Interference Fringes with Maximal Contrast at Finite Coherence Time

M.W. Hamilton*

Department of Physics and Mathematical Physics, Adelaide University, Adelaide SA5005, Australia
(Received 22 May 2002; published 2 October 2002)

Interference fringes can result from the measurement of four-time fourth-order correlation functions of a wave field. These fringes have a statistical origin and, as a consequence, they show the greatest contrast when the coherence time of the field is finite. A simple acoustic experiment is presented in which these fringes are observed, and it is demonstrated that the contrast is maximal for partial coherence. Random telegraph phase noise is used to vary the field coherence in order to highlight the problem of interpreting this interference; for this noise, the Gaussian moment theorem may not be invoked to reduce the description of the interference to one in terms of first-order interference.

DOI: 10.1103/PhysRevLett.89.173901 PACS numbers: 42.25.Hz, 42.25.Kb, 42.50.Ar, 43.20.+g

The familiar interference that is observed in interferometers such as Fabry-Perot and Michelson interferometers is just the first in a hierarchy of orders of interference and is described by the autocorrelation function of the wave field, $G^{(2)}(t;0) = \langle E^*(t)E(0)\rangle$. Second-order interference results when the signal depends on a fourth-order correlation function, such as the intensity autocorrelation function $G^{(4)}(0,t;t,0) = \langle E^*(0)E^*(t)E(t)E(0)\rangle = \langle I(t)I(0)\rangle$. This correlation was measured in the classic optical experiment by Hanbury-Brown and Twiss [1], where a beam was split into two parts and the rate of photon coincidences detected between the two beams was measured. However, one normally would not expect to see interference fringes in $G^{(4)}(0,t;t,0)$ because each of the two physical fields is measured by an intensity detector; the direct measurement of intensity destroys information about the phase difference between the fields. Certain uniquely quantum mechanical light fields provide an important exception to this—with pairs of correlated and indistinguishable photons interference fringes can appear in the photon coincidence rate [i.e., in $G^{(4)}(0,t;t,0)$] even though no first-order interference is observed [2]. For the moment, however, we consider only fields that can be represented classically and have some associated stochastic process which makes them partially coherent.

With four-time correlation functions, one can see interference fringes with purely classical fields. These fringes are particularly interesting because they show greatest contrast when the fields are partially first-order coherent, rather than fully coherent or incoherent. In this Letter, I demonstrate these fringes with a simple acoustic experiment in which the coherence time of the sound can be continuously varied. Although I describe an acoustic experiment, the inspiration for this work comes from nonlinear optics and the theoretical ideas expressed in the introductory paragraphs apply to any scalar wave field. To the extent that polarization can be ignored, both light and sound can be represented as scalar waves.

The four-time correlation function of interest is $G^{(4)}(t,t+\tau;\tau,0) = \langle E^*(t)E^*(t+\tau)E(\tau)E(0)\rangle$. One of the time arguments is taken to be zero because the stochastic processes involved are assumed to be stationary, and ergodicity is assumed also. It is important to note that we are ignoring spatial correlations in what follows: we consider briefly some issues regarding spatial coherence at the conclusion.

In a spectroscopic two-photon absorption experiment that incorporates a retroreflecting mirror [3], so that the laser beam is passed twice through the absorber, the two-photon absorption rate is proportional to the Fourier transform of $G^{(4)}(t,t+\tau;\tau,0)$ [4],

$$W(\omega) = \int_{-\infty}^{\infty} \exp(-i\omega t) G^{(4)}(t,t+\tau;\tau,0)dt. \quad (1)$$

Here τ is the time delay that the light suffers in going from the absorber to the retroreflecting mirror and back again. Now there is only one detector, the two-photon absorber, so the relative phase information of the fields is preserved. As long as there is a stochastic process to introduce some incoherence, there will be a sharp corner in the function $G^{(4)}$ at $t = \tau$ (see Fig. 1). This corner introduces oscillatory terms into $W(\omega)$ with period τ^{-1} [5]. The oscillations in $W(\omega)$ are second-order interference fringes. If one were to monitor the value of $W(\omega)$

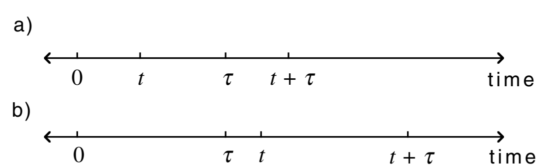

FIG. 1. Two different time regimes are needed for the evaluation of $G^{(4)}(t,t+\tau;\tau,0)$, which leads to a sharp corner in this function. In (a), where $t < \tau$, there are three time intervals in which the fluctuations in the field can be considered statistically independent $[0,t)$, $[t,\tau)$, $[\tau,t+\tau]$. However, if $t > \tau$ the set of intervals shown in (a) ceases to be statistically independent and we must consider the intervals shown in (b), i.e., $[0,\tau)$, $[\tau,t)$, $[t,t+\tau]$. The correlation must be calculated separately for the cases (a) and (b).

at a given frequency ω, one would see this value oscillate as the delay time is varied. This would be a direct demonstration that the fringes satisfy the operational definition of interference fringes shown in Fig. 2, but it can be more practical to simply observe the oscillations in the spectrum $W(\omega)$ itself. The existence of these fringes depends on the stochastic process; if it were absent, the correlation function would be a product of sinusoids, whose Fourier transform is just a sum of delta functions.

The existence of these fringes in two-photon absorption remains only a prediction [5] , but similar fringes have been observed in four-wave mixing and coherent Raman spectroscopy experiments [6–8]. What has never been demonstrated is that these fringes that arise from four-time correlations have maximum contrast if the field is partially coherent; that is, their existence requires the existence of the stochastic process. In the experiments [6,7], the signal was proportional to the Fourier transform of a four-time sixth-order correlation function. The fringes that were seen were termed radiation difference oscillations and were interpreted by appealing to the Gaussian moment theorem [9]. This was possible since in that work pulsed dye lasers were used, and it was reasonable to assume that the underlying stochastic process had a Gaussian density function. The independent fluctuations in the many laser modes that are present give rise to Gaussian statistics according to the central limit theorem. Thus, the sixth-order correlations were reduced to products of second-order correlation functions; that is, of first-order interferences.

However, such an interpretation would not have been possible were the stochastic process non-Gaussian, which is why we have chosen a random telegraph process [10] with a two-peaked density function for the experiment described here. In a random telegraph process, the noisy variable switches randomly between two well defined values and the number of transitions in a given time is a Poisson random variable. The average time that the variable spends at either of the values is the dwell time T. If the noise is applied to the phase of the field and the size of the phase jumps is π, $T/2$ can be identified with the coherence time.

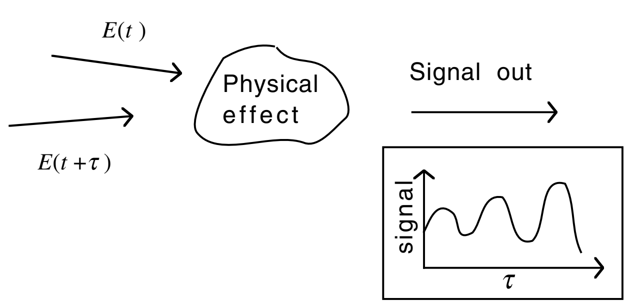

FIG. 2. The operational definition of interference fringes; two wave fields E_1 and E_2, one retarded by a time τ, are combined in physical effect which is measurable through some output signal. The generation of this output signal might also involve some computation, as happens when an intensity autocorrelation is measured. If the signal varies nonmonotonically with τ we have interference fringes.

A relatively low frequency wave, such as an acoustic or radio wave, allows one to directly measure each of the fields simultaneously and then compute $W(\omega)$, which is defined by Eq. (1). If there is only one source of the field, one could, of course, measure time series data for the field at just one place and still compute $W(\omega)$. However, we choose to measure the field from a single source at two places, which is equivalent to two times. This allows the second-order fringes to be used to measure the difference in the distances between two points and the source, provided the wave speed is known. Thus, the fact that the periodicity of the fringes is determined by the delay τ suggests that measuring the Fourier transform of $G^{(4)}(t, t+\tau; \tau, 0)$ might have some utility as a ranging technique.

Accordingly, a speaker and two microphones were set up in an anechoic chamber such that the distance from the speaker to each of the two microphones differed by 50 cm. The speaker was a moving-coil dome "tweeter" with a frequency range from 1.5 to 20 kHz and rms input power rating of 5 W. The microphones were both simple electret microphones. The sinusoidal signal from a function generator (Hewlett Packard HP3325) was applied to the speaker with a frequency of 9 kHz and an amplitude of 40 mV at the speaker (nominal impedance 8Ω). Applied to the signal, via the phase modulation input of the HP3325, was a random telegraph signal of variable dwell time T. The size ϕ of the phase jump was also variable. This modulation signal was produced by a voltage noise generator that has been previously described in the literature [11]. The two microphone signals were amplified (Tektronix 7A22 amplifiers) and recorded simultaneously at 50×10^3 samples/s and 12-bit resolution. From the recorded microphone signals, $W(\omega)$ was calculated. The delay time τ is now the time that the sound takes to travel the 50 cm spacing between the microphones. This calculation is speeded up by noting that we, in fact, calculate the power spectrum of the product of fields $H(t, \tau) = E(t)E(t+\tau)$; this is just the modulus squared of the transform of $H(t, \tau)$, which is rapidly computed with a standard fast Fourier transform algorithm. Figure 3 shows a part of one of these spectra. The oscillations seen in the wings of the spectrum are the second-order fringes that we seek.

The spacing of the fringes in Fig. 3 is 720 ± 20 Hz, which is in reasonable agreement with the reciprocal of the delay time $\tau = 1.43$ ms, as expected from the theory [5]. It can be seen that the peak is somewhat asymmetric; the fringes are much less visible on the high frequency side. According to Ref. [5], for π radian phase jumps the spectrum has the form of a Lorentzian with modulated wings;

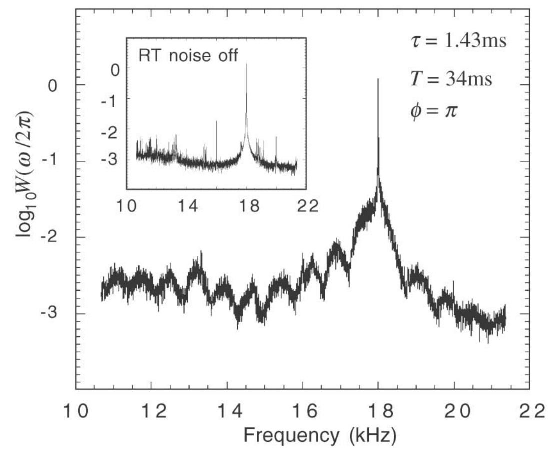

FIG. 3. The Fourier transform of $G^{(4)}(t, t+\tau; \tau, 0)$, for $\tau = 1.34$ ms, $T = 34$ ms, and $\phi = \pi$. The transform shown is the average of four transforms, each using 32×10^3 samples. The peak at 18 kHz is due to the 9 kHz frequency of the sound. The second-order interference fringes are the oscillations on the wings of this peak. The inset shows $W(\omega)$ when the random telegraph phase modulation is switched off.

$$W(\omega) \propto -\frac{\exp(-4\tau/T)}{\Delta}\sin(\Delta\tau) + \frac{4/T}{(4/T)^2 - \Delta^2} \times \{1 - \exp(-4\tau/T)[\cos(\Delta\tau) - (\Delta T/4)\sin(\Delta\tau)]\}, \quad (2)$$

where $\Delta = \omega - 2\omega_0$ and $\omega_0 = 2\pi \times 9$ kHz, i.e., the frequency of the sound wave. This spectrum is symmetric. The observed asymmetry is attributed to the inability of the speaker to properly reproduce the phase fluctuation. The amplitude fluctuation that results is correlated with the phase jump; the correlated amplitude and phase modulations lead to an asymmetric spectrum. The theoretical relation shown is the two-photon absorption spectrum, in whose derivation the atomic spectral response plays a role that is analogous to that of a window function, often used when Fourier transforming a finite series of data. As we use no window function when using the fast Fourier transform to calculate the spectra shown in Fig. 3, we must consider the two-photon absorption spectrum in the limit where the width of the atomic response tends to zero in order to arrive at Eq. (2).

The key evidence that the fringes shown in Fig. 3 are those associated with $G^{(4)}(t, t+\tau; \tau, 0)$ is that the fringe contrast reaches a maximum at finite T. Rather arbitrarily we choose two maxima (at 14.7 and 16.8 kHz) and the minima immediately to the high frequency side of these maxima. The ratio of maximum to minimum can be read off graphs such as Fig. 3, and the logarithm of these ratios is shown as a function of $\log_{10}T$ in Fig. 4. It is apparent that the coherence time for maximum fringe contrast depends on the distance of the fringe from the main peak at 18 kHz. The contrast maximum of the fringe at 16.8 kHz is not seen as clearly as that for the fringe at 14.7 kHz, because in the current apparatus we have an upper limit on T of about 300 ms. The random telegraph voltage noise source incorporates a photomultiplier which detects photons from a small incandescent bulb. The detection of a photon triggers a phase jump in the voltage output which then is used as the modulation input for the HP3325. The dark current of this photomultiplier sets the upper limit on T. In any case, larger values of T would require much more data because the time series data sets must be long enough to incorporate some phase jumps, and also there should be enough data sets to provide some averaging. We can, however, simply turn off the random telegraph noise source so that T is effectively infinite. The result of this is shown in the inset in Fig. 3. The fringes vanish. In order to test that the assumption of stationarity for the stochastic process was valid, we explicitly calculated the four-time fourth-order correlation function for several consecutive subsets of many full data sets (i.e., the time series for given dwell times). The correlation functions from the subsets were essentially the same, and that the average of their Fourier transforms was indistinguishable from the function W calculated in the manner described above. This is as one would expect if the correlation functions were stationary.

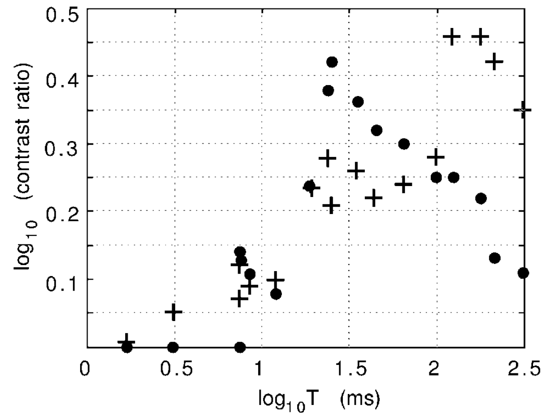

FIG. 4. The ratios (expressed logarithmically) between the maxima and nearest minima for the two fringes (see Fig. 3) at 14.7 kHz (circles) and 16.8 kHz (crosses) as a function of the random telegraph dwell time T. The values of τ and ϕ for all data points are the same as in Fig. 3.

We have confirmed the existence of the fringes for other values of ϕ, but we concentrate on the case of $\phi = \pi$ because in this case the (first-order) coherence is able to vary from complete to nonexistent. Random telegraph phase modulation results in the modulated wave having a power spectrum consisting of a delta function carrier component and a broadband Lorentzian shaped component. From Ref. [12] this is

$$S(\omega) = \cos^2\left(\frac{\phi}{2}\right)\delta(\omega - \omega_0) + \sin^2\left(\frac{\phi}{2}\right)\frac{4/T}{(4/T)^2 - (\omega - \omega_0)^2}. \quad (3)$$

The relative amounts of power in these components varies with ϕ; for $\phi = \pi$ all of the power is in the broadband component, and the coherent component is completely suppressed. At other values of ϕ a coherent component always remains, regardless of how short the dwell time T might be.

An analogy to Fraunhofer diffraction [13], developed in the context of multiphoton absorption, is of some help in interpreting our experiment. In this analogy the correlation function $G^{(4)}(t, t+\tau; \tau, 0)$ represents the transmission function of an aperture which might be formed by a coating of varying transmission applied to a transparent substrate. The "transmission function" has sharp corners and its Fourier transform, i.e., the "diffraction pattern," will therefore show fringes. Although this analogy indicates that the fringes will exist if $G^{(4)}(t, t+\tau; \tau, 0)$ has a sharp corner (which follows from the presence of a stochastic process), it does not, however, provide a satisfactory explanation for the maximum contrast occurring at finite correlation time. A good heuristic explanation is lacking still.

The four-wave mixing experiments [6,7] used the existence of the fringes to extract data about dephasing times of the molecules in the liquid samples. In essence, the fringes are modulated by the material properties. The results of this acoustic experiment imply that picking a field coherence time which maximizes the fringe contrast will improve the accuracy of physical parameters that are extracted from the fringe data. In the analysis of the four-wave mixing experiments, there was excellent agreement between theory and experiment. To a certain extent this agreement is unsurprising because the second-order fringes are predicted to occur with any type of stochastic field [5]. On the other hand, the analysis and interpretation of the four-wave mixing experiments depends on factorizing the higher-order correlation functions with the Gaussian moment theorem: but now we see that such fringes will occur even if the stochastic processes are non-Gaussian. This raises issues which go beyond the scope of this Letter: to what extent would deviations from Gaussian statistics make the measured quantities vary, and how are the fringes then to be interpreted?

The wave fields that we have considered so far are purely classical, but it is worthwhile to look at fields that are uniquely quantum mechanical to better put this experiment in context. It is possible to observe interference fringes in experiments with correlated photon pairs [2], where the modes into which the two photons are emitted are mixed at a beam splitter. When the photon coincidence rate is measured between the two fields leaving the beam splitter, interference fringes are seen as the beam splitter is moved. This is interpreted as a first-order interference of quantum probability amplitudes even though it is a second-order interference measurement from an operational point of view. In contrast, our experiment does not admit, in its interpretation, such an easy reduction of the order of interference, unless it is to the product of fields $E(t)E(t+\tau)$ which has no intuitively obvious significance. Now we return to the issue of spatial correlations. If the intensity spatial autocorrelation function is considered, second-order interference fringes may be observed if the field has underlying first-order interference fringes [14]. Similarly, we would also expect to see, in the temporal correlation, second-order fringes due to beats if the field had two or more discrete frequency components. In neither of these cases would the second-order fringes be expected to show the behavior illustrated in Fig. 4.

I thank the Defence Science and Technology Organisation for the donation of anechoic material for this experiment and K-H Bae for experimental assistance.

*Electronic address: mwh@physics.adelaide.edu.au

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