Faddeev-Yakubovsky Search for ${}_{\Lambda\Lambda}^{4}$H

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Evidence for particle stability of ${}_{\Lambda\Lambda}^{4}$H has been suggested by the BNL-AGS E906 experiment. We report on Faddeev-Yakubovsky calculations for the four-body $\Lambda\Lambda pn$ system using ΛN interactions which reproduce the observed binding energy of ${}_{\Lambda}^{3}$H$(\frac{1}{2}^+)$ within a Faddeev calculation for the Λpn subsystem. No ${}_{\Lambda\Lambda}^{4}$H bound state is found over a wide range of $\Lambda\Lambda$ interaction strengths, although the Faddeev equations for a three-body $\Lambda\Lambda d$ model of ${}_{\Lambda\Lambda}^{4}$H admit a 1^+ bound state for as weak a $\Lambda\Lambda$ interaction strength as required to reproduce $B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{6}\text{He})$.

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Information on hyperon-hyperon interactions is not readily available from experiments in free space. It is almost exclusively limited to the study of strangeness $S = -2$ hypernuclear systems, only a handful of which have been identified to date. This information is crucial for extrapolating into multistrange hadronic matter, for both finite systems and in bulk (Ref. [1] and references cited therein). Until recently only three candidates, identified in emulsion experiments [2–4], existed for $\Lambda\Lambda$ hypernuclei. The $\Lambda\Lambda$ binding energies deduced from these events indicated that the $\Lambda\Lambda$ interaction is strongly attractive in the ${}^1S_0$ channel [5], in fact, considerably stronger than the ΛN interaction deduced from single-Λ hypernuclei, and this seemed at odds with the natural expectation borne out in one-boson-exchange models using flavor SU(3) symmetry or within the naive quark model. For example, the recent Nijmegen soft-core (NSC97) model [6,7] yields

$$\bar{V}_{\Lambda\Lambda} \ll \bar{V}_{\Lambda N} \ll \bar{V}_{NN} \quad (1)$$

for the strength $\bar{V}$ of these essentially attractive interactions. It is gratifying then that the recent unambiguous identification of ${}_{\Lambda\Lambda}^{6}$He in the KEK hybrid-emulsion experiment E373 [8], yielding binding energy substantially lower than that deduced from the older dubious event [3], is consistent with a scattering length $a_{\Lambda\Lambda} \sim -0.5$ fm [9], indicating a considerably weaker $\Lambda\Lambda$ interaction than that specified by $a_{\Lambda N} \sim -2$ fm [6] for the ΛN interaction. With such a relatively weak $\Lambda\Lambda$ interaction, and since the three-body system $\Lambda\Lambda N$ is unbound (comparing it with the unbound Λnn system [10]), the question of whether or not the onset of binding in the $S = -2$ hadronic sector occurs at $A = 4$ becomes highly topical.

The Brookhaven alternating-gradient synchrotron (AGS) experiment E906, studying Ξ^- capture following the (K^-, K^+) reaction on ^{9}Be, has recently given evidence for excess pions that defied known single-Λ hypernuclear weak decays and were conjectured as due to the formation of ${}_{\Lambda\Lambda}^{4}$H ($I = 0$, $J^\pi = 1^+$) [11]. A subsequent theoretical study [12] of the weak-decay modes available to ${}_{\Lambda\Lambda}^{4}$H does not support this conjecture and, in our opinion, the question of whether or not ${}_{\Lambda\Lambda}^{4}$H is particle stable remains experimentally open. If it is confirmed in a future extension of E906 or of a related experiment, then this four-body system $\Lambda\Lambda pn$ would play as a fundamental role for studying theoretically the hyperon-hyperon forces as the ${}_{\Lambda}^{3}$H bound state of the three-body system Λpn has played for studying theoretically the hyperon-nucleon forces (Ref. [13] and references cited therein). Our aim in this Letter is to search theoretically for a possible ${}_{\Lambda\Lambda}^{4}$H bound state by solving the appropriate Faddeev-Yakubovsky equations for the four-body system $\Lambda\Lambda pn$, particularly for $\Lambda\Lambda$ interactions which reproduce the recently deduced binding energy of ${}_{\Lambda\Lambda}^{6}$He [9]. This is the first ever systematic Faddeev-Yakubovsky calculation done for the $A = 4$, $S = -2$ problem. It has the virtue of taking into account properly *all* the rearrangement channels (or, equivalently, clusters) into which the $\Lambda\Lambda pn$ system may be split. We note that a ${}_{\Lambda}^{3}$H bound state does not necessarily imply, for attractive $\Lambda\Lambda$ interactions, that ${}_{\Lambda\Lambda}^{4}$H is particle stable.

The ΛN and $\Lambda\Lambda$ interaction potentials used as input were of a three-range Gaussian s-wave form similar to that used by Hiyama *et al.* [14,15]:

$$V^{(2S+1)}(r) = \sum_{i}^{3} v_i^{(2S+1)} \exp\left(-\frac{r^2}{\beta_i^2}\right). \quad (2)$$

The values of the range parameters β_i and of the singlet- and triplet-strength parameters $v_i^{(2S+1)}$ are listed in Table I. The $\Lambda\Lambda$ interaction, respecting the Pauli principle, is limited to the singlet s-wave channel. The short-range term ($i = 3$) provides for a strong soft-core repulsion and the long-range term ($i = 1$) for attraction. The parameter γ, which controls the strength of the midrange attractive term ($i = 2$), was chosen such that the potential (2) reproduces the scattering length and the effective range for a given model as close as possible. Its appropriate values for ΛN are listed in Table II for two

TABLE I. Range (β) and strength (v) parameters of the three-range Gaussian potential (2).

i	β_i (fm)	$v_i^{(1)}$ (MeV)	$v_i^{(3)}$ (MeV)
1	1.342	-21.49	-21.39
2	0.777	$-379.1 \times \gamma^{(1)}$	$-379.1 \times \gamma^{(3)}$
3	0.350	9324	11359

versions of model NSC97 [6] considered realistic ones. For $\Lambda\Lambda$ we listed the value which was determined in Ref. [9] to reproduce the recently reported $B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{\ 6}\mathrm{He})$ [8]. Also listed are the values of the scattering lengths for these ΛN and $\Lambda\Lambda$ model interactions which obviously satisfy Eq. (1). For the pn triplet interaction we multiplied the ΛN potential (2) by a factor $\alpha = 2.0685$, using $\gamma_{pn}^{(3)} = 1.0498$, in order to reproduce the NN low-energy scattering parameters in this channel plus the binding energy of the deuteron. We used, for comparison, also the Malfliet-Tjon potential MT-III [16]. Our results are insensitive to which form is used.

We solved the differential Faddeev equations under the s-wave approximation [17] for the $I = 0$, $J^\pi = \frac{1}{2}^+, \frac{3}{2}^+$ ground-state doublet levels of ${}_{\Lambda}^{3}\mathrm{H}$ viewed as a Λpn system. Similar calculations for three-body systems are discussed in Ref. [9]. Some of our results are displayed in Table III. The $\frac{1}{2}^+$ ground state is bound and the calculated binding energies of the Λ hyperon (B_Λ) are in rough agreement with that observed. For model NSC97f, for example, our calculated $B_\Lambda = 0.19$ MeV agrees with that of the recent Hiyama *et al.* [19] where no s-wave approximation was invoked. The impact of the higher partial waves for ${}_{\Lambda}^{3}\mathrm{H}$ was estimated by Cobis *et al.* [20] to be of order 0.02 MeV, well within the error of the measured binding energy. Our B_Λ values satisfy the effective-range expansion in terms of Λd low-energy parameters which are close to those derived using effective field theory (EFT) methods [18]. The convergence of the Faddeev calculation using model NSC97f for the ΛN interaction is exhibited in Fig. 1 as a function of the number N of basis functions. The corresponding curve, marked "Λpn," gives the B_Λ value with respect to the horizontal straight line marked "$\Lambda + d$ threshold." The Λpn asymptote serves then for defining the lowest particle-stability threshold, that of $\Lambda + {}_{\Lambda}^{3}\mathrm{H}$, in the four-body $\Lambda\Lambda pn$ calculation described below. The $\frac{3}{2}^+$ (unobserved and probably unbound) excited state of ${}_{\Lambda}^{3}\mathrm{H}$ comes out very weakly bound in our Faddeev calculation in both versions e and f of model NSC97. In order to check the sensitivity of the four-body calculation to the location of ${}_{\Lambda}^{3}\mathrm{H}(\frac{3}{2}^+)$, we give below results also for model NSC97f', where f' coincides with f for the $\frac{1}{2}^+$ channel but slightly departs from it for the $\frac{3}{2}^+$ channel as shown in Table III.

TABLE II. Values of the parameter $\gamma^{(2S+1)}$ appropriate for simulating the Λp potentials of model NSC97 and for a $\Lambda\Lambda$ potential reproducing $B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{\ 6}\mathrm{He})$. The resulting scattering lengths a (in fm) are also listed.

Model	$\gamma^{(1)}$	${}^1a$	$\gamma^{(3)}$	${}^3a$
ΛN: NSC97e	1.0133	-2.10	1.0629	-1.84
ΛN: NSC97f	1.0581	-2.50	1.0499	-1.75
$\Lambda\Lambda$: ${}_{\Lambda\Lambda}^{\ 6}\mathrm{He}$ [9]	0.6598	-0.77	$\cdots$	$\cdots$

TABLE III. $B_\Lambda[{}_{\Lambda}^{3}\mathrm{H}(\frac{1}{2}^+)]$ and Λd low-energy doublet scattering parameters (${}^2B_\Lambda$ in MeV; ${}^2a$, ${}^2r$ in fm) calculated for the $I = 0$ Λpn system. ${}^4B_\Lambda^{\mathrm{scatt}}$ (in MeV) for ${}_{\Lambda}^{3}\mathrm{H}(\frac{3}{2}^+)$ was obtained using the effective-range expansion in the quartet Λd channel.

Model	${}^2a$	${}^2r$	${}^2B_\Lambda$	${}^4B_\Lambda^{\mathrm{scatt}}$
NSC97e	20.7	2.61	0.069	0.015
NSC97f	13.1	2.46	0.193	0.003
NSC97f'	13.1	2.46	0.193	-0.003
EFT [18]	$16.8^{+4.4}_{-2.4}$	2.3 ± 0.3	0.13 ± 0.05	
exp.			0.13 ± 0.05	

Focusing on the $\Lambda\Lambda pn$ Faddeev-Yakubovsky calculation, we note that for two identical hyperons and two essentially identical nucleons (upon introducing isospin), as appropriate to the $I = 0$, $J^\pi = 1^+$ ground state of ${}_{\Lambda\Lambda}^{\ 4}\mathrm{H}$, the 18 Faddeev-Yakubovsky components which satisfy coupled equations reduce to seven independent components, in close analogy to the Faddeev-Yakubovsky equations discussed in our recent work [9] for the $\Lambda\Lambda\alpha\alpha$ model of ${}_{\Lambda\Lambda}^{\ 10}\mathrm{Be}$. Six rearrangement channels are involved in our s-wave calculation for ${}_{\Lambda\Lambda}^{\ 4}\mathrm{H}$:

$$(\Lambda NN)_{S=1/2} + \Lambda, \qquad (\Lambda NN)_{S=3/2} + \Lambda, \qquad (\Lambda\Lambda N)_{S=1/2} + N \tag{3}$$

for 3 + 1 breakup clusters, and

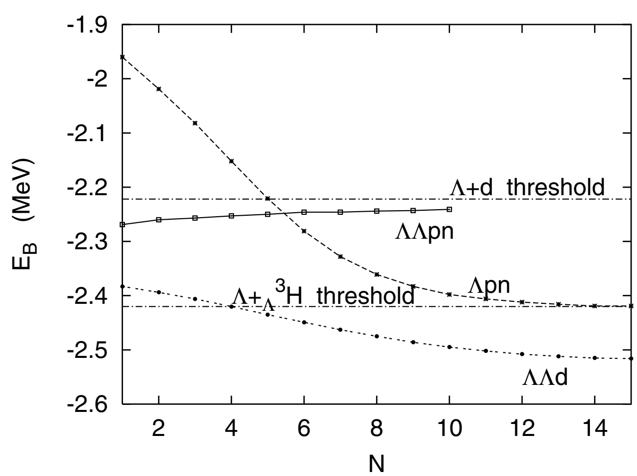

FIG. 1. Convergence of Faddeev-Yakubovsky calculations for the binding energy of the Λpn ($S = 1/2$), $\Lambda\Lambda d$, and $\Lambda\Lambda pn$ ($S = 1$) systems with respect to the number N of basis functions. Values of $R_{\mathrm{cutoff}} = 30$ fm for $\Lambda\Lambda pn$ and Λpn, and 60 fm for $\Lambda\Lambda d$, were used. The $\Lambda\Lambda$ interaction is due to Table II.

$$(\Lambda\Lambda)_{S=0} + (NN)_{S=1}, \qquad (\Lambda N)_S + (\Lambda N)_{S'} \qquad (4)$$

with $(S, S') = (0, 1) + (1, 0)$ and $(1, 1)$ for $2 + 2$ breakup clusters. We find invariably that the three rearrangement channels, in which the two nucleons belong to the same d-like cluster, dominate in actual calculations. This observation, apparently, could justify the use of a $\Lambda\Lambda d$ model for ${}_{\Lambda\Lambda}^{4}$H. However, as we shall see and discuss below, the results of using such a three-body model differ radically from those of the full four-body Faddeev-Yakubovsky calculations which retain the proton and neutron as dynamically independent entities.

Using the $\Lambda\Lambda$ interaction which reproduces $B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{6}\mathrm{He})$ (see Table II), our calculations yield no bound state for the $\Lambda\Lambda pn$ system, as demonstrated in Fig. 1 by the location of the "$\Lambda\Lambda pn$" curve *above* the horizontal straight line marking the "$\Lambda + {}_{\Lambda}^{3}$H threshold" [21]. In fact, our Faddeev-Yakubovsky calculations exhibit little sensitivity to the strength of the $\Lambda\Lambda$ interaction over a wide range, including much stronger $\Lambda\Lambda$ interactions such as the Nijmegen D (ND) model and the extended soft core 2000 (ESC00) model discussed in Ref. [9], the latter one reproducing the (excessive) $B_{\Lambda\Lambda}$ value reported for the "old" ${}_{\Lambda\Lambda}^{6}$He event [3]. For these $\Lambda\Lambda$ interactions, we get a bound ${}_{\Lambda\Lambda}^{4}$H only if the ΛN interaction is made considerably stronger, by as much as 40%. With four ΛN pairwise interactions out of a total of six, the strength of the ΛN interaction (here about half of that for NN binding the deuteron) plays a major role in the four-body $\Lambda\Lambda pn$ problem. In passing, we remark that this is also apparent from the bounds derived in Ref. [22] for the four-body bound-state problem. Put differently, we know of no few-body theorem that would imply, for essentially attractive $\Lambda\Lambda$ interactions and for a *nonstatic* nuclear core d (made out of pn in the present case), the existence of a $\Lambda\Lambda d$ bound state provided that Λd is bound. It is a remarkable outcome of the complete Faddeev-Yakubovsky scheme for four particles that such a natural expectation can be refuted by a specific calculation. However, for a *static* nuclear core d, and disregarding inessential complications due to spin, a two-body Λd bound state does imply binding for the three-body $\Lambda\Lambda d$ system [23]. A discussion of the formal relationship between these four-body and three-body models which do not share a common Hamiltonian is deferred to a subsequent publication.

Our $\Lambda\Lambda d$ model for ${}_{\Lambda\Lambda}^{4}$H uses the $\Lambda\Lambda$ interaction marked "${}_{\Lambda\Lambda}^{6}$He" in Table II plus Λd interactions that reproduce the low-energy parameters of the Λpn Faddeev calculation specified in Table III. The dependence on the functional form chosen for the interpolating Λd interaction potentials proved relatively mild. The results of such a $\Lambda\Lambda d$ three-body Faddeev calculation using model NSC97f for the underlying ΛN interaction are shown in Fig. 1 as function of the number N of basis functions used in the expansion of the Faddeev components. The asymptote of the curve marked $\Lambda\Lambda d$ is now located *below* the horizontal straight line for the $\Lambda + {}_{\Lambda}^{3}$H threshold, so ${}_{\Lambda\Lambda}^{4}$H is particle stable. The figure may suggest that a Λ in ${}_{\Lambda\Lambda}^{4}$H is less bound, by about 0.1 MeV, than a Λ in ${}_{\Lambda}^{3}$H (which in model NSC97f is bound by about 0.2 MeV). However, the $(2J + 1)$ spin-averaged effective $B_{\Lambda}({}_{\Lambda}^{3}\mathrm{H})$ in ${}_{\Lambda\Lambda}^{4}$H is only $\bar{B}_{\Lambda} = 0.07$ MeV and, since $B_{\Lambda\Lambda} \sim 0.3$ MeV, we have $B_{\Lambda\Lambda} > 2\bar{B}_{\Lambda}$, which is equivalent to stating loosely that the second Λ in ${}_{\Lambda\Lambda}^{4}$H is bound even more strongly than the first one. This holds also for model NSC97e and it is a general property of the Faddeev calculation [9].

In Fig. 2 we show $B_{\Lambda\Lambda}$ values calculated for ${}_{\Lambda\Lambda}^{4}$H within this $\Lambda\Lambda d$ Faddeev model as a function of $\bar{V}_{\Lambda\Lambda}$ (quantified by the value of the scattering length $a_{\Lambda\Lambda}$) for two exponential Λd potentials corresponding to versions f and f' of model NSC97 (see Table III). The roughly linear increase of $B_{\Lambda\Lambda}$ holds generally in three-body $\Lambda\Lambda C$ models (C standing for a cluster) over a wide range of values for $\bar{V}_{\Lambda\Lambda}$ [9]. For values $B_{\Lambda\Lambda} \leq 0.2$ MeV, ${}_{\Lambda\Lambda}^{4}$H becomes unstable against emitting a Λ. This onset of particle stability for ${}_{\Lambda\Lambda}^{4}$H requires a minimum strength for the $\Lambda\Lambda$ interaction which is satisfied for our choice of ${}_{\Lambda\Lambda}^{6}$He as a normalizing datum. It is also seen from the figure that the uncertainty in the location of ${}_{\Lambda}^{3}\mathrm{H}(\frac{3}{2}^{+})$ bears serious consequences for the predicted binding of ${}_{\Lambda\Lambda}^{4}$H; this is a particularly strong effect as the $\frac{3}{2}^{+}$ state crosses the $\Lambda + d$ threshold. Yet, we would like to emphasize that no such sensitivity emerges within a genuine four-body model calculation which does not bind ${}_{\Lambda\Lambda}^{4}$H as long as the ΛN interaction is of the size constrained by single-Λ hypernuclear phenomenology.

In cluster models of the type $\Lambda\Lambda C$ and $\Lambda\Lambda C_1 C_2$ for heavier $\Lambda\Lambda$ hypernuclei, where the nuclear-core cluster $C = C_1 + C_2$ is made out of subclusters C_1 and C_2, the

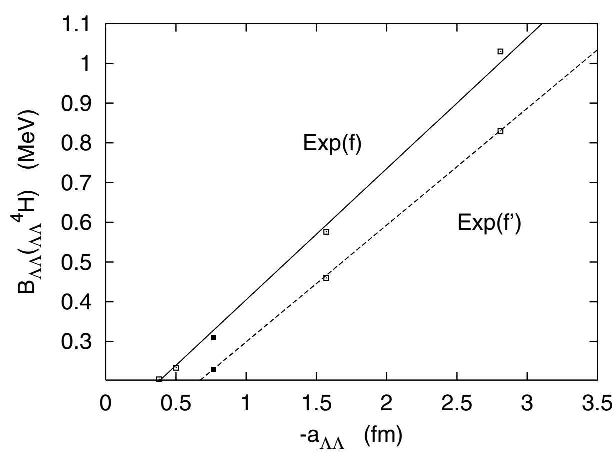

FIG. 2. $B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{4}\mathrm{H})$ calculated in a three-body $\Lambda\Lambda d$ model as a function of the scattering length $a_{\Lambda\Lambda}$, for two exponential Λd potentials corresponding to versions f and f' in Table III. The solid squares correspond to the ${}_{\Lambda\Lambda}^{6}$He $\Lambda\Lambda$ interaction of Table II. The straight lines are drawn only to lead the eye.

ΛC_j interaction (normally producing bound states) is considerably stronger than for ΛN. Our experience with Faddeev-Yakubovsky calculations for ${}^{10}_{\Lambda\Lambda}$Be [9], viewed as a four-body $\Lambda\Lambda\alpha\alpha$ system, is that the relationship between the three-body and four-body models is then opposite to that found here for ${}^{4}_{\Lambda\Lambda}$H: the $\Lambda\Lambda C_1 C_2$ calculation under similar conditions provides *higher* binding than the $\Lambda\Lambda C$ calculation yields. The mechanism behind it is the attraction induced by the ΛC_1-ΛC_2, $\Lambda\Lambda C_1$-C_2, C_1-$\Lambda\Lambda C_2$ four-body rearrangement channels that include bound states for which there is no room in the three-body $\Lambda\Lambda C$ model. The binding energy calculated within the four-body model increases then "normally" with $\bar{V}_{\Lambda\Lambda}$.

In conclusion, we have provided a first four-body Faddeev-Yakubovsky calculation for ${}^{4}_{\Lambda\Lambda}$H using NN and ΛN interaction potentials that fit the available data on the relevant subsystems, including the binding energy of ${}^{3}_{\Lambda}$H. No bound state is obtained for ${}^{4}_{\Lambda\Lambda}$H over a wide range of $\Lambda\Lambda$ interaction strengths, including that normalized to reproduce the binding energy of ${}^{6}_{\Lambda\Lambda}$He. We have traced the origin of this nonbinding as due to the relatively weak ΛN interaction. This is in stark contrast to the results of a "reasonable" three-body $\Lambda\Lambda d$ Faddeev calculation that binds ${}^{4}_{\Lambda\Lambda}$H provided the $\Lambda\Lambda$ interaction is not too weak, say, with $-a_{\Lambda\Lambda} \geq 0.5$ fm.

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