Plasma Wakefield Acceleration for Ultrahigh-Energy Cosmic Rays

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A cosmic acceleration mechanism is introduced which is based on the wakefields excited by the Alfvén shocks in a relativistically flowing plasma. We show that there exists a threshold condition for transparency below which the accelerating particle is collision-free and suffers little energy loss in the plasma medium. The stochastic encounters of the random accelerating-decelerating phases results in a power-law energy spectrum: $f(\epsilon) \propto 1/\epsilon^2$. As an example, we discuss the possible production in the atmosphere of gamma ray bursts of ultrahigh-energy cosmic rays (UHECR) exceeding the Greisen-Zatsepin-Kuzmin cutoff. The estimated event rate in our model agrees with that from UHECR observations.

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Ultrahigh-energy cosmic ray (UHECR) events exceeding the Greisen-Zatsepin-Kuzmin (GZK) cutoff [1] ($\sim 5 \times 10^{19}$ eV for protons originated from a distance larger than $\sim$50 Mps) have been found in recent years [2–5]. Observations also indicate a change of the power-law index in the UHECR spectrum (events/energy/area/time), $f(\epsilon) \propto \epsilon^{-\alpha}$, from $\alpha \sim 3$ to a smaller value at energy around 10^{18}–10^{19} eV. These present an acute theoretical challenge regarding their composition as well as their origin [6].

Thus far, the theories that attempt to explain the UHECR can be largely categorized into the "top-down" and the "bottom-up" scenarios. In addition to relying on exotic particle physics beyond the standard model, the main challenges of top-down scenarios are their difficulty in compliance with the observed event rate and the energy spectrum [6], and the fine-tuning of particle lifetimes. The main challenges of the bottom-up scenarios, on the other hand, are the GZK cutoff, as well as the lack of an efficient acceleration mechanism [6]. To circumvent the GZK limit, several authors propose the "Z-burst" scenario [7], where neutrinos, instead of protons, are the actual messengers across the cosmos. For such a scenario to work, it requires that the original particle, for example, a proton, be several orders of magnitude more energetic than the one that eventually reaches Earth.

Even if the GZK limit can be circumvented through the *Z*-burst scenario, the challenge for a viable acceleration mechanism remains, or becomes even more acute. This is mainly because the existing paradigm for cosmic acceleration, namely, the Fermi mechanism [8], as well as its variants, such as the diffusive shock acceleration [9], are not effective in reaching ultrahigh energies [10]. These acceleration mechanisms rely on the random collisions of the high-energy particle against magnetic field domains or the shock media, which necessarily induce increasingly more severe energy losses at higher particle energies.

From the experience of terrestrial particle accelerators, we learn that it takes several qualifications for an accelerator to operate effectively. First, the particle should gain energy through the interaction with the longitudinal electric field of a subluminous ($v \leq c$) electromagnetic (EM) wave. In such a setting, the accelerated particle can gain energy from the field over a macroscopic distance, much like how a surfer gains momentum from an ocean wave. It is important to note that such a longitudinal field is Lorentz invariant. Second, such a particle-field interaction should be a noncollisional process. This would help to avoid severe energy loss through inelastic scatterings. Third, to avoid excessive synchrotron radiation loss, which scales as particle energy squared, the accelerating particle should avoid any drastic bending. We believe that these qualifications for terrestrial accelerators are also applicable to celestial ones.

The "plasma wakefield accelerator" concepts [11,12] promise to satisfy all the conditions stated above. Collective plasma waves, or "wakefields," can be excited by highly concentrated, relativistic EM energies such as lasers [11] and particle beams [12]. The mutually perpendicular $\vec{E}$ and $\vec{B}$ give rise to a *ponderomotive* force along the direction of EM wave propagation ($\vec{k}$), which induces a longitudinal plasma oscillation with a phase velocity equal to the driving beam group velocity. A trailing particle can then gain energy by riding on this wakefield. Although hard scatterings between the accelerating particle and the plasma medium are inevitable, under appropriate conditions the particle can be collision-free.

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An Alfvén wave propagating in a stationary magnetized plasma has a velocity $v_A = eB_0/(4\pi m_i n_p)^{1/2}$. Here, B_0 is the magnetic field and n_p is the density of the plasma. The relative strength between its transverse fields is $E_A/B_A = v_A/c$ (typically $\ll 1$). Although such a wave is magnetic in nature, it is easy to verify that its ponderomotive force is nonvanishing. Preliminary results from simulations confirmed that Alfvén waves can indeed excite plasma wakefields with $v_{\rm ph} = v_A$ [13]. For the purpose of ultrahigh-energy acceleration, such a slow wave would not be too useful, as the particle can quickly slip out of the acceleration phase in the wakefield. This can be circumvented, however, if the plasma itself has a relativistic bulk flow, so that $v_{\rm ph} \to c$.

With our applications to astrophysical problems in mind, the Alfvén-wave-plasma interaction relevant to us is in the nonlinear regime. The nonlinearity of the plasma wakefield is governed by the Lorentz-invariant normalized vector potential $a_0 = eE/mc\omega$ of the driving EM wave [14]. When this parameter exceeds unity, nonlinearity is strong [11] so that additional important physics incurs. In the frame of a stationary plasma, the maximum field amplitude that the plasma wakefield can support is

$$E_{\rm max} \approx a_0 E_{\rm wb} = a_0 \frac{m_e c \omega_p}{e}, \qquad (1)$$

which is enhanced by a factor a_0 beyond the cold wavebreaking limit, $E_{\rm wb}$, of the linear regime. Transform this to a frame of relativistic plasma flow; the cold wavebreaking field is reduced by a factor $\Gamma_p^{1/2}$ due to Lorentz contraction while a_0 remains unchanged. The maximum "acceleration gradient" G experienced by a singly charged particle riding on this plasma wakefield is then

$$G = eE'_{\rm max} \approx a_0 m_e c^2 \left(\frac{4\pi r_e n_p}{\Gamma_p}\right)^{1/2}, \qquad (2)$$

where $r_e = e^2/m_e c^2$ is the classical electron radius.

At ultrahigh energies once the test particle encounters a hard scattering or bending, the hard-earned kinetic energy would most likely be lost. The scattering of an ultrahigh-energy proton with the background plasma is dominated by the proton-proton collision. In our system, even though the UHE proton is in the ZeV (10^{21} eV) regime, the center-of-mass energy of such a proton colliding with a comoving background plasma proton is in the TeV range, so for our discussion we assume a constant total cross section, $\sigma_{pp} \sim 30$ mb. Since in astrophysical settings an out-bursting relativistic plasma dilutes as it expands radially, its density scales as $n_p(r) = n_{p0}(R_0/r)^2$, where n_{p0} is the plasma density at a reference radius R_0. The UHE proton mean-free path can be determined by integrating the collision probability, $\sigma_{pp} n_p(r)/\Gamma_p$, up to unity from radius R_0 to $R_0 + R_{\rm mfp}$. We find that the solution to $R_{\rm mfp}$ does not exist unless $\sigma_{pp} n_{p0} R_0/\Gamma_p > 1$. That is, there exists a threshold condition below which the system is collision-free:

$$\frac{\sigma_{pp} n_{p0} R_0}{\Gamma_p} \le 1. \qquad (3)$$

In astrophysical settings, the Alfvén shocks are typically stochastic. A test particle would then face random encounters of accelerating and decelerating phases of the plasma wakefields excited by Alfvén shocks. Such a stochastic process can be described by the distribution function $f(\epsilon, t)$ governed by the Chapman-Kolmogorov equation [15,16]. As we will demonstrate later, the astrophysical environment that we invoke is below the collision threshold condition. In addition, the particle acceleration is collinear to the electrostatic wakefield [14] and is thus radiation-free. We can thus ignore energy dissipation and reduce the Chapman-Kolmogorov equation to the Fokker-Planck equation:

$$\frac{\partial}{\partial t} f = \frac{\partial}{\partial \epsilon} \int_{-\infty}^{+\infty} d(\Delta\epsilon) \Delta\epsilon W(\epsilon, \Delta\epsilon) f(\epsilon, t) + \frac{\partial^2}{\partial \epsilon^2} \int_{-\infty}^{+\infty} d(\Delta\epsilon) \frac{\Delta\epsilon^2}{2} W(\epsilon, \Delta\epsilon) f(\epsilon, t). \qquad (4)$$

We now assume the following properties of the transition rate $W(\epsilon, \Delta\epsilon)$ for a purely stochastic process: (a) W is an even function; (b) W is independent of ϵ; (c) W is independent of $\Delta\epsilon$.

Property (a) follows from the fact that in a plasma wave there is an equal probability of gaining and losing energy. In addition, since the wakefield amplitude is Lorentz invariant, the chance of gaining a given amount of energy, $\Delta\epsilon$, is independent of the particle energy ϵ. Finally, under a purely stochastic white noise, the chance of gaining or losing any amount of energy is the same. Based on these arguments, we deduce that $W(\epsilon, \Delta\epsilon) = \text{const}$. We note that there is a stark departure of the functional dependence of W in our theory from that in Fermi's mechanism, in which the energy gain $\Delta\epsilon$ per encounter scales linearly and quadratically in ϵ for the first-order and the second-order Fermi mechanism, respectively.

To look for a stationary distribution, we put $\partial f/\partial t = 0$. Since W is an even function, the first term on the right-hand side of Eq. (4) vanishes. To ensure the positivity of particle energies before and after each encounter, the integration limits are reduced from $(-\infty, +\infty)$ to $[-\epsilon, +\epsilon]$, and we have

$$\frac{\partial^2}{\partial \epsilon^2} \int_{-\epsilon}^{+\epsilon} d(\Delta\epsilon) \frac{\Delta\epsilon^2}{2} W(\epsilon, \Delta\epsilon) f(\epsilon) = 0. \qquad (5)$$

Since W is constant, we arrive at the energy distribution function that follows power-law scaling,

$$f(\epsilon) = \frac{\epsilon_0}{\epsilon^2}, \qquad (6)$$

where the normalization factor ϵ_0 is taken to be the mean energy of the background plasma proton, $\epsilon_0 \sim \Gamma_p m_p c^2$.

We note that a power-law energy spectrum is generic to all purely stochastic, collisionless acceleration processes.

This is why both the first- and the second-order Fermi mechanisms also predict power-law spectrum, if the energy losses, e.g., through inelastic scattering and radiation (which are severe at ultrahigh energies), are ignored. The difference is that in the Fermi mechanism the stochasticity is due to random collisions of the test particle against magnetic walls or the shock medium, which necessarily induce *reorientation* of the momentum vector of the test particle after every diffusive encounter, and therefore should trigger inevitable radiation loss at high energies. The stochasticity in our mechanism is due instead to the random encounters of the test particle with different accelerating-decelerating *phases*. As we mentioned earlier, the phase vector of the wakefields created by the Alfvén shocks in the relativistic flow is nearly unidirectional. Thus the particle's momentum vector never changes its direction but only magnitude, and is therefore radiation-free.

We now apply our acceleration mechanism to the problem of UHECR. Gamma ray bursts (GRBs) are by far the most violent release of energy in the universe, second only to the big bang itself. Within seconds (for short bursts), about $\epsilon_{\rm GRB} \sim 10^{52}$ erg of energy is released through gamma rays with a spectrum that peaks around several hundred keV. Existing models for GRB, such as the relativistic fireball model [17], typically assume either neutron-star–neutron-star (NS-NS) coalescence or supermassive star collapse as the progenitor. The latter has been identified as the origin for the long burst GRBs (with time duration ~10–100 s) by recent observations [18]. The origin of the short burst GRBs, however, is still uncertain, and NS-NS coalescence remains a viable candidate. While both candidate progenitors can, in principle, accommodate our plasma wakefield acceleration mechanism, for the sake of discussion, we will invoke the former as our explicit example. Neutron stars are known to be compact ($R_{\rm NS} \sim O(10)$ km) and carrying intense surface magnetic fields ($B_{\rm NS} \sim 10^{12}$ G). Several generic properties are assumed when such compact objects collide. First, the collision creates sequence of strong magnetoshocks (Alfvén shocks). Second, the tremendous release of energy creates a highly relativistic out-bursting fireball, most likely in the form of a plasma.

The fact that the GRB prompt (photon) signals arrive within a brief time window implies that there must exist a threshold condition in the GRB atmosphere where the plasma becomes optically transparent beyond some radius R_0 from the NS-NS epicenter. Applying our collision-free threshold condition to the case of out-bursting GRB photons, the optical transparency implies that $\sigma_c \leq \Gamma_p/n_{p0}R_0$, where $\sigma_c \approx 2 \times 10^{-25}$ cm^2 is the Compton scattering cross section for $\omega_{\rm GRB} \approx m_e$. Since $\sigma_{pp} < \sigma_c$, the UHECRs are also collision-free in the same environment. There is clearly a large parameter space where this condition is satisfied. To narrow down our further discussion, it is not unreasonable to assume that $R_0 \sim O(10^4)$ km. A set of self-consistent parameters can then be chosen: $n_{p0} \sim 10^{20}$ cm^{-3}, $\Gamma_p \sim 10^4$, and $\epsilon_0 \sim 10^{13}$ eV $\equiv \epsilon_{13}$.

To estimate the plasma wakefield acceleration gradient, we first derive the value of the a_0 parameter. We believe that the magnetoshocks constitute a substantial fraction, for example, $\eta_a \sim 10^{-2}$, of the total energy released from the GRB progenitor. The energy Alfvén shocks carry is therefore $\epsilon_A \sim 10^{50}$ erg. Because of the pressure gradient along the radial direction, the magnetic fields in Alfvén shocks that propagate outward from the epicenter will develop sharp discontinuities and be compactified [19]. The estimated shock thickness is $\sim O(1)$ m at $R_0 \sim O(10^4)$ km. From this and ϵ_A, one can deduce the magnetic field strength in the Alfvén shocks at R_0, which gives $B_A \sim 10^{10}$ G. This leads to $a_0 = eE_A/mc\omega_A \sim 10^9$. Under these assumptions, the acceleration gradient G [cf. Equation (2)] is as large as

$$G \sim 10^{16}\left(\frac{a_0}{10^9}\right)\left(\frac{10^9\text{cm}}{R_0}\right)^{1/2} \text{eV/cm}. \tag{7}$$

Although the UHE protons can, in principle, be accelerated unbound in this system, the ultimate maximum reachable energy is determined by the conservation of energy and our assumption on the population of UHE protons. Since it is known that the coupling between the ponderomotive potential of the EM wave and the plasma wakefield is efficient, we assume that the Alfvén shock energy is entirely loaded to the plasma wakefields after propagating through the plasma. We further assume that the energy in the plasma wakefield is entirely reloaded to the UHE protons through the stochastic process. Thus, the highest possible UHE proton energy, $\epsilon_{\max}$, can be determined by energy conservation:

$$\eta_a\epsilon_{\rm GRB} \sim \epsilon_A \sim \epsilon_{\rm UHE} \sim N_{\rm UHE}\int_{\epsilon_{13}}^{\epsilon_{\max}} \epsilon f(\epsilon)d\epsilon. \tag{8}$$

This provides a relationship between $\epsilon_{\max}$ and the UHE proton population, $N_{\rm UHE}$:

$$\epsilon_{\max} = \epsilon_{13}\exp(\eta_a\epsilon_{\rm GRB}/N_{\rm UHE}\epsilon_{13}). \tag{9}$$

We assume that $\eta_b \sim 10^{-2}$ of the GRB energy is consumed to create the bulk plasma flow, i.e., $\eta_b\epsilon_{\rm GRB} \sim N_p\Gamma_p m_p c^2 \sim N_p\epsilon_{13}$, where N_p is the total number of plasma protons. We further assume that $\eta_c \sim 10^{-2}$ of the plasma protons are trapped and accelerated to UHE, i.e., $N_{\rm UHE} \sim \eta_c N_p$. Then we find $\epsilon_{\max} \sim \epsilon_{13}\exp(\eta_a/\eta_b\eta_c)$. We note that this estimate of $\epsilon_{\max}$ is exponentially sensitive to the ratio of several efficiencies, and therefore should be handled with caution. If the values are indeed as we have assumed, $\eta_a/\eta_b\eta_c \sim O(10^2)$, then $\epsilon_{\max}$ is effectively unbound until additional limiting physics enters. Whereas if the ratio is $\sim O(10)$ instead, the UHE cannot even reach the ZeV regime. The validity of our assumed GRB efficiencies then relies on the consistency check against observations.

In addition to the energy production issue, equally important to a viable UHECR model is the event rates.

Based on observations, we take $f_{\rm GRB} \sim 10^4/{\rm yr}$ for the GRB event rate. In the Z-burst scenario, an initial neutrino energy above 10^{21} eV [7] or 10^{23} eV [20] is required to reach the Z-boson threshold. For the sake of discussion, we take the necessary neutrino energy as $\epsilon_\nu \geq 10^{22}$ eV. Such ultrahigh-energy neutrinos can, in principle, be produced through the collisions of UHE protons with the GRB background protons: $pp \to \pi + X \to \mu + \nu + X$. All UHE protons with energy $\epsilon_{\geq 22} \geq 10^{22}$ eV should be able to produce such neutrinos. The mean energy [by integrating over the distribution function $f(\epsilon)$] of such protons is $\langle \epsilon_{\geq 22} \rangle \sim O(100)\epsilon_{22}$. Therefore the multiplicity of neutrinos per UHE proton is around $\mu_{(p\to\nu)} \sim O(10)$–$O(100)$. At the opposite end of the cosmic process, we also expect multiple hadrons produced in a Z burst. The average number of protons that Z boson produces is ~ 2.7 [21]. Finally, the population of UHE protons above 10^{22} eV is related to the total UHE population by $N_{\geq 22} \sim (\epsilon_{13}/\epsilon_{22})N_{\rm UHE} \sim \eta_b \eta_c \epsilon_{\rm GRB}/\epsilon_{22}$.

Putting the above arguments together, we arrive at our estimated UHECR event rate on Earth,

$$N_{\rm UHECR \geq 20} \sim f_{\rm GRB} \mu_{(p\to\nu)} \mu_{(Z\to p)} \frac{\epsilon_{\rm GRB}}{\epsilon_{22}} \frac{\eta_b \eta_c}{4\pi R_{\rm GRB}^2}. \quad (10)$$

Typical observed GRB events are at a redshift $z \sim O(1)$, or a distance $R_{\rm GRB} \sim 10^{23}$ km. Therefore

$$N_{\rm UHECR \geq 20} \sim O(1)/100\ {\rm km}^2/{\rm yr}/{\rm sr}, \quad (11)$$

which is consistent with observations; or, in turn, this observed event rate can serve as a constraint on the various assumptions of our specific GRB model.

We have demonstrated that plasma wakefields excited by Alfvén shocks in a relativistic plasma flow can be a very efficient mechanism for cosmic acceleration, with a power-law energy spectrum. When invoking GRBs as the sites for UHECR production with a set of reasonable assumptions, we show that our estimated UHECR event rate is consistent with observations. This cosmic acceleration mechanism is generic, and can, in principle, be applied to other astrophysical phenomena, such as *blazars* [22]. It is generally believed that the active galactic nuclei jets are relativistic plasmas. The observed large density variations in the jet may well serve as the driver to excite plasma wakefields. These wakefields can accelerate electrons as well as protons to multi-TeV energies. Bent by the confining helical magnetic fields in the jet, these high-energy electrons can radiate hard photons in the TeV range, while the protons can cascade into high-energy neutrinos. We will present a more detailed discussion on blazars in a separate paper.

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[1] K. Greisen, Phys. Rev. Lett. **16**, 748 (1966); G.T. Zatsepin and V.A. Kuzmin, Sov. Phys. JETP **4**, 78 (1966).
[2] D.J. Bird *et al.*, Phys. Rev. Lett. **71**, 3401 (1993); Astrophys. J. **424**, 491 (1994); **441**, 144 (1995).
[3] M. Takeda *et al.*, Phys. Rev. Lett. **81**, 1163 (1998); Astrophys. J. **522**, 225 (1999).
[4] T. Abu-Zayyad *et al.*, in *Proceedings of the 26th International Cosmic Ray Conference (ICRC), Salt Lake City, 1999* (University of Utah, Salt Lake City, 1999), Vol. 3, p. 125.
[5] M. A. Lawrence, R. J. O. Reid, and A. A. Watson, J. Phys. G **17**, 773 (1991).
[6] A.V. Olinto, Phys. Rep. **333–334**, 329 (2000).
[7] T. Weiler, Astropart. Phys. **11**, 303 (1999); D. Fargion, B. Mele, and A. Salis, Astrophys. J. **517**, 725 (1999).
[8] E. Fermi, Phys. Rev. **75**, 1169 (1949); Astrophys. J. **119**, 1 (1954).
[9] W. I. Axford, E. Leer, and G. Skadron, in *Proceedings of the 15th International Cosmic Ray Conference, Plovdiv, 1977* (Bulgarian Academy of Science, Plovdiv, 1977), Vol. 11, p. 132; G. F. Krymsky, Sov. Phys. Dokl. **22**, 327 (1977); A. R. Bell, Mon. Not. R. Astron. Soc. **182**, 147 (1978); R. D. Blandford and J. F. Ostriker, Astrophys. J. Lett. **221**, L29 (1978).
[10] A. Achterberg, in *Proceedings of the IAU Symposium*, edited by P. C. H. Martens and S. Tsuruta (Astronomical Society of the Pacific, San Francisco, 2000), Vol. 195.
[11] T. Tajima and J. M. Dawson, Phys. Rev. Lett. **43**, 267 (1979).
[12] P. Chen, J. M. Dawson, R. Huff, and T. Katsouleas, Phys. Rev. Lett. **54**, 693 (1985).
[13] P. Romenesko, P. Chen, and T. Tajima (to be published).
[14] E. Esarey, P. Sprangle, J. Krall, and A. Ting, IEEE Trans. Plasma Sci. **24**, 252 (1996).
[15] K. Mima, W. Horton, T. Tajima, and A. Hasegawa, in *Proceedings of the Nonlinear Dynamics and Particle Acceleration*, edited by Y. H. Ichikawa and T. Tajima (AIP, New York, 1991).
[16] Y. Takahashi, L. Hillman, and T. Tajima, in *High Field Science*, edited by T. Tajima, K. Mima, and H. Baldis (Kluwer Academic/Plenum, New York, 2000).
[17] M. J. Rees and P. Meszaros, Mon. Not. R. Astron. Soc. **158**, P41 (1992); P. Mezsaros and M. J. Rees, Astrophys. J. **405**, 278 (1993).
[18] A. P. Price *et al.*, astro-ph/0203467 (to be published); P. M. Garnavich *et al.*, astro-ph/0204234 (to be published).
[19] A. Jeffrey and T. Taniuti, *Nonlinear Wave Propagation* (Academic Press, New York, 1964).
[20] G. Gelmini and G. Varieschi, UCLA/02/TEP/4, 2002 (unpublished).
[21] Particle Data Group, D. E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).
[22] M. Punch *et al.*, Nature (London) **358**, 477 (1992); J. Quinn *et al.*, Astrophys. J. Lett. **456**, L63 (1996); C. M. Urry, Adv. Space Res. **21**, 89 (1998).