On-Off Intermittency in a Human Balancing Task

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Motion analysis in three dimensions demonstrate that the fluctuations in the vertical displacement angle of a stick balanced at the fingertip obey a scaling law characteristic of on-off intermittency and that $> 98\%$ of the corrective movements occur fast compared to the measured time delay. These experimental observations are reproduced by a model for an inverted pendulum with time-delayed feedback in which parametric noise forces a control parameter across a particular stability boundary. Our observations suggest that parametric noise is an essential, but up until now underemphasized, component of the neural control of balance.

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Two intrinsic properties of neural control mechanisms are the presence of time delays and random, uncontrolled fluctuations (“noise”). The interplay between noise and time-delayed control mechanisms can lead to the appearance of new phenomena that themselves may be beneficial for neural function. For example, state-independent (“additive”) noise can stabilize neural control by postponing bifurcations [1], result in noise-induced switching between coexistent attractors [2] and even stochastic resonance [3]. Considerably less attention has been paid to the possible effects of parametric, i.e., state-dependent, noise on neural function. Variations in parameters can have larger effects on stability than additive noise [4].

An often overlooked limitation of time-delayed feedback for control is that corrective responses to perturbations do not occur until a time τ after the perturbation occurs. This means that, for an attractor with a finite basin of attraction, perturbations could cause destabilization before feedback had time to respond. Since time delays in physiological systems range from milliseconds to days [5], an important question is how can control be maintained on time scales shorter than the delay? Parametric noise may provide one solution to this problem. In particular, stochastic or chaotic forcing of a control parameter across a stability boundary produces on-off intermittency [6]. This stochastic form of intermittency has been observed in a variety of mathematical models [6] and in specialized experimental paradigms [7]. Here we demonstrate that on-off intermittency also occurs in stochastic dynamical systems with retarded variables. The importance is that corrective movements occur on all time scales including those shorter than the delay.

A situation in which overcoming the limitations of delayed feedback becomes critically important to the nervous system is the maintenance of balance: even momentary miscalculations can result in a fall. A balancing task that is amenable to analyses is stick balancing at the fingertip [Fig. 1(a)] [8]. The crucial role played by time-delayed feedback in stick balancing is demonstrated by the observation that longer sticks are much easier to balance than shorter ones: once the stick is sufficiently long, its rate of movement becomes slow relative to the time required by the nervous system to make corrective movements. For an inverted pendulum stabilized by time-delayed feedback, it is possible that the upright position can be stable for certain parameter choices ([9] and this Letter). Parametric noise is an important component of a motor control loop [10]. Thus, the two properties most essential for the occurrence of on-off intermittency, parametric noise and a stability boundary, are present in the task of stick balancing.

The movements of a stick balanced at the fingertip in three dimensions can be monitored noninvasively with high precision [Fig. 1(a)] [11]. During stick balancing, the wrist and fingers are held rigid and the movements occur at the shoulder and elbow (flexion-extension and abduction-adduction). The excursions made by the hand are approximately fivefold larger than those made by the tip of the stick. Light sticks cannot be balanced with the eyes closed [12]. Thus, it is likely that visual estimation of the vertical displacement angle, θ, of the balanced stick is a primary sensory input. The quantity $\Delta z/l$ is equal to $\cos\theta$, where Δz is the difference in the vertical coordinate of the upper and lower ends of the stick and l is the stick length. For a 39 and a 62 cm stick, Δz ranges between, respectively, 0.19–1.95 cm and 0.12–1.24 cm (spatial resolution of the motion analysis system is 0.005 cm when data is collected at 120 Hz).

Figure 1(b) shows the fluctuations in $\Delta z/l$ as a function of time for a single realization. The fluctuations in $\Delta z/l$ exhibit the following characteristics: (i) periods in which small fluctuations occur alternate with shorter periods characterized by larger changes; (ii) the baseline for the fluctuations is not the upright position; i.e., most of the time the balanced stick is slightly deviated from the vertical; and (iii) the power spectrum for the fluctuations contains two scaling regions: one with slope $-\frac{1}{2}$ and another with slope -2.5 [solid lines in Fig. 1(c)]. The $\sim$1–2 Hz peak in the power spectrum is not part of the phenomenon we discuss in this Letter [13].

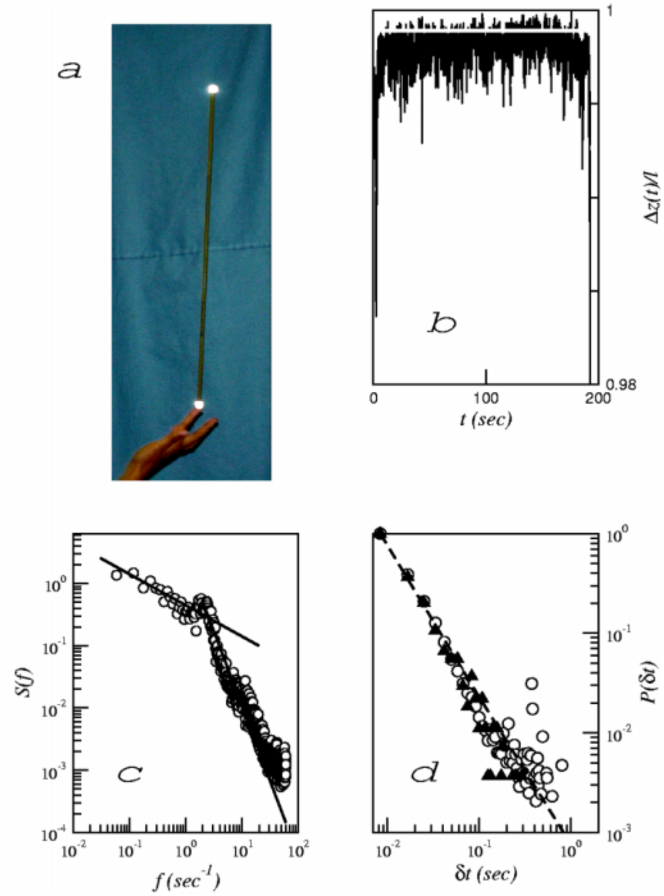

FIG. 1 (color online). (a) Stick balanced at the fingertip. (b) Temporal series for $\frac{\Delta z}{l}$ for a 62 cm stick balanced at the fingertip. The data was sampled at 120 Hz. The horizontal line depicts the threshold position at 1.0005 times the mean value. (c) Log-log plot of the power spectrum for the fluctuations in $\frac{\Delta z}{l}$. It is characterized by two power law regimes with exponents close to $-\frac{1}{2}$ (upper line) and -2.4 (lower line). (d) Normalized laminar phase probability distribution, $P(\delta t)$. The triangles are calculated from a single realization and the circles represent the mean distribution obtained from 48 realizations. The normalization of the $P(\delta t)$ is done to permit comparison between different realizations. The dashed line represents a power law with exponent $-\frac{3}{2}$.

A $-\frac{1}{2}$ scaling law is expected to occur in the power spectra of systems that exhibit on-off intermittency [14]. To further explore this possibility, we choose a threshold [horizontal line in Fig. 1(b)] and measured the time intervals, δt, between the occurrence of corrective movements, i.e., fluctuations that cross the threshold in the upward direction. These time intervals are referred to as the laminar phases. Figure 1(d) shows a double log plot of the normalized probability of having laminar phases of length δt, $P(\delta t)$. The linear relationship over more than two decades implies that the fluctuations in θ are governed by a scaling law with exponent $-\frac{3}{2}$ (identical results were obtained for four subjects). This scaling is a characteristic of on-off intermittency [6,7]. The same result was obtained for stick lengths between 39–150 cm and for other choices of the threshold, provided that it was placed in such a way that there were sufficient crossings to permit good statistics.

The time delay was estimated from the cross correlation between the movements of the tip of the stick and the finger to be ~100 msec (range of 70–120 msec for five subjects balancing a 62 cm stick). This delay is consistent with that estimated for other manual tracking tasks under visual feedback [15]. Thus $\geq 98\%$ of the times between corrective movements are shorter than the latency for the neural reflex that controls stick balance. The slight departure from the scaling law for the larger $\delta(t)$ likely reflects the influence of additive noise [16]. However, since the effects are small and enter for $\delta(t)$ larger than the delay, we do not consider additive noise further in this study.

We investigated a simple model for the stabilization of an inverted pendulum with time-delayed feedback in the presence of parametric noise [17]. An inverted pendulum of mass m and inertia moment $I = \frac{1}{3}ml^2$ moves under the action of three different forces, namely: its weight $m\vec{g}$, friction $\vec{F}_\gamma$, and a restoring force applied by the hand which depends on the angular deviation at time $t - \tau$, $\vec{F}(\theta(t-\tau))$. Determining the net torque acting on the stick results, after proper delay normalization, in the following delay differential equation

$$\ddot{\theta} + \Gamma\dot{\theta} - q\sin\theta + cF(\theta(t-1)) = 0,$$

where $\Gamma \equiv \frac{3\gamma}{m}\tau$, $q \equiv \frac{3g}{l}\tau^2$, and $c \equiv \frac{3}{ml}\tau^2$. The "$-$" sign is because we have taken $\theta = 0$ to be the upright position. Next we expand

$$cF(\theta(t-1)) \sim R_0\theta(t-1) + R_1\dot{\theta}(t-1) + \ldots$$

and take into consideration only the linear term drooping the linear derivative. It is important to note that the effect of the hand movement is not just that of changing θ but also the pendulum's suspension point. This can be modeled by introducing a random force in the parameter of the restoring force, i.e., $R(t) \equiv R_0 + \xi(t)$, thus we obtain

$$\ddot{\theta} + \Gamma\dot{\theta} - q\sin\theta + R(t)\theta(t-1) = 0, \qquad (1)$$

where $\xi(t)$ is a Gaussian white noise and R_0 is an adjustable parameter.

Figure 2(a) shows the stability diagram for (1) as a function of R_0 and τ. Choices of R_0 and τ within the crescent-shaped region result in a stabilized inverted pendulum. On the unstable side of the boundary, the mode of escape to infinity differs between different regions of parameter space: outside the upper (straight) stability boundary there is a slow oscillatory escape [Fig. 2(b)]; outside the lower (convex) stability boundary there is rapid and randomlike escape [Fig. 2(e)]. Time series that most closely resembled those observed for

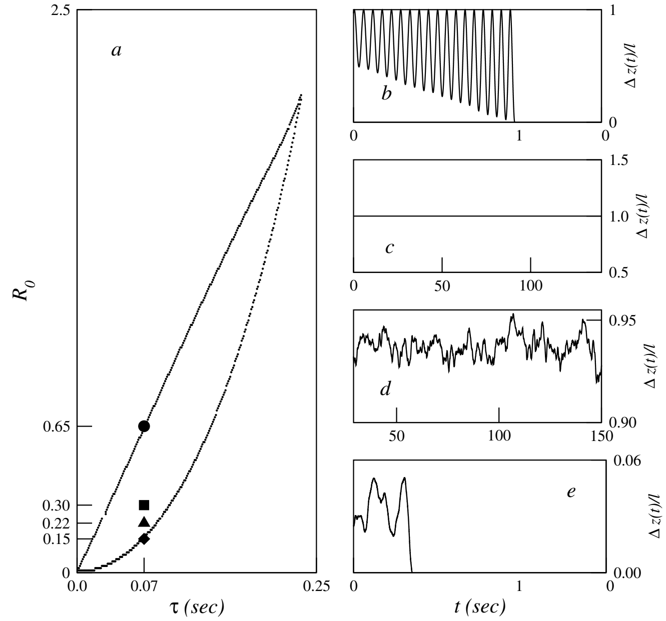

FIG. 2. (a) Stability domain of (1) in the deterministic situation computed as follows: For each pair (R_0, τ), (1) was integrated using an Euler algorithm with step size $h = 10^{-3}$ an initial condition $\theta(s) = 0$ for $s \in [0, -\tau]$. We assumed that the system escaped whenever $\theta(t) > \frac{\pi}{2}$ for $\frac{t}{h} < 2^{23}$; otherwise, we assumed that the parameter pair belonged to the stability domain. The stability boundary determined in this way is expected to approximate the true stability boundary for $t \to \infty$; i.e., the true stability domain is likely smaller than shown. Representative time series for the parameter choices corresponding to the solid symbols in (a) are shown, respectively, from top to bottom, in (b), (c), (d), and (e). Parameter values are indicated on the axis of (a).

stick balancing [Fig. 1(b)] occurred when R_0 and τ were chosen to be close to the lower stability boundary [Fig. 2(d)].

The power spectrum and log-log plot for the laminar phases for (1) are shown in Fig. 3 [parameter choices correspond to Fig. 2(d)]. All of the principle features of the dynamics observed for stick balancing are reproduced, namely: (i) a time series with laminar phases resting on a baseline slightly displaced from vertical [Fig. 2(d)]; (ii) a power spectrum exhibiting a small region of $\sim \frac{-1}{2}$ scaling and a larger region of ~ -2.5 scaling [Fig. 3(a)]; and (iii) threshold crossings that exhibit $-\frac{3}{2}$ scaling [Fig. 3(b)]. The slow convergence of the log-log plot for the laminar phases [Fig. 3(b)] reflects the influence of critical slowing down phenomena due to the proximity of the dynamical system to the stability boundary. For numerical reasons, we performed the calculations using a small delay ($\tau = 70$ msec) and a choice of R_0 on the stable side of the lower stability boundary; i.e., the stick does not fall. In order to achieve balance times resembling those observed experimentally, it is necessary to choose these parameters closer to the boundary. Thus, the observations in Figs. 1–3 strongly argue

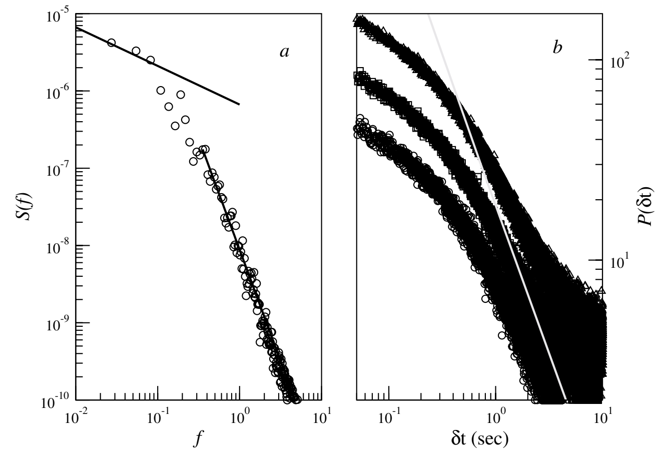

FIG. 3. Log-log plot of the (a) power spectrum characterized by two power law regimes with exponents close to $-\frac{1}{2}$ (upper line) and -2.4 (lower line) and (b) laminar phases distribution for (1) for three different values of the total number of laminar phases, from top to bottom: 3×10^6, 2×10^6, and 10^6. Numerical integration was performed using an Euler algorithm with a step size of 10^{-3}. The initial integration time of 50τ was discarded and (1) was further integrated $> 10^7\tau$ to obtain the data to calculate (a) and (b). Parameter values for the simulations were $\tau = 0.07$ sec, $R_0 = 0.226981$, $\sigma = 0.13$, and $\gamma = 100$.

that in parameter space the neural control for stick balancing is placed to be very near, or perhaps on, a stability boundary.

The limitations of time-delayed feedback for maintaining control in the presence of rapid, random perturbations have received little attention. Our observations suggest that, at least for stick balancing, the nervous system has an elegant solution to this problem. Maneuverability is increased by tuning parameters to place the control system very close to instability; instability is also employed, for example, in the design of high performance aircraft [18] and walking toys [19]. Near a stability boundary parametric noise results in corrective movements of the tip of the stick that occur on all time scales and, in particular, those shorter than the delay (i.e., on-off intermittency). The beneficial effect of on-off intermittency arises because the fluctuations in θ resemble a random walk for which the mean value of θ is approximately zero; i.e., the balanced position is statistically stabilized.

Previous investigators have recognized the importance of closed-loop (i.e., sensory feedback dependent) and open-loop (i.e., sensory feedback independent, including noise) control mechanisms for the maintenance of balance by the nervous system [2,20]. However, our study is the first to draw attention to the crucial importance of parametric noise in allowing control to be extended to time scales shorter than the delay. Moreover, our observations emphasize that in order to understand control in noisy dynamical systems with retarded variables it will be necessary to change the focus from the identification and characterization of attractors to phenomena that occur near stability boundaries.

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[11] Reflective balls were attached to the end of the stick (mass $\sim$0.25 gr). Infrared light reflected from these markers is detected by two specialized motion capture cameras (Qualisys, Inc.). The image projected on the CCD of each camera determines two of the spatial coordinates; the third spatial coordination is determined by using triangulation techniques involving both cameras.
[12] Construction materials (e.g., wood, metal, synthetics) were chosen so that the mass of the stick remained the same (35 gr) as the length varied.
[13] The frequency of this peak is greater than that of the heartbeat ($\sim$ 1.2 Hz) and was present in the power spectra obtained when a subject closed their eyes and moved their hand in a manner analogous to the movements during stick balancing. Under these conditions, the power spectra did not contain a significant region with slope $-\frac{1}{2}$.
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