Near-Threshold Reflectivity Fluctuations in the Independent-Convective-Hot-Spot-Model Limit of a Spatially Smoothed Laser Beam

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In the framework of the independent-hot-spot model, it is shown that the reflectivity resulting from scattering instabilities when a spatially smoothed laser beam interacts with a plasma exhibits large statistical fluctuations near threshold. The importance of the fluctuations is discussed in terms of a confidence interval for the reflectivity, which is more relevant to experimental measurements than the average reflectivity. An analytical model for the fluctuating reflectivity is developed and shown to be in good agreement with numerical simulations. The influence of the transverse size of the interaction region is studied.

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Much experimental and theoretical work has been devoted over the last two decades to studying the influence of laser beam smoothing on scattering instabilities. In the case of spatial smoothing, such as random phase plates (RPP) [1], a good idealized model of the physics into play can be obtained by regarding the laser-plasma system as a stochastic convective amplifier driven by the square of a Gaussian field. The most important result obtained in the linear limit of this model is the concept of critical intensity $I_c(n)$ defined as the average laser intensity at which the nth moment of the *linear* reflectivity diverges [2]. Experimentally, one expects $I_c(1)$ to correspond to the threshold of the instability. Of course, the divergences associated with $I_c(n)$ reveal a breakdown in the validity of the model which neglects both nonlinear saturation and transient time evolution [2,3]. Physically, they can be interpreted as indicating a change in the nature of the amplification from a regime where it is dominated by the bulk of the laser field to a regime where it is dominated by its high overintensities (or hot spots) only [4]. This interpretation has led to a simplified version of the stochastic amplifier model, the so-called "independent hot spot model" (or IHS model) [2], in which the reflectivity is the sum of contributions from hot spots randomly distributed in the interaction region.

Taking the concept of critical intensity seriously, one has to face the following problem: since $I_c(2) = I_c(1)/2 < I_c(1)$, one cannot *a priori* rule out the possibility that large reflectivity fluctuations occur around the threshold of the average reflectivity. In such a case, the validity of the average reflectivity and its associated critical intensity $I_c(1)$ as estimates of the experimental reflectivity and threshold might be questionable. In this Letter, this problem is addressed for the first time by studying near-threshold statistical fluctuations of the reflectivity, i.e., fluctuations from one realization of the RPP field to an other one, in the context of the IHS model. We show that, although the average reflectivity can be significantly different from the experimental value, its critical intensity $I_c(1)$ remains a physically relevant quantity corresponding to the threshold of both the reflectivity fluctuations and the most likely reflectivity. This result, interpreted in terms of intensity fluctuations of the hottest spot, leads us to propose a simple analytical model of the fluctuating reflectivity yielding statistical information easily and for a wide range of plasma and optics parameters. For the sake of completeness, it should be noticed that beside the laser-plasma interaction context, the IHS model is also relevant to the interaction of a smoothed laser beam with other nonlinear media like, e.g., liquids and crystals. As examples, one can mention the problems of optic damaging by a partially incoherent laser, stimulated Brillouin scattering in lens, and stimulated Raman scattering in crystals [5].

In this Letter we consider the case of convective amplification in a weakly inhomogeneous plasma in which the resonance length for a given wave triplet is comparable to the hot spot length. Backscattering in each hot spot can then be treated as in a homogeneous plasma, whereas multiple amplifications in successive hot spots can be neglected due to the fact that the light backscattered in a given hot spot is out of resonance in any other hot spot it encounters on its way out of the interaction region. In this limit, it has been shown in Ref. [6] that backscattering in each hot spot can be obtained by considering an appropriate effective cylindrical hot spot of length L and waist w_0. For a (3D) circular top-hat RPP, one has $L = 1.99z_c$ and $w_0 = 0.64\rho_c$. Here ρ_c and z_c are defined by $\rho_c = f\lambda_0$ and $z_c = \pi f^2\lambda_0$, where f is the f number and λ_0 is the laser wavelength. One can then apply the results of Ref. [7] in the time-independent convective regime. It must be stressed that the weakness of the inhomogeneity is *not* a key assumption of our theory. One could easily make it suitable to the case of a strongly inhomogeneous plasma by replacing the results of Ref. [7] by their inhomogeneous counterparts from Ref. [8]. In the case of stimulated Brillouin backscattering with $0.01 \lesssim \nu_S/\omega_S \lesssim 0.1$, where ν_S and ω_S, respectively, denote the linear damping and the angular frequency of the ion acoustic wave, we have checked that neither self-focusing nor self-induced smoothing occurs in the range of laser intensities we consider. We have also checked that the backscattering in each hot spot saturates nonlinearly at a hot spot intensity lower than the absolute instability threshold, so that only convective hot spots have to be considered. [Note that for low damping ($\nu_S/\omega_S \sim 0.01$) a relatively large f-number is needed (typically, $f \gtrsim 9$)].

According to the results of Ref. [7], the power backscattered by a convective hot spot is given by $P_{\rm HS}(u) = \pi w_0^2 u\langle I\rangle \alpha_1(u)\min[R_{\rm lin}(u), R_{\rm sat}]$, where $\langle I\rangle$ is the average laser intensity, $u \equiv I/\langle I\rangle$ is the normalized hot spot intensity, and $R_{\rm sat} \leq 1$ is the maximum hot spot reflectivity. The linear hot spot reflectivity is given by $R_{\rm lin}(u) = B\alpha_2(u)[\exp(gu) - 1][gu(1 + Q_{\rm 3D})]^{-1}$, where B is the noise level and g is the average hot spot convective gain (for intensity). The geometrical factors α_1 and α_2 are given by $\alpha_1(u) = \min(1, \Delta\Omega_0/\Delta\Omega_{sc})$ and $\alpha_2(u) = (S_{sc}\Delta\Omega_{\rm noise})/(\pi w_0^2\Delta\Omega_0)$. The quantities $Q_{\rm 3D}$, S_{sc}, $\Delta\Omega_0$, $\Delta\Omega_{sc}$, and $\Delta\Omega_{\rm noise}$ are defined in section IV of Ref. [7]. For a given realization of the RPP field, the hot spot contribution to the overall macroscopic reflectivity reads

$$R_{\rm HS} = \int r_{\rm HS}(u)dN(u), \tag{1}$$

where $r_{\rm HS}(u) \equiv P_{\rm HS}(u)/(S_{\rm int}\langle I\rangle)$, $S_{\rm int}$ is the interaction region cross section, and $dN(u) \equiv N(u + du) - N(u)$ where $N(u)$ is the number of hot spots of intensity $I/\langle I\rangle \leq u$. Since $N(u)$ changes from one realization to another, $R_{\rm HS}$ is a random variable which depends on the realization of the RPP field. The statistics of $N(u)$ we have used to study the fluctuations of $R_{\rm HS}$ has been obtained numerically from a sample of 10 000 realizations of a 2D top-hat RPP field. The size of the simulation box was $L_x = 350$ μm and $L_z = 300$ μm. The half-width of the laser spectral density was $k_{\rm max} = (2\pi/\lambda_0)(1 + 4f^2)^{-1/2}$ with $\lambda_0 = 0.35$ μm and $f = 3$. With these parameters there were $\sim$600 hot spots (i.e., local maxima of the laser intensity with $u \gtrsim 3$) for each realization. The fact that the hot spot statistics is 2D introduces some arbitrariness in the definition of $S_{\rm int}$. A reasonable value compatible with both our 3D expression of $P_{\rm HS}(u)$ and the 2D hot spot statistics is $S_{\rm int} = 2L_x w_0$, which corresponds to a flat interaction region with $L_y = 2w_0 \ll L_x \simeq L_z$. The remainder of this Letter is devoted to a study of the statistical fluctuations of $R_{\rm HS}$ as given by Eq. (1).

Figure 1 shows the sample estimates of the average reflectivity, $\langle R_{\rm HS}\rangle \equiv (1/n)\sum_{i=1}^n R_i$, and the standard deviation (normalized to $\langle R_{\rm HS}\rangle$), $\Delta R_{\rm HS} \equiv [\sum_{i=1}^n (R_i - \langle R_{HS}\rangle)^2/(n-1)]^{1/2}$, as a function of g for typical parameters $B = 10^{-9}$ and $R_{\rm sat} = 1$. Here $n = 10\,000$ is the number of realizations of the RPP field and R_i is given by Eq. (1) for each realization of $N(u)$. It can be seen that $\Delta R_{\rm HS}/\langle R_{\rm HS}\rangle$ starts to grow at $g = 1/2$, which corresponds precisely to the theoretical value of $I_c(2)$. It remains negligible until $g \simeq 1$ [corresponding to $I_c(1)$], where it passes $\Delta R_{\rm HS}/\langle R_{\rm HS}\rangle = 1$. Then, it reaches a maximum of $(\Delta R_{\rm HS}/\langle R_{\rm HS}\rangle)_{\rm max} = 33$ at $g = 1.48$. The subsequent decrease is due to the finite maximum possible value $R_{\rm sat}$ of the hot spot reflectivity. We have observed very similar increases of $\Delta R_{\rm HS}/\langle R_{\rm HS}\rangle$ near the threshold of the average reflectivity [with $(\Delta R_{\rm HS}/\langle R_{\rm HS}\rangle)_{\rm max} \gtrsim 20$] over the whole range $10^{-10} \leq B \leq 10^{-6}$, $10^{-2} \leq R_{\rm sat} \leq 1$, with $B/R_{\rm sat} \leq 10^{-6}$.

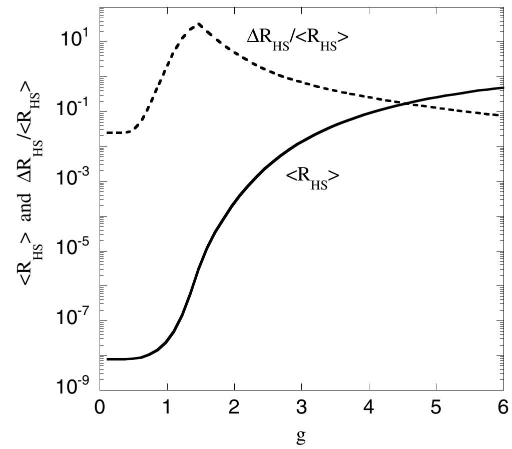

FIG. 1. Average reflectivity (solid line) and relative standard deviation (dotted line) obtained from 10 000 realizations of a 2D RPP field as a function of the average hot spot convective gain g. Geometrical and plasma parameters are given in the text.

These results suggest that $\langle R_{\rm HS}\rangle$ might not be a good estimate of the experimental reflectivity near the amplification threshold. To clarify this point, we consider the confidence interval obtained from the histogram of the reflectivity, and defined as the smallest reflectivity interval containing 90% of the R_is. Experimentally, the measured reflectivity lies in this confidence interval 9 times out of 10. Figure 2 shows the bounds of the confidence interval for the same typical parameters as in Fig. 1, as well as the median reflectivity defined such that half of the R_is in the confidence interval lies above and below it. It can be seen that for $1 < g \lesssim 2$ the average reflectivity lies *outside* the confidence interval, significantly overestimating the experimental reflectivity. Such a strong discrepancy between the average and most likely reflectivity must be attributed to the very asymmetrical shape of the reflectivity histogram near the amplification threshold. It can also be seen in Figs. 1 and 2 that, although the fluctuations start to grow at $g = 1/2$, they are

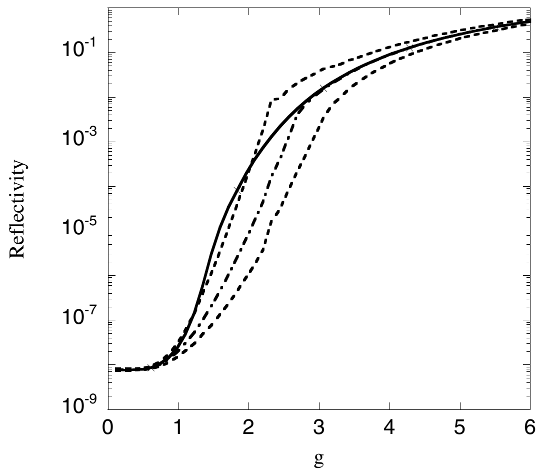

FIG. 2. The 90% confidence interval (dotted lines), median reflectivity (dot-dashed line), and average reflectivity (solid line) as a function of g. Parameters are the same as in Fig. 1.

not significant until the laser intensity reaches the threshold for the reflectivity, $g \simeq 1$, corresponding to $\langle I \rangle \simeq I_c(1)$. Again, we have observed similar behaviors over a wide range of B and $R_{\rm sat}$ (cf. discussion of Fig. 1). These results lead us to the following conclusions: (i) for typical parameters, $\langle R_{\rm HS} \rangle$ is not a good estimate of the experimental reflectivity near the amplification threshold; and (ii) $I_c(1)$ remains a physically relevant quantity as the threshold of both the reflectivity fluctuations and the most likely reflectivity.

As previously mentioned, at $g = 1$ [or $\langle I \rangle = I_c(1)$] there is a transition to a regime where the reflectivity is determined by the rare high intensity hot spots. Experimental results obtained by Baldis *et al.* in the strongly inhomogeneous limit [4] confirm this leading role of the hottest spot contribution to the macroscopic reflectivity in the unsaturated regime. This suggests that the fluctuations seen in Fig. 2 originate from intensity fluctuations of the hottest spot. From this remark it is possible to get a useful analytical description of the reflectivity releasing the usual constraints inherent in numerical simulations. Namely, assuming that the statistics of the reflectivity near the amplification threshold is mainly determined by the statistics of the *hottest* spot, one can neglect the fluctuations of $N(u < u_{\max})$ and replace $dN(u)$ by $[|dM(u)/du| + \delta(u - u_{\max})]du$ in Eq. (1), where $u_{\max}$ is the (random) normalized intensity of the hottest spot, and $M(u)$ is the average number of hot spots of intensity $u \leq I/\langle I \rangle \leq u_{\max}$. By doing so, one obtains the following simple approximation for the reflectivity of a given realization of the RPP field:

$$\tilde{R}_{\rm HS} = r_{\rm HS}(u_{\max}) + \int_3^{u_{\max}} r_{\rm HS}(u) \left| \frac{dM(u)}{du} \right| du, \qquad (2)$$

which depends now on a single random variable $u_{\max}$ instead of on the whole random field $N(u)$. Using first the statistics of $u_{\max}$ yielded by our simulations, we have observed a strong correlation between $R_{\rm HS}$ and $\tilde{R}_{\rm HS}$ over the whole range $1 < g \lesssim 2$–3 in which we found $\rho(R_{\rm HS}, \tilde{R}_{\rm HS}) \simeq 1$, where $\rho(R_{\rm HS}, \tilde{R}_{\rm HS})$ is the sample estimate of the correlation coefficient. Figure 3 shows a typical scatter plot of $R_{\rm HS}$ and $\tilde{R}_{\rm HS}$ for $g = 1.6$ and the same parameters as in Fig. 1. For the sake of legibility, we show a plot corresponding to 1000 realizations only. It can be seen that the data are scattered over a very narrow strip around $\tilde{R}_{\rm HS} = R_{\rm HS}$, which shows that $\tilde{R}_{\rm HS}$ is a good approximation to $R_{\rm HS}$.

In order to make Eq. (2) practically useful, one needs *analytical* expressions for $|dM(u)/du|$ and the probability distribution of $u_{\max}$. Such expressions can be found in Refs. [9,10], respectively. In the latter reference, it is shown that $u_{\max}$ can be written as $u_{\max} = \Gamma - \ln Z$ where Z is a random variable the probability distribution of which does not depend on the specific geometry of the RPP [see Eq. (8) of Ref. [10]]. The only RPP dependent quantity is the geometrical factor C_d defining the deterministic quantity $\Gamma \equiv \ln C_d + [(d-1)/2]\ln(\ln C_d)$, where d is the space dimension. In order to compare with our numerical results and validate the approximation (2), we consider the 2D top-hat RPP case in which one has $C_2 \simeq 0.48(1+4f^2)^{-3/2} L_x L_z / \lambda_0^2$ and $|M(u)'| \simeq C_2(10.85\,u - 1.13)\exp(-u)$. Figure 4 shows the confidence interval and the median reflectivity as obtained from Eq. (1) and our numerical statistics of $N(u)$ (solid lines), and from Eq. (2) and the analytical results of Refs. [9,10] (dashed lines). The parameters are the same as in Fig. 1. One observes a very good quantitative agreement up to $g \simeq 2.5$ where the analytical calculation slightly underestimates the fluctuations. This small discrepancy observed for $g \gtrsim 2.5$ must be attributed to the fluctuations of the bulk of $N(u)$, which are not taken into account in Eq. (2). In view of these results, in the

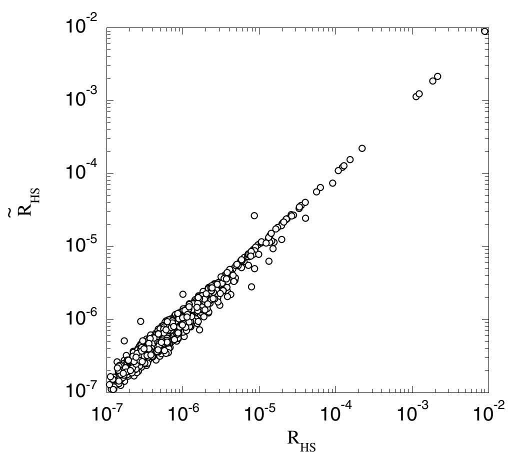

FIG. 3. Scatter plot of $R_{\rm HS}$ [Eq. (1)] and $\tilde{R}_{\rm HS}$ [Eq. (2)] for $g = 1.6$ and 1000 realizations of the RPP field. In this regime, the macroscopic reflectivity of a given realization is mainly determined by $u_{\max}$.

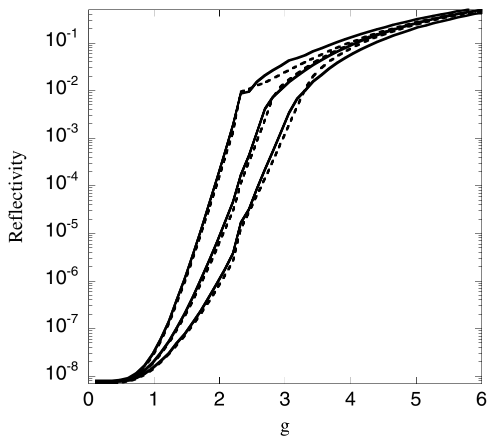

FIG. 4. The 90% confidence interval (upper and lower curves) and median reflectivity (middle curves) for R_{HS} in the case of 10 000 realizations of the RPP field (solid lines) and for $\tilde{R}_{HS}$ with the theoretical hot spots statistics for $M(u)$ and u_{max} (dashed lines).

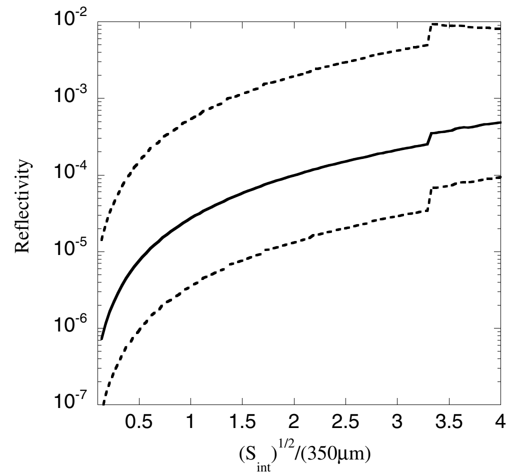

FIG. 5. The 90% confidence interval (dotted lines) and median reflectivity (solid line) as a function of $(S_{int})^{1/2}$ for a 3D circular top-hat RPP field with $f = 8$, $L_z = 350\ \mu$m, and $g = 1.6$.

near-threshold regime where fluctuations in u_{max} determine fluctuations of the reflectivity, we propose Eq. (2), together with Eq. (6) of Ref. [9] and Eqs. (8), (27), and (28) of Ref. [10], as a good analytical model for the fluctuating reflectivity in the case of a fully three-dimensional RPP field.

As a direct application of this model, one can estimate the effect on the reflectivity of varying the interaction region cross section S_{int}. In the case of a 3D circular top-hat RPP, Refs. [9,10] give $C_3 \simeq 2.76(1 + 4f^2)^{-2} S_{int} L_z/\lambda_0^3$ and $|M(u)'| \simeq 5.33 \times 10^{-2} C_3 (u^{3/2} - 1.8u^{1/2} + 0.15u^{-1/2}) \exp(-u)$. Figure 5 shows the confidence interval and the median reflectivity as a function of $(S_{int})^{1/2}$ for $g = 1.6$, $f = 8$, $L_z = 350\ \mu$m, and $\lambda_0 = 0.35\ \mu$m. It can be seen that in this near-threshold regime, using a reduced simulation box can lead to a significant underestimation of the experimental reflectivity. This remark applies also to the extrapolation of present experimental measurements done with smaller focal spot to future larger ones. The discontinuity observed at $(S_{int})^{1/2} \simeq 1.15$ mm is a consequence of the generic shape of the histogram of the reflectivity which has a wide bump and a sharp peak at a higher reflectivity corresponding to the realizations in which the hottest spot saturates. As S_{int} increases, the number of realizations in this peak increases to the detriment of the bump, leading to a discontinuous shift of the confidence interval and the median toward the peak.

In conclusion, in the independent-convective-hot-spot model limit of a spatially smoothed laser beam, we have found that the reflectivity exhibits large statistical fluctuations near the threshold of the amplification. We have discussed the statistics of the reflectivity in terms of median reflectivity and confidence interval, which are more relevant to experimental measurements than average reflectivity and standard deviation. We have shown that the critical intensity for the reflectivity remains a physically relevant quantity as the threshold of the reflectivity fluctuations. This led us to attribute these fluctuations to intensity fluctuations of the hottest laser spot and develop an analytical model for the fluctuating reflectivity. Finally, we have shown that considering a reduced-size interaction region can lead to a strong underestimation of the actual reflectivity.

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