Perturbative QCD Analysis of Local Duality in a Fixed W^2 Framework

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We study the Q^2 dependence of large x F_2 nucleon structure function data, with the aim of providing a perturbative QCD based, quantitative analysis of parton-hadron duality. As opposed to previous analyses at fixed x, we use a framework in fixed W^2. We uncover a breakdown of the twist-4 approximation with a renormalon type improvement at $\mathcal{O}(1/Q^4)$ which affects the initial evolution of parton distributions.

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One of the key challenges in quantum chromodynamics (QCD) today is to formulate a connection between the description of the hard, or short-distance, scattering processes, which can be calculated in terms of quark and gluon degrees of freedom using perturbative methods, and the physical asymptotic states, i.e., the spectra of hadrons which are not calculable within perturbative QCD (pQCD) and are in principle only remotely related to parton dynamics [1]. Yet, a substantial number of observations indicate that QCD has manifestly a *dual* parton-hadron nature. Recent investigations of hadronic jets at e^+e^- [2], ep [3], and $p\bar{p}$ [4] colliders show that several infrared-sensitive features of the inclusive hadron distributions can be reproduced by a pQCD description of the parton shower down to $Q_o \approx \Lambda_{\rm QCD}$, Q_o being an effective cutoff defining the onset of the perturbative regime and $\Lambda_{\rm QCD}$ being the scale of QCD. (Notice, however, that detailed angular correlation measurements have been found to be in disagreement with available pQCD estimates [5].) Similarly, deep inelastic scattering (DIS) experiments on F_2 at very low Bjorken x show a smooth and fast transition between the perturbative and nonperturbative regimes at $Q^2 \approx 1$ GeV^2, just before the real photon scattering limit—$F_2(x, Q^2) \to 0$ for $Q^2 \to 0$—is reached. The question of the role and nature of nonperturbative corrections to all of these processes naturally emerges.

In this Letter we focus on yet another set of experiments, namely, inclusive ep and eD scattering in the resonance region, where a pQCD description seems to hold in spite of the low values of the invariant mass squared, W^2, produced in the final state. Here the observation of Bloom-Gilman (BG) duality [6], or the similarity between the behavior of the resonance contribution to the nucleon structure function and DIS, can be formulated theoretically, as the equivalence between the moments of the structure function in the low W^2 kinematical region dominated by resonances and in the DIS one, *modulo* perturbative corrections and expectedly small power corrections [7]. Furthermore, because of the large body of highly accurate data that has recently been made available [8], the data in the resonance region can now be fitted to a smooth curve which, once evolved according to pQCD, can be compared with the DIS data.

There has been a growing theoretical interest in this intriguing phenomenon, where the quasiexclusive nature, or low inelasticity of the scattering process would be expected to hinder a direct observation of the partonic structure of the target. The present analysis is motivated by the expectation that a more precise understanding of the mechanisms behind BG duality might provide a handle on the type of hadronic configurations that are present in a "semi-hard" regime, before confinement sets in. It is therefore of paramount importance to perform a detailed study of both the logarithmic corrections and of the size and nature of the power corrections in the pQCD expansion, in order to ascertain whether the apparent weak Q^2 dependence of the data is coincidental, or a cancellation of higher twist (HT) terms, possibly understandable within parton-hadron duality models. The analysis conducted here involves a number of steps similar to recent extractions of power corrections from inclusive data [9–12], namely, the form

$$F_2^{\rm exp}(x, Q^2) = F_2^{\rm pQCD+TMC}(x, Q^2) + \frac{H(x, Q^2)}{Q^2} + \mathcal{O}(1/Q^4) \tag{1}$$

is adopted, where $F_2^{\rm pQCD+TMC}(x, Q^2)$ is the twist-2 contribution, including kinematical power corrections from the target mass (TMC); the other terms in the formula are the dynamical power corrections, formally arising from higher order terms in the twist expansion. Both $F_2^{\rm pQCD+TMC}$ and H can be extracted from the data at large x, by taking care of aspects of pQCD evolution peculiar to this region.

Uncertainties are introduced at each step. In this Letter, we address them one by one with the important addition that we perform, for the first time in the literature, an analysis of the scale dependence of the resonance region ($W^2 < 4$ GeV^2) at sufficiently high Q^2. Recent studies have, in fact, provided a verification of the original parton-hadron duality observations where HTs seem to be "small or canceling" [8]. Here we uncover a much richer structure of the Q^2 dependence behind this apparent cancellation that does not seem to follow current expectations from the twist expansion with terms $\propto 1/W^2, 1/W^4 \cdots$, appearing as Q^2 is lowered. Analyses using moments of F_2 [13] are affected at lower values of Q^2 by elastic scattering, rendering the extraction of higher twists ambiguous. We choose, therefore, an alternative method using data on F_2 in fixed W^2 bins.

The data in the resonance region [8] have been fitted to a smooth curve for $F_2^{p(D)}(\xi)$, where $\xi = 2x/(1 + \sqrt{1 + 4M^2x^2/Q^2})$ [14] is the Nachtmann scaling variable. The fit is applied to different bins in W^2, shown in Fig. 1. The χ^2/d.o.f. for these fits varies from 0.8 to 1.1. The uncertainty for the scaling curves is estimated to be better than 10%, taking into account uncertainties in the experimental data, and in the averaging and fitting procedures. The error in each W^2 bin is represented by the hatched areas in Fig. 1. Notice that the spectra at fixed W^2 require that Q^2 increases with x as $Q^2(x) = (W_R^2 - M^2)x/(1 - x)$.

Next, we compare these smooth curves to the pQCD predictions for F_2 at the same kinematics in order to extract potential HT contributions to the structure functions. Only valence quarks contribute to $F_2^{p(D)}$, at $W^2 \lesssim 10$ GeV^2 and $x \geq 0.3$. pQCD evolution to $Q^2 \equiv Q^2(x)$ for

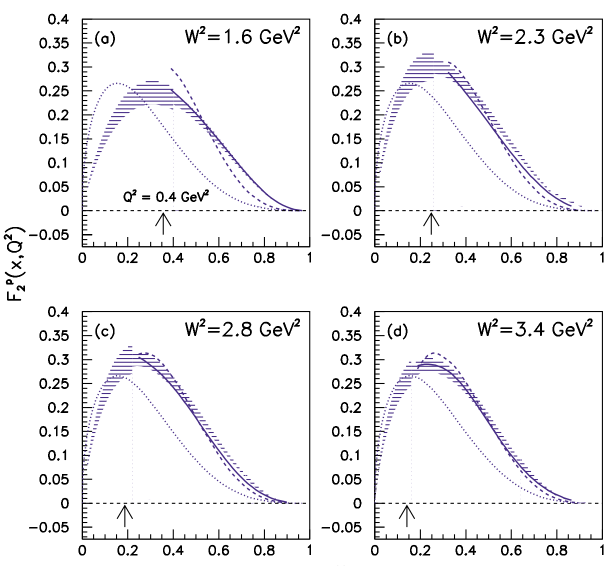

FIG. 1 (color online). Comparison of NLO pQCD calculations (dashed lines), and NLO pQCD + TMC (solid lines) with the data on F_2^p (hatched areas) at fixed values of $W^2 = W_R^2$, vs x: (a) $W_R^2 = 1.6$ GeV^2, (b) $W^2 = 2.3$ GeV^2, (c) $W^2 = 2.8$ GeV^2, (d) $W_R^2 = 3.4$ GeV^2. The dotted curve shows for comparison the DIS calculation, obtained at $Q^2 = 200$ GeV^2. The data are averaged with the procedure described in the text and reference. The pQCD curves were obtained using parametrizations for the NS distributions at the input value of $Q_o^2 = 0.4$ GeV^2 [15], indicated by the arrows. Other parametrizations [16] give similar results, starting from their input value of $Q_o^2 \approx 1$ GeV^2.

the nonsinglet (NS) distributions at next-to-leading order (NLO) is given by

$$q_i^{(-)}(x, Q^2) = \int_{Q_o^2}^{Q^2(x)} \frac{dQ'^2}{Q'^2} \frac{\alpha_S(Q'^2)}{2\pi} \times \int_x^1 \frac{dy}{y} P_{qq}\left(\frac{x}{y}, \alpha_S(Q'^2)\right) q_i^{(-)}(y, Q'^2), \quad (2)$$

where $q_i^{(-)} = q_i - \bar{q}_i \equiv q_i^v$, $i = u, d$. The expressions for the splitting function $P_{qq}(z, \alpha_S(Q^2))$ and for the corresponding end points at NLO in the $\overline{MS}$ scheme can be found in [17,18]; α_S is the strong coupling constant. The structure function, F_2, is obtained by convoluting Eq. (2) with the quark coefficient function, $B_2^{\rm NS}$ [17]. We fix the values of the initial parton distribution functions (PDFs), at $Q_o^2 \approx 0.4$–1 GeV^2, to the ones taken from NLO global fits to world data [15,16,19], and we solve the evolution equations directly in x space, with $\alpha_S(M_Z^2) = 0.117$. The shape of the initial NS PDFs is practically constrained [16], at variance with the singlet and gluon distributions at low Q^2, whose shape is strongly correlated with the value of α_S. Both the coefficient functions and the splitting functions for the NS part, are known up to next-to-next-to-leading order (NNLO). Detailed studies of the impact of NNLO corrections and beyond, on the determination of power corrections for the NS structure functions, have been performed in [9,10]. The question of whether these can "mimic" the contributions of higher twists, including the uncertainties due to the well known scale/scheme dependence of calculations, within the current precision of data is currently under intense investigation [20]. Here we single out the contributions that are expected to dominate the higher order perturbative predictions at large x, namely, powers of $\ln(1 - z)$ terms, where $z = x/y$. These terms can be resummed to all orders. We perform the resummation in x space by replacing the Q^2 scale with a z-dependent one, $\widetilde{W}^2 = Q^2(1 - z)/z$ [21]. It is well known that such a procedure introduces in principle an ambiguity in the evaluation of the running coupling at low values of the scale $\widetilde{W}^2$, i.e., as $z \to 1 - \Lambda^2/Q^2$ [22]. This ambiguity is lessened in our analysis because at fixed W^2, $\Lambda^2/Q^2(x)$ is very close to zero. Our results for NS evolution are in good agreement with more recent resummation calculations performed in n space and anti-Mellin transformed as in [23].

Finally, we take into account TMC leading to a modification of the leading twist term, $F_2^{\rm pQCD}$, as in [7]. Ambiguities in this procedure are known to arise because of the neglect of higher order and higher twist corrections. We therefore use as a minimal criterion that only the kinematical points yielding low values of the parameter x^2M^2/Q^2 in the TMC expansion [7] are kept.

Comparisons of pQCD + TMC predictions with the resonance average data are shown in Figs. 1 and 2. In Fig. 1, (i) TMC (full lines) modify substantially the pQCD behavior (dotted lines) rendering a better agreement with the data; (ii) the curve calculated using NLO pQCD at $Q^2 = 200\ \mathrm{GeV}^2$ shown for comparison demonstrates the large effect of pQCD corrections above $x \sim 0.2$. In Fig. 2, we show the low W^2 data extracted here, along with large W^2 data from [24,25]. Note that the data in the resonance region smoothly blend to the deep inelastic—another manifestation of BG duality. The curves correspond to our calculations including pQCD + TMC at NLO (dashes), and pQCD + TMC with resummation (full). The dots in each curve represent regions where TMC are uncertain. The effect we find is qualitatively similar to that found in [9,10], in that over the range $0.45 \leq x \leq 0.85$, higher order perturbative contributions, in this case large x resummation, improve the agreement with the data. Substantial discrepancies remain, which we interpret in terms of dynamical HT corrections. We parametrize $H(x, Q^2)$ as

$$H(x, Q^2) = F_2^{\mathrm{pQCD+TMC}}(x, Q^2) C_{\mathrm{HT}}(x). \quad (3)$$

Equation (3) is motivated by the lack of knowledge of the anomalous dimensions of the twist-4 operators, a reasonable assumption within the precision of the data (see also [26]). Our fixed W^2 approach enables us to extract C_{HT} from the resonance region and from the DIS region, separately.

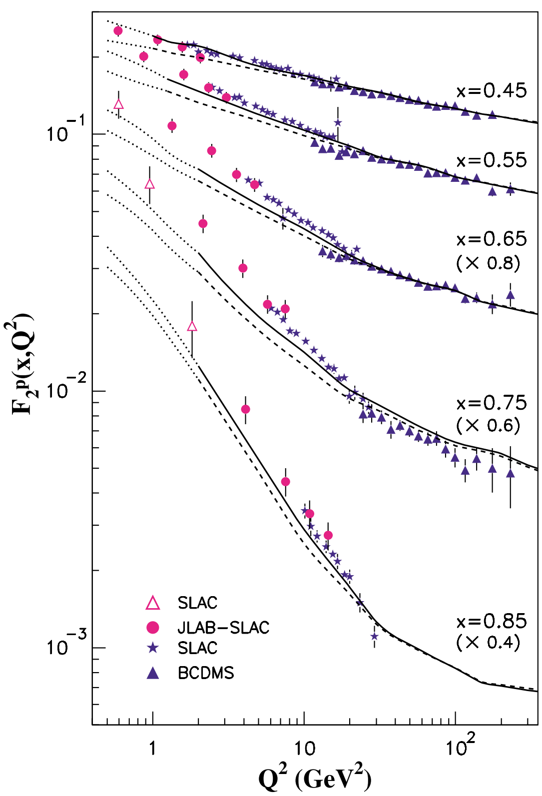

FIG. 2 (color online). Comparison of pQCD + TMC calculations at NLO (dashed lines) and with resummation (full lines), with current large x data. The solid dots are in the resonance region, $1.3 \leq W^2 \leq 3.4\ \mathrm{GeV}^2$; the open triangles correspond to $W^2 \leq 1.3\ \mathrm{GeV}^2$. The dotted lines represent the regions where TMC contributions are uncertain.

In Fig. 3(a) we show the coefficient C_{HT}, Eq. (3), extracted from the following: (i) DIS data with $W^2 \geq 4\ \mathrm{GeV}^2$, (ii) the resonance region, $W^2 < 4\ \mathrm{GeV}^2$, as well as (iii) averaged over the entire range of W^2. The figure also shows the range of extractions previous to the current one [11,27]. We observe in all three cases, values for C_{HT} smaller than the ones in [11,27], because of the effect of large x resummation. We have checked that our results without resummation are consistent with a previous extraction using moments of the structure function [12]. Most importantly, while the large W^2 data track a curve that is consistent with the $1/W^2$ behavior expected from most models [28], the low W^2 data yield a much smaller value for C_{HT}, and they show a bend over of the slope vs x, already predictable from a similar behavior of the slopes at low W^2 in Fig. 2. This surprising effect is not a consequence of the interplay of higher order corrections and the HT terms, but just of the extension of our detailed pQCD analysis to the large x, low W^2 kinematical region. In order to ascertain whether the discrepancy between the low W^2 and large W^2 values of C_{HT} are due to $\mathcal{O}(1/Q^4)$ terms in the twist expansion, Eq. (1), which could become more important at low W^2, we have extracted for each resonance the quantity $\Delta H(x, Q^2)$, defined as

$$\frac{F_2^{\mathrm{exp}}}{F_2^{\mathrm{pQCD+TMC}}} = 1 + \frac{C_{\mathrm{HT}}(x)}{Q^2} + \Delta H(x, Q^2), \quad (4)$$

where $C_{\mathrm{HT}}(x)$ coincides with the value fitted at large W^2. From Fig. 3(b) one sees that $\Delta H(x, Q^2)$ is negative for all lower W^2 ($\leq 3.4\ \mathrm{GeV}^2$) bins, as expected if a cancellation among higher order inverse powers were to occur, consistent with the requirement of parton-hadron duality. However, we uncover a nontrivial Q^2 dependence of this term: one can see a sharp change between the behavior of the higher mass resonances and that of the $N - \Delta$ transition region which shows a distinctively steeper fall with

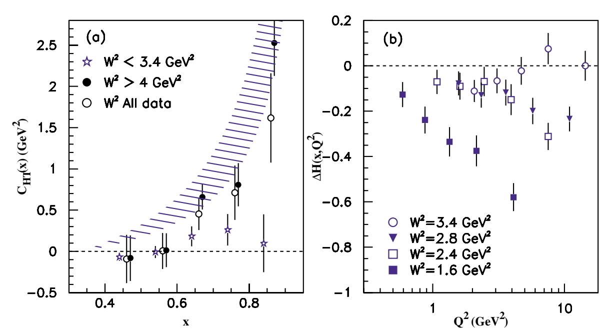

FIG. 3 (color online). (a) Coefficient C_{HT}, Eq. (3), extracted from DIS data with $W^2 \geq 4\ \mathrm{GeV}^2$ (solid dots), from the resonance region, $W^2 < 4\ \mathrm{GeV}^2$ (stars) and averaged over the entire range of W^2 (open dots). The shaded area summarizes extractions previous to the current one. A dotted line at zero is added to guide the eye; (b) ΔH, Eq. (4), extracted at fixed values of W^2 as described in the text, and plotted vs Q^2. The figure further elucidates a breakdown of the twist expansion at low W^2, already visible in (a).

Q^2. Furthermore, the high mass resonances seem to be in fair agreement with a simple $-1/W^4$ fit inspired by infared renormalons (IRR) calculations [28], which would produce a constant line at a decreasing height for increasing W^2. The $N - \Delta$ transition region shows a departure from this behavior which cannot be accounted for by fitting the first few terms of an inverse power expansion. A similar conclusion was reached in our analysis in moment space [13].

To summarize, the size of the power corrections obtained at $W^2 > 3.4$ GeV^2 is comparable to recent analyses of DIS data [9–11,26,27], the inclusion of large x resummation producing a reduction of the coefficient $C_{\rm HT}$. At lower invariant mass, $1.9 \leq W^2 \leq 3.4$ GeV^2, an extra term of order $\mathcal{O}(1/Q^4)$ with a negative coefficient is necessary to fit the data, in line with the asymptotic nature of the twist expansion and with current predictions from IRR calculations [28]. A similar expansion does not reproduce; however, the scale dependence of data at even lower masses, $1.2 \leq W^2 \leq 1.9$ GeV^2, yet for Q^2 values where pQCD is expected to apply. Therefore, the experimental observation of a rather flat Q^2 dependence cannot be taken as a signature of parton-hadron duality for this region, but on the contrary, it shows the limits of applicability of this idea.

Having this determination in hand, one can now attempt to provide theoretical interpretations. In particular, the fact that the pattern of dynamical power corrections to resonance production is comparable to the one for the large Q^2 deep inelastic data, i.e., that power corrections remain small in the resonance region, suggests that color confinement is more likely to happen locally, with a smooth transition between partonic and hadronic configurations, a mechanism supported also by recent studies of hadron spectra in jet measurements [4]. This mechanism seems to break down in the $N - \Delta$ transition region, providing a threshold where cooperative effects among partons similar to the ones in cluster hadronization models [29] might dominate the structure function.

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