

## Paramagnetic and Diamagnetic States in Two-Dimensional Josephson-Junction Arrays

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Many experiments on high-temperature superconductors have shown paramagnetic behavior when the sample is field cooled. The paramagnetism was attributed to a  $d$ -wave order parameter creating  $\pi$ -junctions in the samples. However, the same effect was later discovered in traditional low-temperature superconductors and conventional Josephson-junction arrays which are  $s$  wave. By simulating *both* conventional and mixed  $\pi$ /conventional Josephson-junction arrays we determine that differences exist which may be sufficient to clearly identify the presence of  $\pi$  junctions. In particular, the  $\pi$  junctions cause a symmetry breaking providing a measurable signature of their presence.

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In recent years a significant discussion in the superconductivity field was devoted to the paramagnetic response in field-cooled samples, the so-called paramagnetic Meissner effect (PME) [1]. PME was first observed in high- $T_c$  ceramic materials [1–3] and then also in low- $T_c$  samples [4,5]. A mechanism to explain the effect was introduced by Sigrist and Rice [6] where PME was due to the presence of  $\pi$  junctions in the multiple-connected network of junctions formed by the superconducting grains in the high- $T_c$  ceramic.  $\pi$  junctions are Josephson junctions formed between superconductors with unconventional pairing which cause a  $\pi$  shift in the phase-current relation [7,8]. Unconventional pairing, such as a  $d$ -wave order parameter, was introduced to explain the properties of high- $T_c$  ceramic materials [9]. PME was taken as strong evidence for a  $d$ -wave order parameter [6,10]. Multiple-connectiveness and  $\pi$  junctions were soon shown to give a paramagnetic response in simulated Josephson-junction arrays by Dominguez, Jagla, and Balseiro [11].

The discovery of PME in conventional low- $T_c$  (LTC) samples shows that PME cannot always be attributed to  $\pi$  junction because they are present only in  $d$ -wave superconductors (or in unconventional superconductor-ferromagnet systems [8]). New explanations such as a giant flux state [12], flux compression [13], and surface states [14] have been introduced. The common feature was that PME in LTC samples is described as nonequilibrium phenomenon.

Very recently a new experiment was devised to test the connection between multiconnectiveness and PME in conventional systems. A square array of LTC junctions was field cooled and shown to be predominantly paramagnetic [15]; this paper also proposed a qualitative explanation for the effect. A subsequent analysis performed by numerical simulations of the arrays confirmed the presence of PME in LTC arrays [16]. On this basis the essential ingredient of PME appears to be multiple-

connectiveness rather than the presence of  $\pi$ -junction or nonequilibrium effects.

It is clear that simulated Josephson-junction arrays both with [11] and without [16]  $\pi$  junctions show PME. Here we investigate by means of numerical simulations the differences between the two cases and the true role of the  $\pi$  junctions. We also suggest an experimental signature of the presence of  $\pi$  junctions in field-cooled array samples.

The array is described by means of a full mutual inductance model similar to that used in Ref. [16]. The main difference is the presence of some  $\pi$  junction, i.e., Josephson junctions where supercurrent is proportional to  $\sin(\varphi + \pi)$  instead of  $\sin\varphi$  (cf. Fig. 1). The dynamics of an  $N \times N$  array are described by the following equations [16–18]:

$$\frac{\beta_L}{2\pi} (1 + \delta) \sin(\vec{\varphi} + \vec{\psi}) + \sqrt{\frac{\beta_L}{\beta_C}} \vec{\varphi} + \vec{\varphi} = \hat{K}\hat{L}^{-1}\vec{m}. \quad (1)$$

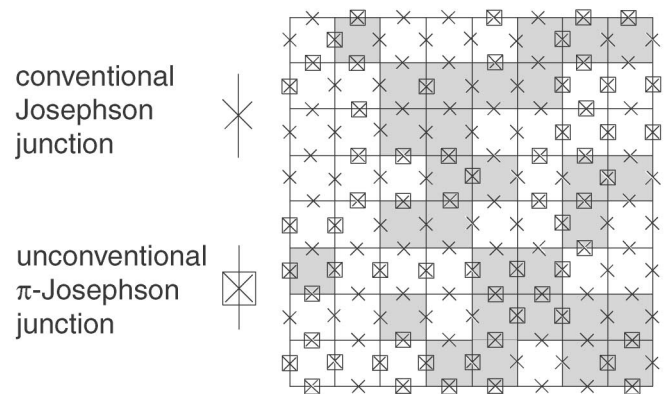


FIG. 1. Two-dimensional Josephson-junction array with a random distribution of  $\pi$  junctions. The gray loops represent the  $\pi$  loops, i.e., loops with an odd number of  $\pi$  junctions. The figure is just the top left corner of the mixed array in Fig. 2(b).

Here time is normalized to a cell frequency ( $\omega^{-2} = L'C$ ), with  $L'$  the self-inductance of the elementary loops formed of the array  $C$  the junction capacitance.  $\beta_C$  is the Stewart-McCumber parameter  $\beta_C = 2\pi I_0 C / \Phi_0 G^2$ , where  $I_0$  is the (mean) Josephson critical current for the junctions in the array,  $\Phi_0$  is the flux quantum, and  $G$  is the junction conductance.  $\beta_L$  is equal to  $2\pi L' I_0 / \Phi_0$ .  $\vec{m}$  represents the normalized loop magnetization (in unit of  $\Phi_0$ ) which depends on frustration  $f$ , phase vector  $\vec{\varphi}$ , and the initial distribution of flux quanta in the array loops  $\vec{n}$ , here chosen to be a random integer vector [16].  $\hat{K}$  and  $\hat{L}^{-1}$  are the matrices carrying the mutual inductance coupling. The vector  $\vec{\psi}$  has values of 0 or  $\pi$ , providing a  $\pi$  shift to a subset of the junctions in the array. The concentration  $c$  of  $\pi$  junction [11] is the ratio between the number of  $\pi$  junctions and the total number of junctions. The vector  $\vec{\delta}$  is a Gaussian variable with zero mean which we use to introduce disorder in the distribution of critical currents.

Here we suppose that in the array the contribution of the screening currents is not negligible, i.e.,  $\beta_L > 1$  (cf. [3]). In particular we choose  $\beta_L = 30$  and  $\beta_C = 63$  (cf. Refs. [15,16]).

In both experiments [15] and previous simulations [16] it was shown that multiple-connectiveness effects are dominant for large field, i.e., for large values of frustration  $f$ . To understand the effect of the  $\pi$  shift we studied smaller values of frustration  $0 < f < 1$ . In this region the behavior of an array without disorder and with all normal or all  $\pi$  junctions roughly follows the single-loop behavior [15,16,19] which is described by

$$\frac{\Phi}{\Phi_0} - f = \frac{\beta_L}{2\pi} \sin\left(\frac{\pi}{2}n - \frac{\psi}{4} - \frac{\Phi}{\Phi_0}\right), \quad (2)$$

where  $\Phi$  is the total flux through the loop. The solution of Eq. (2) gives the possible states for the total flux  $\Phi$  (or the current) in the single loop. For  $\psi = 0$ , Eq. (2) describes a single loop made by four conventional Josephson junctions. There are four independent solutions which are obtained varying the integer  $n$  from zero to three. The (stable) lowest Gibbs energy state is diamagnetic for  $0 < f < 1/2$ , i.e., the loop current is negative, generating a magnetic moment opposite to the external field, and paramagnetic for  $1/2 < f < 1$ , i.e., the loop current is positive [15,16]. On the other hand, if  $\psi = \pi$  the  $\pi$  shift will reverse the behavior: for the  $\pi$  loop (a loop with an odd number of  $\pi$  junctions) the lowest Gibbs energy state is paramagnetic for  $0 < f < 1/2$  and diamagnetic for  $1/2 < f < 1$ .

In Ref. [16] we showed that conventional-array behavior can be qualitatively understood in terms of a model based on the single-loop behavior. We found that each loop in the array had a magnetization that was close to one of the lowest energy solutions of Eq. (2) for the isolated loop, with the number of loops of each type

determining the overall magnetization. This simple picture also works for arrays with both conventional and  $\pi$  junctions, with the number of possible solutions being increased because that Eq. (2) has additional solutions for the  $\pi$  loops.

In order to compare conventional arrays with mixed conventional/ $\pi$  arrays, we first simulate the conventional arrays using the method of Ref. [16]. In Fig. 2(a) the magnetization of a  $20 \times 20$  conventional array with  $f = 0.2$  is shown. There are two single-loop magnetization states with a predominance of diamagnetic states (light gray) over which a set of paramagnetic loops (dark gray) are randomly distributed. The histogram of loop magnetizations is shown in Fig. 3(a) and shows clearly only two peaks corresponding to two loop magnetizations. The mean magnetization is negative and small as expected for such small values of frustration accordingly to Eq. (2) for the single loop; the exact value is  $m = -0.0027$ . As said above the values of the single peak averages can be obtained from Eq. (2); the small spread of the peaks is due to mutual inductance coupling.

In Fig. 2(b) a similar  $20 \times 20$  array with  $f = 0.2$  is shown in which 380  $\pi$  junctions have been introduced at random. This is equivalent to a concentration of  $\pi$  junctions of  $c = 0.45$ . The main difference with the case of conventional array is the presence of four magnetization states, two for each type of loop. We note that the new  $\pi$ -loop states are predominantly paramagnetic according to the symmetry shift discussed above. This, together with the lower number of conventional loops, is causing the reversal of the mean magnetization (here being  $m = +0.0018$ ). This is the way in which a mixed array becomes paramagnetic for low values of frustration. Lower concentrations will make the array diamagnetic (i.e., the dependence on concentration is similar to that reported in Fig. 3b of Ref. [11] for a 3D system). For other values of  $f$  we note that the single-loop symmetry is broken, i.e., for

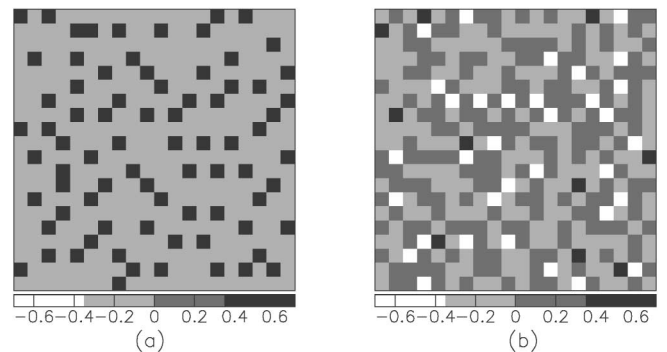


FIG. 2. Simulated magnetization of a  $20 \times 20$  array at  $f = 0.2$  with  $\beta_L = 30$  and  $\beta_C = 63$ : (a) without  $\pi$  junctions, light gray represents diamagnetic loops, gray paramagnetic ones; (b) with  $\pi$  junctions at a concentration of  $c = 0.45$  white and light gray represents diamagnetic loops, gray and dark gray paramagnetic ones.

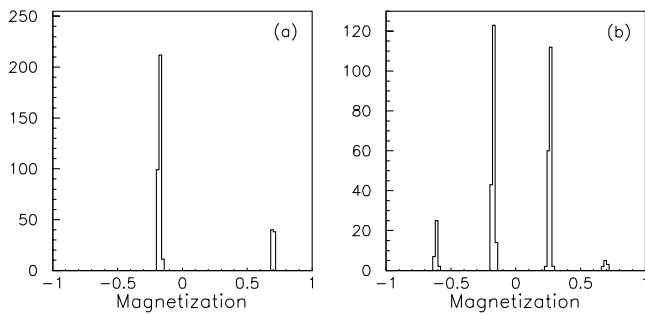


FIG. 3. Magnetization histogram of a  $20 \times 20$  plane array at  $f = 0.2$  with  $\beta_L = 30$  and  $\beta_C = 63$ : (a) distribution without  $\pi$  junctions; (b) distribution with  $\pi$  junctions at a concentration of  $c = 0.45$ .

$f < 1/2$  the array can be diamagnetic or paramagnetic depending on the distribution and number of  $\pi$  loops.

From the preceding discussion, it is clear that there are at best only subtle differences between a conventional array and an array containing a mixture of conventional and  $\pi$  junctions. In the remainder of the paper we will show that when  $f = 1/2$  these differences are the most pronounced, and therefore  $f = 1/2$  is probably the best place to look experimentally in searching for evidence of  $\pi$  junctions in nonzero magnetic field. When  $f = 1/2$  the problem is simplified by the fact that the four lowest states shrink to three because the  $\pi$ -loop magnetization becomes zero. So one is confronted with only three peaks which are symmetrically distributed around the zero magnetization. In Fig. 4(a) this case is shown in a magnetization histogram. The central peak is the zero magnetization of  $\pi$  loops and two symmetric lateral peaks are due to conventional loops. Symmetrical histogram structure is an optimal “signature,” it is easily recognized with SQUID scanning microscope and statistical filtering or image filtering techniques can be applied knowing the true distribution [20].

The central peak intensity is related to presence of  $\pi$  loops. At  $c = 0.3$  (250  $\pi$  junctions) it becomes thinner preserving the same height. Then it decreases by about 10% at  $c = 0.15$  (126  $\pi$  junctions). Finally at  $c = 0.05$  (42  $\pi$  junctions) it decreases of about 60%.

In Fig. 4(a) a weak Gaussian disorder with a standard deviation of  $\sigma = 20\%$  was introduced in the critical currents of the arrays similarly to Ref. [21]. This was added to show that weak disorder does not change the three-peak structure of the magnetization histogram. For high values of the parameter  $\beta_L$  the nature of the magnetization states is discrete and this is preserved for weak disorder in the currents. Peak structure exists also with stronger disorder in the critical currents. In Fig. 4(b) we set  $\sigma = 80\%$  [22] and we see peaks enlarging until the tails of the single three distributions overlap. It is observed that the remaining discrete structure is due again to the large  $\beta_L$  loops. Also we have simulated the array with the same parameters of Fig. 4(b) using a mean value

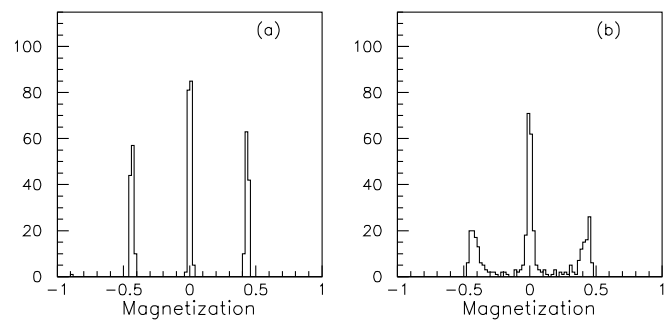


FIG. 4. Magnetization histogram of a  $20 \times 20$  plane array at  $f = 0.5$  with  $\beta_L = 30$ ,  $\beta_C = 63$ , and  $\pi$ -junctions distribution at  $c = 0.45$ : (a) effect of a weak disorder in the current distribution ( $\sigma = 20\%$ ); (b) the same for a stronger disorder ( $\sigma = 80\%$ ).

of  $1/3$  for the  $\pi$ -junction critical current. The result is again similar to Fig. 4(b) but that central peak is lowered by roughly 13% and the histogram appears more enlarged.

In conclusion, we would stress again how both conventional and mixed conventional/ $\pi$  arrays show a similar response when field cooled. This response is related to the single-loop solutions of Eq. (2). In low field  $f < 0.5$  paramagnetism in mixed conventional/ $\pi$  arrays is the result of the paramagnetic behavior of  $\pi$  loops. Quantifying this effect for a disorder system is difficult because it depends on distribution and the number of  $\pi$  loops. On the other hand, a noteworthy point is  $f = 1/2$ . Here the  $\pi$  loops sets to zero current so it is possible from a measurement of the magnetization histogram to trace back the presence of  $\pi$  loops in the sample. For High- $T_c$  materials this experiment can be an important complement of the search for spontaneous currents in zero field [23]. Correlation between  $f = 0$  and  $f = 1/2$  would be of great interest permitting one to estimate the  $\pi$ -loops content of the sample.

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