

# Aharonov-Bohm Magnetization of Mesoscopic Rings Caused by Inelastic Relaxation

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(Received 11 April 2002; published 2 October 2002)

The magnetization of a system of many mesoscopic rings under nonequilibrium conditions is considered. The corresponding disorder-averaged current in a ring  $I(\phi)$  is shown to be a sum of the “thermodynamic” and “kinetic” contributions both resulting from the electron-electron interaction. The thermodynamic part can be expressed through the diagonal matrix elements  $J_{\mu\mu}$  of the current operator in the basis of exact many-body eigenstates and is a generalization of the equilibrium persistent current. The novel kinetic part is present only out of equilibrium and is governed by the off-diagonal matrix elements  $J_{\mu\nu}$ . It has drastically different temperature and magnetic field behavior.

DOI: 10.1103/PhysRevLett.89.176601

PACS numbers: 72.15.Rn, 72.20.Ht, 72.70.+m, 73.23.-b

Recently there has been considerable interest in mesoscopic phenomena under nonequilibrium conditions. The relaxation of electron energy distribution function due to inelastic processes of electron interaction has been measured in mesoscopic wires [1]. The Kondo effect in quantum dots with finite bias has been intensively discussed theoretically [2,3] and observed experimentally [4].

The nonequilibrium effects are also important for the magnetic response of a system of mesoscopic rings [5,6]. In equilibrium, a weak constant magnetic field gives rise [7] to a magnetization of a system of many normal-metal mesoscopic rings that corresponds to the average persistent current per ring

$$I^{(\text{PC})}(\phi) = -\frac{\partial \mathcal{E}(\phi)}{\partial \phi} = \sum_{n=1}^{\infty} I_n^{(\text{PC})} \sin\left(\frac{4\pi n \phi}{\phi_0}\right), \quad (1)$$

where  $\phi$  is the magnetic flux through a ring,  $\mathcal{E}(\phi)$  is the total energy of interacting electrons in a ring averaged over the disorder and the thermal ensembles, and  $\phi_0 = hc/e$  is the flux quantum.

The remarkable feature of the flux dependence in Eq. (1) is its periodicity and odd character. The latter is due to the dissipationless nature of the equilibrium persistent current. Indeed, in equilibrium both directions of time are equivalent, so that time-reversal symmetry requires  $I^{(\text{PC})}(\phi) = -I^{(\text{PC})}(-\phi)$ . The periodic and odd in  $\phi$  magnetization of the form Eq. (1) has been indeed observed [8] in a sample containing  $10^7$  mesoscopic copper rings.

However, the *disorder-averaged* current in a ring must respect another symmetry related with the space homogeneity of the disorder-averaged system. This is space reflection about the ring diameter. It is easy to see that both the current and the magnetic flux change sign under such a reflection, so that the symmetry relationship

$I(\phi) = -I(-\phi)$  should hold even in the case where equilibrium and time-reversal symmetry is not assumed. We thus arrive at the statement that the odd character of  $I(\phi)$  cannot be used as evidence that the disorder-averaged current observed in [8] is an *equilibrium* persistent current. The same is true for the  $\phi_0/2$  periodicity.

It has been shown by straightforward calculations [5,6] that a *nonequilibrium* dc current  $I^{(\text{dc})}(\phi)$  of the same form as Eq. (1) indeed arises when the ring is driven out of equilibrium by an external ac electric field.

A nonequilibrium current is sustainable for a reasonably long time only if there is an *external force* acting on the electron system. This force can produce a nonequilibrium dc current either by a *direct* action on electrons in the ring or indirectly. The case of the external ac electric noise in Ref. [6] can be considered as a representative example of the direct effect.

In this Letter we consider another, indirect mechanism of a nonequilibrium current which is related with the *relaxation* of the *given nonequilibrium* electron energy

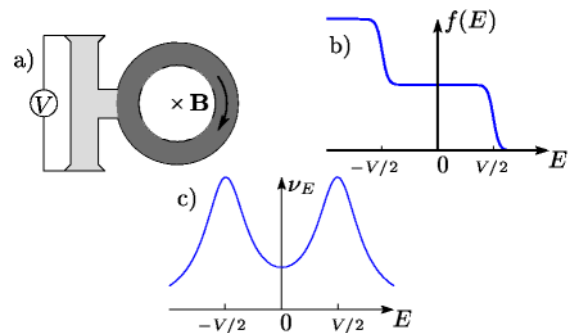


FIG. 1 (color online). (a) The experimental geometry. (b) Nonequilibrium energy distribution function with two steps. (c) DOS for a Kondo system [4].

distribution created by an external force. The similar relaxation-induced mesoscopic photovoltaic effect has been considered in Ref. [9]. However, in contrast to Ref. [9], we do not assume an electron-phonon mechanism of relaxation which is extremely weak at low temperatures [10]. Instead, we consider much more effective inelastic processes due to *electron-electron interaction*. One can imagine the experimental geometry where the direct action of the external force on electrons in the ring is absent while the one-particle energy distribution deviates from the Fermi-Dirac form. Consider a mesoscopic ring weakly coupled to the center of a wire connecting two reservoirs with the temperature  $T$  and the chemical potential difference  $V$  [see Fig. 1(a)] maintained by an applied voltage. For a sufficiently short wire, the distribution of one-particle energies in the ring  $f(E)$  is roughly a superposition of two Fermi-Dirac distributions corresponding to two reservoirs [1] [see Fig. 1(b)].

Below we identify the disorder-averaged dc current  $I^{(r)}(\phi)$  in a ring which is entirely due to the relaxation of this distribution resulting from the *electron-electron* interaction. We show that this relaxation-induced current can be represented in the form Eq. (1) with the flux harmonics  $I_n^{(r)}$  expressed through the electron energy distribution function  $f(E) = \frac{1}{2}(1 - h_E)$ :

$$I_n^{(r)} = C_n \frac{e}{hg} \int dE dE' \left( \frac{\delta D_{E'}}{D_0} \right) \left( \frac{\partial R_\omega(E', E)}{\partial \omega} \right)_{\omega=0}, \quad (2)$$

where  $-e$  is the electron charge,  $g$  is the dimensionless conductance of the ring with the perimeter  $L$  and the cross-section area  $S$ ,  $C_n = \frac{1}{(2\pi)^2} \sum_{m=1}^{\infty} \frac{1}{m^2} (1 - e^{-2\pi nm})$ , and

$$R_\omega(E, E') = [(h_E - h_{E-\omega})(1 - h_{E'} h_{E'-\omega}) - (h_{E'} - h_{E'-\omega})(1 - h_E h_{E-\omega})], \quad (3)$$

The diffusion coefficient  $D_E = D_0 + \delta D_E$  is supposed to have a small but essentially energy-dependent part  $\delta D_E \ll D_0$ . It is also assumed that the characteristic energy scale of the energy dependence in  $\delta D_E$  is larger than the typical energy transfer  $\omega \sim E_T = \hbar D_0 / L^2$ .

The function  $R_\omega(E, E')$  in Eq. (3) is exactly the combination of the electron energy distribution function  $h_E$  that enters in the inelastic collision integral [11]:

$$St(E) = \int dE' d\omega P(\omega) R_\omega(E, E'). \quad (4)$$

For the Fermi-Dirac distribution  $h_E = \tanh(E/2T)$  we have identically  $R_\omega(E, E') = 0$ , and both the relaxation rate Eq. (4) and the nonequilibrium current Eq. (2) vanish. This quantifies an intimate relationship between them.

Equations (2) and (3) are the main result of this Letter. They are valid for the case of a pure potential disorder with no spin-orbit interaction (orthogonal symmetry class) for  $g \gg 1$  as long as  $T \gg E_T$  and  $n \ll L_\phi / L$ ,

where  $L_\phi = [D_E \tau_\phi^{(\text{ring})}]^{1/2} \gg L$  is the dephasing length, and  $\tau_\phi^{(\text{ring})}$  is the dephasing time in the ring.

The non-Fermi-Dirac form of  $h_E$  is the necessary but not the sufficient condition for  $I_n^{(r)}$  to be nonzero. The global balance in Eq. (4) requires  $\int dE S t(E) = 0$  which results from the obvious identity  $\int dE dE' R_\omega(E, E') = 0$ . Then one concludes from Eq. (2) that  $I_n^{(r)} = 0$  unless the electron diffusion coefficient  $D_E = v^2 \tau_E / 3$  is *energy-dependent*.

For the white-noise impurity potential within the noncrossing (self-consistent Born) approximation, the product of the elastic scattering time  $\tau_E$  and the disorder-averaged one-electron density of states (DOS)  $\nu_E$  is energy independent  $\tau_E \nu_E = \text{const}$ . If the energy dependence of  $\nu_E$  is stronger than the dependence of the electron velocity  $v(E)$ , we obtain  $D_E = D_0 \nu / \nu_E$  where  $\nu$  is the (one-spin) DOS outside the region (near the Fermi energy) of the strong energy dependence.

Though the nature of the  $E$  dependence of  $\nu_E$  goes beyond the scope of this Letter, we note that the electron-electron interaction results in the strong energy dependence of the one-particle *tunnel* density of states  $\nu_E^{(\text{tun})}$  exactly at the Fermi level [12–14]. In our problem, the corresponding DOS  $\nu_E^{(r)}$  should be obtained from the nonperturbative treatment of the electron-electron interaction and will be considered elsewhere. An alternative mechanism of the energy dependence [see Fig. 1(c)] is the Abrikosov-Suhl [15] peaks in  $\nu_E$  at  $E = \pm V/2$  that arise [4] because of the Kondo effect.

Now we outline the derivation of Eq. (2). We start with the expression for current density  $J$  in terms of the components of the matrix Green's function  $\underline{G} = \begin{pmatrix} G^R & G^K \\ 0 & G^A \end{pmatrix}$  in the Keldysh technique [11,16]

$$J = \frac{-i}{2} \text{Tr} \{ \hat{J} \underline{G}^K \}, \quad (5)$$

where  $\hat{J}$  is the current density operator,  $G^{R,A}$  are the retarded (advanced) electron Green's functions, and  $G^K$  is the Keldysh function. The matrix Green's function  $\underline{G} = \underline{G}_0 + i \underline{G}_0 \underline{\Sigma}^F \underline{G}_0 + i \underline{G}_0 \underline{\Sigma}^H \underline{G}_0$  is calculated in the first order in the screened electron interaction  $\underline{U} = \begin{pmatrix} U^R & U^K \\ 0 & U^A \end{pmatrix}$ , where  $\underline{G}_0$  is the matrix Green's function without electron interaction in the presence of a static disorder potential, and  $\underline{\Sigma}^{F,H}$  are the Fock and the Hartree self-energy parts due to electron interaction.

The electron energy distribution function  $h_E$  enters in the theory through the Keldysh component of the *unperturbed* matrix Green's function  $\underline{G}_0$  via the ansatz [11,16]

$$G_0^K(E) = h_E [G_0^R(E) - G_0^A(E)]. \quad (6)$$

The effective interaction  $\underline{U}$  is calculated in the random-phase approximation which after averaging over disorder in the limit  $g \gg 1$  yields:

$$(U^R - U^A)_{\omega, \mathbf{q}} = \int dE' U_{\omega, \mathbf{q}}(E') (h_{E'} - h_{E'-\omega}), \quad (7)$$

where  $2ivD_0q^2U_{\omega,\mathbf{q}}(E') = (D_0^2q^4 + \omega^2)/(D_{E'}^2q^4 + \omega^2)$ . It reduces to the well known expression [11] in the case of an energy-independent diffusion coefficient  $\delta D_{E'} = 0$  if one uses the identity  $\int dE'(h_{E'} - h_{E'-\omega}) = 2\omega$ . The Keldysh component of the effective interaction  $U_{\omega,\mathbf{q}}^K$  is given by Eq. (7), where  $(h_{E'} - h_{E'-\omega})$  is replaced by  $(1 - h_{E'}h_{E'-\omega})$ .

Rewriting the linear in  $\sum^{F,H}$  terms explicitly in terms of the components  $G_0^{R,A,K}$  and using the ansatz Eq. (6), one identifies [10] three different interaction-induced contributions to the current Eq. (5):

$$\begin{aligned} \mathbf{I}_1 &= (\mathbf{AAR} - \mathbf{RAR})[(h_E - h_{E-\omega})U_\omega^K \\ &\quad - (1 - h_Eh_{E-\omega})(U_\omega^R - U_\omega^A)], \\ \mathbf{I}_2 &= \mathbf{ARR}(U_\omega^R - pU_0^R)(1 - h_Eh_{E-\omega}) \\ &\quad + \mathbf{RAA}(U_\omega^A - pU_0^A)(1 - h_Eh_{E-\omega}), \\ \mathbf{I}_3 &= (\mathbf{RRR} - \mathbf{AAA})h_EU_\omega^K - (1 - h_Eh_{E-\omega}) \\ &\quad \times [\mathbf{RRR}(U_\omega^R - pU_0^R) + \mathbf{AAA}(U_\omega^A - pU_0^A)], \end{aligned} \quad (8)$$

where  $\mathbf{AAR} \equiv G_0^A(E - \omega)G_0^A(E)\hat{J}G_0^R(E)$ , and the integral over all  $E$  and  $\omega$  is assumed in Eq. (8). The degeneracy factor  $p = 2$  for unpolarized electron spins and  $p = 1$  if spins are fully polarized by the parallel magnetic field.

The three contributions are very different in character. Since  $\mathbf{I}_3$  contains only retarded or only advanced Green's functions, it vanishes after averaging over disorder at a constant chemical potential. The contribution  $\mathbf{I}_2$  amounts to  $I_n^{(\text{PC})}$ . In particular, the result of Ref. [17] follows from this contribution if one assumes the equilibrium Fermi-Dirac distribution and the multiplicity  $p = 2$ . Indeed, using the identity  $L\frac{\partial}{\partial\phi}G_0^{R,A}(E) = -G_0^{R,A}(E)\hat{J}G_0^{R,A}(E)$ , one concludes that  $\mathbf{I}_2$  can be expressed in terms of the flux derivative of the effective energy functional  $\mathcal{E}_{\text{eff}}(\phi)$  that in the absence of equilibrium stands for the total energy  $\mathcal{E}(\phi)$  in Eq. (1). In order to understand the physical meaning of the additional contribution  $\mathbf{I}_1$ , we invoke the basis of exact *many-body* electron states  $\Psi_\mu$  and the corresponding matrix elements of the current operator  $J_{\mu\nu}$ . In this representation, the ‘‘thermodynamic’’ contribution  $\mathbf{I}_2$  is expressed in terms of the *diagonal* matrix elements  $J_{\mu\mu}$  only. In contrast to that, the ‘‘kinetic’’ contribution  $\mathbf{I}_1$  contains only *off-diagonal* matrix elements  $J_{\mu\nu}$ . One can check that replacing the current operator  $\hat{J}$  in the expression for  $\mathbf{I}_1$  by the unit operator results in vanishing of the whole expression, as is required by the orthogonality of the *different* many-body wave functions. Thus, the contribution  $\mathbf{I}_1$  contains information on the *overlap* of the different many-body wave functions that is totally absent in  $\mathbf{I}_2$ .

We also note that  $\mathbf{I}_1$  comes entirely from the Fock-type diagrams, while  $\mathbf{I}_2$  contains both the Hartree and the Fock contributions. The two contributions have opposite signs and the balance between them is controlled by the multiplicity factor  $p$ . Therefore, the thermodynamic con-

tribution  $\mathbf{I}_2$  is very sensitive to the parallel magnetic field that leads to the cancellation of the main part of the persistent current which is due to the *real* part of the screened interaction  $U^{R,A}$ . At the same time, the effect of the parallel magnetic field on the kinetic contribution reduces merely to a possible variation of the  $E$  dependence of DOS.

The contribution  $\langle \mathbf{I}_1 \rangle$  averaged over disorder is exactly the relaxation-induced nonequilibrium current  $I^{(r)}(\phi)$ . Substituting Eq. (7) into the first of Eqs. (8), we obtain

$$I_n^{(r)} = \int_{-\infty}^{+\infty} \frac{dE dE'}{(2\pi)^2} \int_{-\infty}^{+\infty} d\omega J_n(E, E', \omega) R_\omega(E, E'), \quad (9)$$

where  $R_\omega(E, E')$  is given by Eq. (3) and  $J_n(E, E', \omega)$  is the  $n$ th flux harmonic of

$$\sum_{q \neq 0} \langle (G_0^A - G_0^R)_{E-\omega} G_0^A(E) \hat{J} G_0^R(E) \rangle_q U_{\omega,\mathbf{q}}(E'). \quad (10)$$

The disorder average  $\langle \dots \rangle$  in Eq. (10) is done within the impurity diagrammatic technique. In the leading approximation in  $1/g$ , the result is described by the diagram Fig. 2.

The quasi-one-dimensional geometry of rings is taken into account by the quantization of momenta of diffusion propagators  $[D_E \mathbf{q}^2 - i\omega]^{-1}$  and the electron interaction  $U_{\omega,\mathbf{q}}(E')$ , such that the transverse momentum  $\mathbf{q}_\perp = 0$ , and the longitudinal momentum  $q_\parallel = (2\pi/L)(m - 2\phi/\phi_0)$  for the Cooperons and  $q_\parallel \equiv q = (2\pi m/L)$  for the diffusons and the electron interaction, where  $m = 0, \pm 1, \pm 2, \dots$  is an integer. The only constraint is that the electron interaction must be zero at zero momentum. Performing the Poisson summation over  $\mathbf{k}$ , we obtain

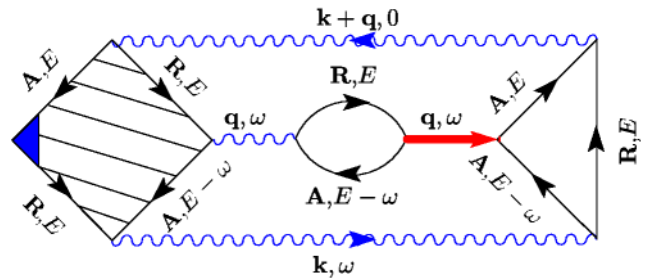


FIG. 2 (color online). Disorder averaging of the AAR term in Eq. (10). Solid lines labeled by  $R(A)$  are disorder-averaged electron Green's functions  $\langle G_0^{R(A)}(E) \rangle$  with the energy indicated; the wavy lines are the diffusion propagators (diffusons and Cooperons) with the momentum and frequency indicated; the dashed square is the Hikami box with the vector vertex  $\hat{J}$  denoted by the bold triangle; the bold solid line is the screened electron interaction  $U_{\omega,\mathbf{q}}(E')$ .

$$J_n(E, E', \omega) = \frac{4eD_E}{gL^2} \text{Im} \sum_{q \neq 0} \frac{(1 - e^{-nL/L_\omega})(D_0^2 q^4 + \omega^2)}{(D_E q^2 - i\omega)^3 (D_{E'}^2 q^4 + \omega^2)}, \quad (11)$$

where  $L_\omega = [D_E/(-i\omega)]^{1/2}$  and  $g = \nu DS/L$ .

The energy dependence of  $J_n(E, E', \omega)$  originates (i) from the  $E$  dependence of the triangle of Green's functions in Eq. (10) and (ii) from the  $E'$  dependence of the polarization bubble in the effective interaction  $U_{\omega, \mathbf{q}}(E')$ . It is easy to see that only the latter is important. Indeed, let us expand  $R_\omega(E, E')$  in Eq. (9) up to the linear in  $\omega$  term. If  $D_{E'} = D_0$  one can perform the Wick rotation  $-i\omega \rightarrow \omega$  that immediately gives  $\int_0^\infty J_n(E, E', \omega) \omega d\omega = 0$  because of the Im sign in front of the sum in Eq. (11). This is not true if  $\delta D_{E'} \neq 0$ , as in this case the Wick rotation leads to divergency. A careful analysis shows that the term proportional to  $\delta D_E$  in Eq. (11) makes a contribution to Eq. (9) that is small by the parameter  $(E_T/T)^{1/2} \ll 1$  compared to that resulting from  $\delta D_{E'}$ . Neglecting this contribution by setting  $D_E = D_0$ , we arrive at a finite result [Eq. (2)].

Note that  $J_n(E, E', \omega)$  is *not* exponentially small at  $\omega \gg E_T$ . This can be traced back to the structure  $G_0^{R,A}(E - \omega) \hat{J} G_0^R(E) \hat{J} G_0^A(E)$  of the kinetic term  $\mathbf{I}_1$  that allows one to build a Cooperon at zero frequency [10]. This is impossible for the thermodynamic term  $\mathbf{I}_2$  in which both Green's functions with the same energy  $E$  are either retarded or advanced. Therefore, the corresponding kernel for the thermodynamic term is proportional to  $\exp[-L/L_\omega] \sim \exp[-\sqrt{T/E_T}]$ . This is the reason why the kinetic term wins over the thermodynamic one for  $T \gg E_T$ .

In conclusion, using the Keldysh formalism we identified two different contributions, thermodynamic and kinetic, to the disorder-averaged magnetization of mesoscopic rings with a nonequilibrium distribution of one-electron energies. Both contributions are caused by the electron-electron interaction. However, the kinetic contribution is present only out of equilibrium provided that the one-electron density of states is not constant near the Fermi energy. This contribution is proportional to the same combination of the one-electron energy distribution function as the inelastic relaxation rate and is thus in-

timately related with the relaxation. In the basis of exact many-body wave functions of the weakly interacting electron gas, the kinetic contribution is strictly off-diagonal in contrast to the thermodynamic contribution (persistent current) that depends only on the diagonal matrix elements of the current operator. The sign of the kinetic contribution is not fixed by the basic symmetry of the problem (orthogonal or symplectic) but depends on the nature of the energy dependence of the one-electron DOS.

We are grateful to Igor Aleiner for critical comments and to B. L. Altshuler for fruitful discussions clarifying the physical meaning of the kinetic contribution to the current.

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