin 1/2. The island is attached via a tunnel junction to a bulk superconducting electrode, leading to the form

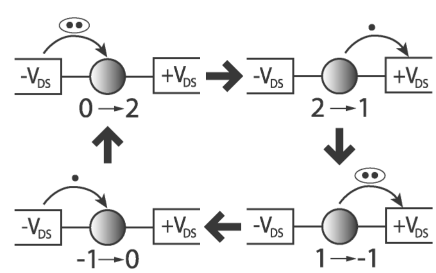

FIG. 1. Schematic showing the four steps of the double Josephson quasiparticle process which can occur in a superconducting single-electron transistor. Circles represent the central island of the SSET, while the rectangles are the electrodes. Numbers indicate the charge of the SSET island.

$$\mathcal{H}_Q = -\tfrac{1}{2}\{[4E_{CQ}(1-\mathcal{N}_Q)]\sigma_z + E_{JQ}\sigma_x\}, \tag{1}$$

where E_{CQ} is the charging energy of the box, E_{JQ} is the Josephson coupling energy of the box, and $\mathcal{N}_Q$ is the dimensionless gate voltage applied to the box. The SSET consists of a superconducting, Coulomb-blockaded island which is attached via tunnel junctions to two superconducting electrodes (Fig. 1). The SSET Hamiltonian $\mathcal{H}_S = H_K + H_C + H_V + H_T$ has a term H_K describing the kinetic energy of source, drain, and central island electrons, a term H_V which describes the work done by the voltage sources, and a tunneling term H_T. The charging term is $H_C = E_{CS}(n_S - \mathcal{N}_S)^2$, where E_{CS} is the SSET charging energy, n_S is the number of electrons on the central island, and $\mathcal{N}_S$ is the dimensionless gate voltage applied to the island. Finally, the qubit is capacitively coupled to the SSET: $H_{\rm int} = 2E_{CQ}\frac{C_C}{C_\Sigma}\sigma_z n_S \equiv E_{\rm int}\sigma_z n_S$. Here C_C is the cross capacitance between the box and the central island of the SSET, and C_Σ is the total capacitance of the SSET island. Note that we neglect the coupling of the qubit to its environment, as we are interested here in the intrinsic effect of the SSET on the qubit [15]. We also assume a SSET with identical tunnel junctions, whose dimensionless conductance g satisfies $g/(2\pi) \ll 1$.

The DJQP process occurs when the SSET gate voltage $\mathcal{N}_S$ and drain-source voltage $2V_{DS}$ are such that two Cooper-pair tunneling transitions (one in each junction) are resonant. We label these transitions as $n_s = 0 \to 2$ (left junction) and $n_s = 1 \to -1$ (right junction) (see Fig. 1). Resonance thus requires $eV_{DS} = E_{CS}$ and $\mathcal{N}_S = 1/2$. In addition, E_{CS}/Δ_S (where Δ_S is the superconducting gap of the SSET) must be chosen so that the quasiparticle transitions linking the two Cooper-pair resonances are energetically allowed (i.e., $n_S = 2 \to 1$ and $n_S = -1 \to 0$), whereas transitions which end the cycle (i.e., $n_S = 0 \to 1$) are not. We take $E_{CS} = \Delta_S$ to satisfy these conditions; this corresponds to the experiment of Ref. [13]. The two quasiparticle transitions which occur in the DJQP are characterized by a rate Γ, which is given by the usual expression for quasiparticle tunneling between two superconductors [16]. The effective Cooper-pair tunneling rate γ_J emerging from our description [i.e., Eq. (3) below] is given by [8]

$$\gamma_J(\delta) = \frac{E_{JS}^2\Gamma}{4[\delta^2 + (\Gamma/2)^2]}. \tag{2}$$

Here δ is the energy difference between the two charge states involved in tunneling, E_{JS} is the Josephson energy of the SSET, and we set $\hbar = 1$.

Calculation approach.—We consider the reduced density matrix ρ of the qubit plus SSET system obtained by tracing out the SSET fermionic degrees of freedom. The evolution of ρ is calculated perturbatively in the tunneling Hamiltonian H_T, keeping only the lowest order terms; this corresponds to the neglect of cotunneling processes, which is valid for small g and near the DJQP resonance. Using an interaction representation where only H_T (and not $H_{\rm int}$) is viewed as a perturbation, the equation of motion of ρ takes the standard form:

$$\frac{d}{dt}\rho(t) = -\int_{-\infty}^{t} dt' \langle[\mathcal{H}_T(t), [\mathcal{H}_T(t'), \rho(t')\otimes\rho_F]]\rangle. \tag{3}$$

The angular brackets denote the trace over SSET fermion degrees of freedom; as we work at zero temperature, ρ_F is the density matrix corresponding to the ground state of these degrees of freedom in the absence of tunneling.

To make further progress, we treat the Josephson coupling emerging from Eq. (3) as energy independent and given by the Ambegaokar-Baratoff value $E_{JS} = g\Delta_S/8$. We also use the smallness of g to neglect logarithmic renormalization terms, as was done in Ref. [4]. One can then solve for the time-independent solution of Eq. (3), which describes the state achieved by the system after all mixing and dephasing of the qubit by the SSET has occurred. To describe the dynamics of mixing (i.e., the relaxation of the qubit state populations to their stationary value), we also calculate the corresponding eigenmode of Eq. (3). A Markov approximation is made which involves replacing $\rho(t')$ by $\rho(t)$ on the right-hand side of Eq. (3). This approximation is justified as long as the time dependence of ρ in the mixing mode is weak compared to typical frequencies appearing in the correlators of Eq. (3), requiring here that $\Gamma_{\rm mix} \ll E_{CS}$ and $E_{JS} \ll E_{CS}$ [17].

Backaction.—We focus here primarily on the mixing effect of the measurement backaction; dephasing will be discussed more extensively in Ref. [17]. The mixing rate $\Gamma_{\rm mix} = \Gamma_{\rm rel} + \Gamma_{\rm exc}$ is set by the rates at which the measurement relaxes and excites the qubit. Let Ω denote the $\mathcal{N}_Q$-dependent energy difference between the two qubit states. For weak coupling ($E_{\rm int} \ll \Omega$), Fermi's golden rule relates $\Gamma_{\rm rel}$ and $\Gamma_{\rm exc}$ to the quantum charge noise of the SSET island $S_Q(\omega) = \int dt e^{-i\omega t}\langle n_S(t) n_S(0)\rangle$:

$$\Gamma_{\rm rel/exc} = E_{\rm int}^2\left(\frac{E_{JQ}}{\Omega}\right)^2 S_Q(\pm\Omega). \tag{4}$$

In our approach, these rates may be directly obtained by using the stationary solution (which gives the postmixing occupancies of the box eigenstates) and the mixing eigenvalue of Eq. (3). In the limit of weak coupling, one can then use Eq. (4) to extract $S_Q(\Omega)$. Our method for calculating the quantum noise, which uses the qubit as a spectrum analyzer, is physically intuitive and no more difficult to implement than standard approaches [6]; in addition, we are able to calculate $\Gamma_{\rm rel}$ and $\Gamma_{\rm exc}$ when the coupling is not weak, and Eq. (4) fails.

Figure 2 displays the quantum charge noise obtained at zero temperature, using SSET parameters which correspond to Ref. [13]. The solid curve in Fig. 2 is for the center of the DJQP resonance—$\mathcal{N}_S = 1/2$, $eV_{DS} = E_{CS}$. Note the sudden asymmetry that develops between

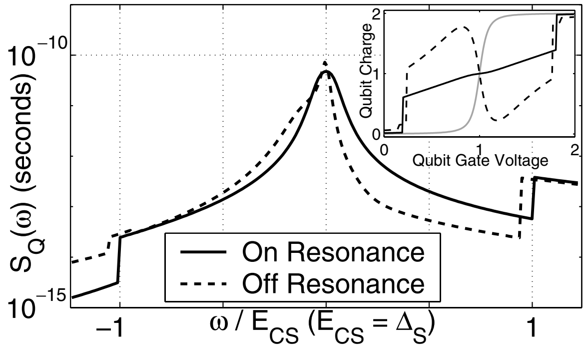

FIG. 2. Quantum charge noise associated with the DJQP process. The solid curve corresponds to $\mathcal{N}_S$, V_{DS} tuned to the center of the DJQP resonance; the dashed curve corresponds to moving eV_{DS} away from resonance by $+\Gamma/4$. We take $g = 0.5$ and $\Delta_S \simeq E_{CS} = 0.25$ meV in the SSET, corresponding to the device of Ref. [13]; this gives $E_{JS}/(h\Gamma) \simeq 0.04$. Inset: average qubit charge after mixing has occurred for weak coupling ($E_{\rm int}/E_{JQ} = 0.01$), as a function of qubit gate voltage $\mathcal{N}_Q$; see text for details. We take $E_{CQ} \simeq 77$ μeV and $E_{JQ} \simeq 27$ μeV. The frequency range probed by tuning $\mathcal{N}_Q$ matches the range of the main plot; the sharp steps in the average charge occur at $\Omega(\mathcal{N}_Q) \simeq E_{CS}$.

absorption [i.e., $S_Q(+|\omega|)$] and emission [i.e., $S_Q(-|\omega|)$] when $|\omega|$ increases beyond E_{CS}. These jumps correspond to the opening and closing of transport channels in the SSET, and their sharpness is a result of the singularity in the quasiparticle density of states. For example, as ω rises past E_{CS}, transitions which are normally forbidden in the DJQP cycle (i.e., $n_S = 0 \rightarrow 1$) suddenly become energetically allowed *if* they absorb energy from the qubit, causing a sudden increase in $S_Q(\omega)$.

The effect of the SSET quantum charge noise on the qubit is shown in the inset of Fig. 2, where the average qubit charge $\langle N_B \rangle \equiv 1 + \langle \sigma_z \rangle$ for $t \gg \tau_{\rm mix}$ is shown as a function of $\mathcal{N}_Q$. Changing $\mathcal{N}_Q$ tunes the qubit splitting frequency Ω, allowing one to probe the frequency dependence of the noise. The solid black curve corresponds to being at the center of the DJQP feature, and the grey curve corresponds to the unperturbed qubit ground state. The features in the quantum noise manifest themselves in $\langle N_B \rangle$, a quantity which is accessible in experiment.

Even more interesting is the situation when one tunes $\mathcal{N}_S$ or V_{DS} slightly off the DJQP resonance center. Unlike the case of a SET, where noise asymmetries are weak for $|\omega| \ll E_{CS}$ [6], there are strong features here that result from the resonant nature of Cooper-pair tunneling. By treating the mixing terms in Eq. (3) perturbatively, analytic expressions can be obtained for the quantum noise in this regime when $E_{JS} < \Gamma$ [in Ref. [13], $E_{JS}/(h\Gamma) \simeq 0.04$]. If one moves away from the DJQP center by tuning only V_{DS} (i.e., $\mathcal{N}_S = 1/2$, $eV_{DS} = E_{CS} + \delta_V/2$), we find ($|\omega| < E_{CS}$)

$$S_Q(\omega) = \gamma_J(\delta_V)\frac{[\gamma_J(\delta_V + \omega)/\gamma_J(\delta_V - \omega)]}{[4\gamma_J(\delta_V + \omega)\gamma_J(\delta_V - \omega)] + \omega^2}. \quad (5)$$

In the limit where $\omega \ll \Gamma/2$, Eq. (5) simply corresponds to classical telegraph noise (the SSET spends only appreciable time in the states $n_S = 0$ and $n_S = 1$). However, for finite δ_V and ω, Eq. (5) indicates that the noise develops a pronounced asymmetry, even though $|\omega| \ll E_C$. In particular, if $\delta_V > 0$, one has $S_Q(-|\omega|) > S_Q(+|\omega|)$, implying that *emission by the SSET exceeds absorption.* This behavior is shown by the dashed curves in Fig. 2, which correspond to $\mathcal{N}_S = 1/2$, $\delta_V = +\Gamma/4$. This effect is a direct consequence of the resonant nature of Cooper-pair tunneling—by emitting energy, *both* Cooper-pair tunneling processes in the DJQP cycle become more resonant, while absorbing energy pushes them even farther from resonance. The result is a population inversion in the qubit at zero temperature, which in turn leads to a striking, nonmonotonic dependence of qubit charge on $\mathcal{N}_Q$ (dashed curve in the inset of Fig. 2) [15]. Note that if one moves away from the center of the DJQP resonance by changing only the $\mathcal{N}_S$, no asymmetry in the noise results, as now emission (or absorption) moves one of the Cooper-pair transitions in the DJQP process farther *towards* resonance, while it moves the other transition farther *away* from resonance. Letting $\delta_V = 0$ and $\delta_{\mathcal{N}} = 4E_{CS}(\mathcal{N}_S - 1/2)$, we have for $E_{JS} < \Gamma$:

$$S_Q(\omega) = \gamma_J(\delta_{\mathcal{N}})\frac{1 + \frac{(8\delta_{\mathcal{N}}\omega)^2}{\Gamma^2 E_{JS}^4/2}\gamma_J(\delta_{\mathcal{N}} - \omega)\gamma_J(\delta_{\mathcal{N}} + \omega)}{[4\gamma_J(\delta_{\mathcal{N}} + \omega)\gamma_J(\delta_{\mathcal{N}} - \omega)] + \omega^2}. \quad (6)$$

Measurement rate.—To determine the measurement time $\tau_{\rm meas}$, we extend our density-matrix description to also include m, the number of electrons that have tunneled through the left SSET junction [4,14]. We are thus able to calculate the distribution of tunneled electrons $P(m, t|i)$, where $i = \uparrow, \downarrow$ denotes the initial state of the qubit. $\tau_{\rm meas}$ is defined as the minimum time needed before the two distributions $P(m, t|\uparrow)$ and $P(m, t|\downarrow)$ are statistically distinguishable [4]:

$$\frac{1}{\tau_{\rm meas}} = \left(\frac{I_\uparrow - I_\downarrow}{\sqrt{2f_\uparrow I_\uparrow} + \sqrt{2f\downarrow I_\downarrow}}\right)^2. \quad (7)$$

Here $I_\uparrow$ and $I_\downarrow$ are the average SSET currents associated with the two qubit states, and $f_\uparrow$ and $f_\downarrow$ are the associated Fano factors which govern the zero-frequency shot noise in the current. In the absence of the qubit, the density-matrix equations for the SSET yield the following for the single Fano factor f:

$$f(\delta) = \frac{3}{2}\left[1 - \frac{1}{2}\frac{E_{JS}^2[3(\Gamma/2)^2 - \delta^2]}{([\Gamma/2]^2 + \delta^2 + E_{JS}^2/2)^2}\right], \quad (8)$$

where we take $eV_{DS} = E_{CS}$, $\delta = \delta_{\mathcal{N}} = 4E_{CS}(\mathcal{N}_S - 1/2)$. Equation (8) indicates that the effective charge of the carriers in the DJQP process is $3e/2$ in the limit where $\Gamma \gg E_{JS}$. In this limit, Cooper-pair tunneling is the rate-limiting step in the cycle; electrons effectively tunnel in clumps of e or $2e$, leading to an average charge of $3e/2$.

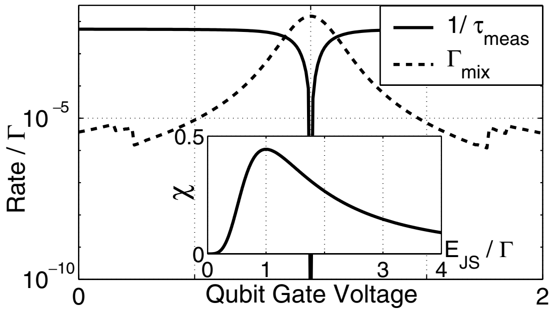

FIG. 3. $1/\tau_{\text{meas}}$, Γ_{rel}, and Γ_{exc} vs qubit gate voltage $\mathcal{N}_Q$ for a strongly coupled system, where $E_{\text{int}}/E_{JQ} \simeq 0.3$ (i.e., $C_C/C_\Sigma = 0.05$). A good measurement is possible for a wide range of gate voltages. Inset: Heisenberg efficiency $\chi = \tau_\varphi/\tau_{\text{meas}}$ at weak coupling, as a function of E_{JS}/Γ.

We consider τ_{meas} in the limit of weak coupling ($E_{\text{int}} \ll \Omega$) and weak mixing ($E_{JQ} \ll \Omega$). Taking $\delta_V = 0$ and $\delta\mathcal{N} = \Gamma/2$ for near optimal gain, and using Eqs. (6)–(8), we find that the intrinsic signal-to-noise ratio $(\tau_{\text{meas}}\Gamma_{\text{mix}})^{-1/2}$ of the measurement, in the relevant regime $E_{JS} < \Gamma$, is given by

$$(S/N)_{\text{DJQP}} = \sqrt{\frac{4}{3}}|\cot\theta|\frac{\Omega}{\Gamma/2}. \qquad (9)$$

Here $\cot\theta \equiv 4E_{CQ}(1 - \mathcal{N}_Q)/E_{JQ}$, and we take $\gamma_J(0) \ll \Omega < E_{CS}$. If a SET in the sequential-tunneling regime is used for the qubit measurement, it was found in Refs. [3,4] that the optimal S/N is given by ($\Omega < E_{CS}$)

$$(S/N)_{SET} = \lambda|\cot\theta|\sqrt{\left(\frac{\Omega}{eV_{DS}}\right)^2 + \frac{g^2}{\pi^2}}, \qquad (10)$$

where λ is of order unity. As the quasiparticle transition rate $\Gamma \sim \frac{g}{2\pi}eV_{DS}$, we see that the S/N achieved using DJQP is parametrically larger (in $2\pi/g \gg 1$) than that obtained for the SET. This enhancement results largely from the narrow width of the DJQP feature—the energy scale over which the current changes (and thus the gain) is set by Γ rather than V_{DS}. The gain and S/N ratio of the SET could be improved by working in the cotunneling regime; however, this would result in a much larger τ_{meas} ($\tau_{\text{meas}} \propto g^{-2}$), making one more susceptible to unwanted environmental effects. In contrast, the DJQP feature has both a large gain *and* a short τ_{meas} (i.e., $\tau_{\text{meas}} \propto 1/g$). Shown in Fig. 3 as a function of $\mathcal{N}_Q$ are τ_{meas}, Γ_{rel}, and Γ_{exc} for a strongly coupled device ($C_C/C_\Sigma = 0.05$), with all other parameters as listed in the caption of Fig. 2. We have taken $\delta_V = 0$ and $\delta\mathcal{N} = \Gamma/2$ for optimal gain. Figure 3 confirms that an excellent measurement is indeed possible, with $(S/N)^2 > 100$.

We have also studied the efficiency $\chi = \tau_\varphi/\tau_{\text{meas}}$ of measurement using DJQP for a weak coupling ($E_{\text{int}} \ll E_{JS}, \Gamma$) and $\Omega < E_{CS}$, where τ_φ is the measurement-induced dephasing time [17]. Unlike an SET in the sequential-tunneling regime, where $\chi \propto g^2$ is always much less than the quantum limit $\chi = 1$ [3,4], here χ is controlled by the ratio E_{JS}/Γ. As shown in the inset of Fig. 3, by tuning this ratio, *χ can be made to approach the quantum limit.* Here, for each value of E_{JS}/Γ, we have set V_{DS} and $\mathcal{N}_S$ to optimize the gain. Measurement using DJQP is able to reach a high efficiency when $E_{JS} \simeq \Gamma$ both because of the symmetry of the process and because of the coherent nature of Josephson tunneling; the large gain of the process is also important [17]. Clearly, the DJQP process allows for a far superior measurement of a Cooper-pair box qubit than a SET.

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[16] See, e.g., M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1996).
[17] τ_φ can be obtained in a manner analogous to that used in Ref. [4]. A full derivation and discussion will be presented in A. A. Clerk *et al.* (to be published).