Test of Local Realism with Entangled Kaon Pairs and without Inequalities

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We propose the use of entangled pairs of neutral kaons, considered as a promising tool to close the well known loopholes affecting generic Bell's inequality tests, in a specific Hardy-type experiment. Hardy's contradiction without inequalities between local realism and quantum mechanics can be translated into a feasible experiment by requiring ideal detection efficiencies for *only one* of the observables to be alternatively measured. Neutral kaons are near to fulfill this requirement and therefore to close the efficiency loophole.

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Bell's theorem [1] proved the incompatibility between quantum mechanics (QM) and local realism (LR) and opened the attractive possibility to solve the famous Einstein-Bohr debate from a purely experimental point of view. The various forms of Bell's inequalities [1–4], which are strict consequences of LR but can be violated by QM, have been the usual tool for such an experimental discrimination. This requires the use of entangled systems, such as the singlet state of two photons or two spin-half particles, as first considered by Bohm [1], or the formally equivalent two-kaon state:

$$\Phi_0 = [K^0\bar{K}^0 - \bar{K}^0K^0]/\sqrt{2}, \tag{1}$$

also discussed in other recent analyses. *Maximally* entangled bipartite states such as these show [5] the maximum QM violation of Bell's inequalities. A lot of experiments violating Bell's inequalities have been carried out. Unfortunately, none of them has been loophole-free [6]: the so-called *locality* and *detection* loophole affected these Bell-type tests. Important steps forward have been done very recently: the experiments with entangled photons of Refs. [7,8] closed the locality loophole, while, by employing beryllium ions [9], it has been possible to close the efficiency loophole. However, a test closing simultaneously both loopholes is lacking.

In 1992 Hardy [10] proved Bell's theorem without using inequalities for any *nonmaximally* entangled state composed of two two-level subsystems. Such a proof allows, at least in principle, for a clear-cut discrimination between LR and QM for a fraction ($\lesssim 9\%$), usually called "Hardy fraction," of the single experimental runs. The independent demonstration of Bell's theorem without inequalities provided by Greenberger *et al.* [11] applies to every single experimental run; i.e., it is an "all vs nothing" proof. Unfortunately, it requires entangled states consisting of three or more two-level subsystems, which are difficult to produce and control, whereas the bipartite systems of Hardy's proof offer an easier use. The only problem is that, being Hardy's proof is related to a certain lack of symmetry of the state, it cannot work for familiar maximally entangled states such as (1). Although this feature complicates the issue, Hardy's treatment has been discussed and generalized [12,13].

A few experiments with polarization-entangled photon pairs [12,14,15] tested LR vs QM by means of Bell's inequalities derived from Hardy's argument. Being that these Hardy tests were affected by the same loopholes previously mentioned, they were not conclusive; i.e., they could not refute all versions of LR. In addition, a new and specific difficulty, the need to perform a "postselection" of events, affects these experiments. It comes from the fact that true nonmaximally entangled states are not easily produced. One thus starts with a factorizable state and, by a selective and *a posteriori* choice of the events to be considered (the rest are discarded), one attempts to reproduce the partial entanglement required by Hardy's argument. True nonmaximally entangled states have been produced very recently by using a spontaneous-down-conversion photon source [16]. They have been used for a measurement of the Hardy fraction, confirming QM.

The unsatisfactory situation due to the previous loopholes could be improved by using entangled neutral kaon pairs. Such pairs are copiously produced in ϕ-resonance decays into state (1) [17] as well as in $p\bar{p}$ annihilation processes [18]. The two kaons then fly apart from each other at relativistic velocities and easily fulfill the condition of spacelike separation. Moreover, kaons as well as their decay products are strongly interacting particles, thus allowing for high detection efficiencies [19]. Compared to photons (ions), neutral kaons seem thus to offer a more promising situation to close the efficiency (locality) loopholes. For these reasons, several papers on Bell's inequalities for the $K^0\bar{K}^0$ system have appeared in the last several years [19–30]. The fact that kaons are massive objects quite different from the massless photons usually considered adds further interest to these analyses. However, two specific problems appear when dealing with neutral kaons. The first comes from the interplay between strangeness oscillations and weak decays, which makes it very difficult to deduce Bell's inequalities violated by QM. The other problem is that, contrary to photons, whose polarization can be measured along any chosen direction, the choice in the kaon case reduces to measure either its lifetime or its strangeness [20,21,25,30]. Nevertheless, a few versions of QM violated and experimentally testable Bell's inequalities have been proposed for maximally [21,25,27,28] and nonmaximally [30,31] entangled kaon pairs.

The aim of this Letter is to explore the possibility to discriminate between LR and QM by applying Hardy's proof to entangled kaons. There are two good reasons for doing this: (1) the fact that genuine QM measurements for kaons are only of two types is not a drawback for Hardy's tests, since the latter require *two* distinct measurement possibilities on each kaon; (2) kaon pairs produced in $p\bar{p}$ annihilation processes [18] (in ϕ decays [17]) already appear in (can be converted, by means of a kaon regenerator, into) nonmaximally entangled states and are thus unaffected by postselection problems. As we will see, clear progress is then achieved in closing all the mentioned loopholes.

Let us start by considering the following nonmaximally entangled state:

$$\Phi = \frac{K_S K_L - K_L K_S + R K_L K_L + R' K_S K_S}{\sqrt{2 + |R|^2 + |R'|^2}}, \qquad (2)$$

which was originally discussed in Ref. [30] and where

$$R = -r \exp\{[-i\Delta m + (\Gamma_S - \Gamma_L)/2]T\},$$
$$R' = -r^2/R,$$

and $\Delta m \equiv m_L - m_S$ is the difference between the K_L and K_S masses, while Γ_S and Γ_L are their respective decay widths.

At a ϕ factory, state Φ can be prepared in the following way [30]. ϕ decays produce the antisymmetric state (1) which, ignoring small CP violation effects ($|\epsilon| \ll 1$), can also be written as $\phi_A = (K_S K_L - K_L K_S)/\sqrt{2}$. A thin (few mm's) neutral kaon regenerator placed along the right beam, close to the pair creation point, converts state ϕ_A into $\phi_r \propto K_S K_L - K_L K_S + r K_S K_S - r K_L K_L$, r being the regeneration parameter. Values of r are known to be rather small [typically, $|r| = (1 \div 5) \times 10^{-3}$ for 1 mm of material and kaon momenta below 1 GeV]. The state of Eq. (2) is then obtained from the unitary evolution of ϕ_r in free space up to a proper time T, after normalizing to undecayed pairs. To this aim, kaon pairs showing the decay of one (or both) member(s) before T have to be detected and excluded. Since this occurs prior to any measurement employed in Hardy's test, ours is a "preselection" (as opposed to postselection) procedure.

In experiments on $p\bar{p}$ annihilation at rest, the state preparation could be less complicated. One simultaneously has a dominant contribution of s-wave annihilation into the previous $J^{PC} = 1^{--}$ antisymmetric state, ϕ_A, plus a contamination of p-wave annihilation into 0^{++} and 2^{++}, i.e., with kaon pairs in the symmetric state $\phi_S = (K_S K_S - K_L K_L)/\sqrt{2}$. The coherent addition of these two annihilation amplitudes leads again to a state such as ϕ_r, where r now measures the relative strength of the p- to s-wave channels. Values for this new r of the same order of magnitude as before could be achieved by using appropriately polarized $\bar{p}$'s and modifying the target densities [32]. Unitary evolution in free space up to time T leads again to the desired state (2).

As in Ref. [30], we consider two mutually exclusive measurements of either strangeness or lifetime to be performed, at will, on each one of the two kaons of state (2) at a time $T \gtrsim 10\tau_S$, i.e., when the two kaons are reasonably far away from each other to fulfill the locality requirement (for details on how these measurements must be carried out, see Ref. [30]). Such intervals of T imply a very small value (neglected in what follows) of $|R'|$: $|R'| \leq 7 \times 10^{-3}|r| \ll 1$, and $|R| = \mathcal{O}(1)$. The following alternative joint measurements will be considered in our argumentation: (i) strangeness on both left and right beams; (ii) strangeness on the left and lifetime on the right; (iii) lifetime on the left and strangeness on the right; (iv) lifetime on both left and right beams. Being weak and strong interaction eigenstates related by $K_S = (K^0 + \bar{K}^0)/\sqrt{2}$ and $K_L = (K^0 - \bar{K}^0)/\sqrt{2}$ ignoring CP-violation effects, state (2) can be conveniently rewritten for settings (i), (ii), and (iii) as follows:

$$\Phi_{(i)} = \frac{1}{2\sqrt{2 + |R|^2}} [R K^0 K^0 + R \bar{K}^0 \bar{K}^0 + (2 - R) \bar{K}^0 K^0 - (2 + R) K^0 \bar{K}^0], \qquad (3)$$

$$\Phi_{(ii)} = \frac{1}{\sqrt{2(2 + |R|^2)}} [-K^0 K_S + \bar{K}^0 K_S + (1 + R) K^0 K_L + (1 - R) \bar{K}^0 K_L], \qquad (4)$$

$$\Phi_{(iii)} = \frac{1}{\sqrt{2(2 + |R|^2)}} [K_S K^0 - K_S \bar{K}^0 - (1 - R) K_L K^0 - (1 + R) K_L \bar{K}^0], \qquad (5)$$

while, for setting (iv), one has $\Phi_{(iv)} \equiv \Phi$ with $R' = 0$.

Now, let us consider the particular case in which $R = -1$ [33]. We refer to the corresponding QM state as *Hardy's state*. For it, QM predicts [34,35]

$$P_{\rm QM}(K^0, \bar{K}^0) = \eta\bar{\eta}/12, \qquad (6)$$

$$P_{\rm QM}(K^0, K_L) = 0, \qquad (7)$$

$$P_{\rm QM}(K_L, \bar{K}^0) = 0, \qquad (8)$$

$$P_{\rm QM}(K_S, K_S) = 0, \qquad (9)$$

where η ($\bar{\eta}$) is the K^0 ($\bar{K}^0$) detection efficiency of the experiment. The proof of Bell's theorem without inequalities consists in showing that this set of QM results is incompatible with LR. This we do by adapting Hardy's argument to our specific case.

It is easy to reproduce the prediction of Eq. (6) under LR. Introduce a local hidden-variable model with a normalized probability distribution $\rho(\lambda)$ [$\int_\Lambda d\lambda\rho(\lambda) = 1$] for which, if the pair is created in the state $\lambda \in \Lambda_{0,\bar{0}}$, the single kaon probabilities to detect a K^0 on the left and a $\bar{K}^0$ on the right are $0 < p_l(K^0|\lambda) \leq 1$ and $0 < p_r(\bar{K}^0|\lambda) \leq 1$, respectively. These functions and the hidden-variable distribution can be chosen such that

$$P_{\rm LR}(K^0, \bar{K}^0) \equiv \int_\Lambda d\lambda\rho(\lambda)p_l(K^0|\lambda)p_r(\bar{K}^0|\lambda) = \eta\bar{\eta}/12 \leq \int_{\Lambda_{0,\bar{0}}} d\lambda\rho(\lambda) \equiv \mu(\Lambda_{0,\bar{0}}). \quad (10)$$

The necessity to reproduce, in LR, predictions (7) and (8) has the following effects. Suppose that in a run of an experiment measuring strangeness on both sides at proper time T, a detection of a K^0 on the left and a $\bar{K}^0$ on the right occurred. If quantum mechanical prediction (6) is correct, such an event will be actually observed sometimes. From the fact that a K^0 has been observed on the left for this specific event, through Eq. (7) we can infer that if lifetime (and not strangeness) had been measured on the right with an ideal (efficiency one) detector, one would have observed a K_S. Following the Einstein-Podolsky-Rosen condition for the existence of an *element of physical reality* (EPR) [36], the above prediction, made with certainty and without disturbing the right going particle, permits us to assign an EPR to the kaon on the right, the fact of being a K_S [37]:

$$p_r(K_S|\lambda) = 1, \qquad \forall\ \lambda \in \Lambda_{0,\bar{0}}. \quad (11)$$

For these λ values, such an EPR existed independently of any measurement performed on the left going kaon. In fact, according to the *locality assumption,* when the two kaons are spacelike separated, the EPR's belonging to one kaon cannot be created or influenced by a measurement made on the other kaon. From the fact that a $\bar{K}^0$ has been observed on the right, by applying a similar argument to the prediction of Eq. (8) one concludes that the kaon on the left has an EPR, again corresponding to the fact of being a short living kaon:

$$p_l(K_S|\lambda) = 1, \qquad \forall\ \lambda \in \Lambda_{0,\bar{0}}. \quad (12)$$

Imposing locality, the same EPR on the left would have existed if lifetime (and not strangeness) had been measured on the right. For *all* the runs of the joint strangeness measurements which gave the result $(K^0, \bar{K}^0)$ (a fraction $\eta\bar{\eta}/12$ of the total), we then expect that if one had instead measured lifetime along both beams with ideal detectors, one would have obtained the outcome (K_S, K_S). This contradicts QM prediction (9) since, through Eqs. (10)–(12), LR requires

$$P_{\rm LR}(K_S, K_S) \equiv \int_\Lambda d\lambda\rho(\lambda)p_l(K_S|\lambda)p_r(K_S|\lambda) \geq \mu(\Lambda_{0,\bar{0}}) \geq \eta\bar{\eta}/12. \quad (13)$$

To prove whether LR is refuted by Nature, the quantities of Eqs. (6)–(9) must then be measured. Such a Hardy-type experiment requires perfect $K_{L,S}$ detection, but only moderate (strictly, nonvanishing) K^0 and $\bar{K}^0$ detection efficiencies. Let us consider then the real experimental possibilities. As discussed in Ref. [30], to measure the strangeness of a neutral kaon, a piece of ordinary, nucleonic matter has to be placed at the appropriate (time-of-flight) distance T, thus inducing distinct K^0N and $\bar{K}^0N$ strangeness-conserving strong reactions which allow for unambiguous K^0 vs $\bar{K}^0$ identification [18,20]. The situation is then quite analogous to that encountered in photon polarization measurements, including the low detection efficiency effects. Indeed, the need to perform the strangeness measurement at a given instant T requires that the piece of matter which induces the kaon-nucleon reaction has to be rather thin. But then the probability of interaction is considerably reduced [18], at least for ordinary materials and kaon velocities. However, even if this translates into η, $\bar{\eta}$ well below 1, our argumentation remains valid. It requires only an ideal efficiency for the alternative, lifetime measurements at the same instant T. In this case, the previous piece of material has to be removed in such a way that the neutral kaon continues its propagation in free space after time T. If it is observed to decay shortly after T (mostly into two easily detectable pions) the neutral kaon has been measured to be a K_S not only at the decay point but also at the relevant time T, since there are no K_S-K_L oscillations in free space. As discussed in Refs. [21,30], some K_S-K_L misidentifications will appear (moreover, the two states K_S and K_L are not strictly orthogonal), but only at an acceptably low level. Therefore, the $K_{L,S}$ detection efficiencies seem to be sufficiently close to 1.

Neutral kaon pairs in the state (2) seem thus to offer an excellent opportunity to discriminate between QM and LR in experimental tests quite close to the original proposal by Hardy. This requires the measurement of the four joint probabilities $P(K^0, \bar{K}^0)$, $P(K^0, K_L)$, $P(K_L, \bar{K}^0)$, and $P(K_S, K_S)$, using alternative experimental setups fulfilling the conventional locality conditions. In order to confirm QM, the latter three probabilities have to be compatible with zero within experimental errors; i.e., no events should survive after background subtraction [38] in the three corresponding runs. Once these null results are (most probably) confirmed one has to look at events corresponding to strangeness measurements on both sides. The compatibility or not with zero of the (similarly corrected) number of $(K^0, \bar{K}^0)$ events decides either against QM or against LR. At a ϕ factory the test requires kaon pairs surviving up to a detection time $T = 11.09\tau_S$ [33]. This occurs to one over 66×10^3 created pairs, to be compared with the 190×10^3 $\phi \to K^0\bar{K}^0$ decays presently produced per hour at DaΦne.

Needless to say, null measurements cannot be strictly performed but have allowed for a conceptually very simple Hardy-type test. More realistic but more involved treatments would require the use of inequalities, quite in line with the conventional Bell-type tests and will be discussed elsewhere [31]. Our purpose here was another one: to emphasize the usefulness of kaon pairs in this kind of discussion and particularly in those based on Hardy's argument. On the one hand, no postselection is required, in contrast to other (photonic) Hardy experiments. On the other hand, thanks to the fact that in order to complete Hardy's argument perfect efficient detection is needed for *just one* measurement type (at variance with Bell-type analyses), a promising possibility to close the efficiency (as well as the locality) loophole has been opened.

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[33] It is possible to obtain $R = -1$ if a beryllium (carbon) regenerator with thickness $d = 2.83$ mm ($d = 0.78$ mm) is used in addition to a detection time $T = 11.09\tau_S$ ($T = 11.31\tau_S$) and kaon pairs created at a ϕ factory ($p\bar{p}$ machine).

[34] In the standard Hardy-like proofs [10,12,13], the probabilities corresponding to our (7)–(9) are perfectly vanishing. We have $P_{\rm QM}(K^0, K_L)/(\eta\eta_\tau) = P_{\rm QM}(K_L, \bar{K}^0)/(\bar{\eta}\eta_\tau) \simeq (2/3)({\rm Re}\epsilon)^2 \simeq 2 \times 10^{-6}$ (because of CP nonconservation), η_τ being the efficiency for lifetime measurements, and $P_{\rm QM}(K_S, K_S)/\eta_\tau^2 = |R'|^2/3 = 8 \times 10^{-13}$ (5×10^{-11}) for the experiment at a ϕ factory ($p\bar{p}$ machine). Nevertheless, this will not prevent us from deriving a contradiction between LR and QM.

[35] Note that for $R = 1$, a similar contradiction is obtained between LR and the following predictions of QM: $P_{\rm QM}(\bar{K}^0, K^0) = \eta\bar{\eta}/12$, $P_{\rm QM}(\bar{K}^0, K_L) = P_{\rm QM}(K_L, K^0) = P_{\rm QM}(K_S, K_S) = 0$. Unfortunately, a test corresponding to this situation is more difficult to realize [31]. $R = \pm 1$ are the only values that allow to prove the contradiction.

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[38] The level of background should be moderately low because the signal consists of jointly detected and *collinear* events.