Collective Directed Transport of Symmetrically Coupled Lattices in Symmetric Periodic Potentials

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A mechanism responsible for the directed transport of symmetrically coupled lattices in a symmetric periodic potential is proposed. Under the action of an external wave that breaks the spatiotemporal symmetry and introduces inhomogeneities of the lattice, a net unidirectional current can be observed, originating from a collaboration of the lattice (periodic potential and mutual interactions) and the wave (amplitude, frequency, and phase shifts). Mode-locking induced resonant steps are observed and predicted. The directed transport can be optimized and furthermore controlled by suitably adjusting the parameters of the system.

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The fundamental property of detailed balance excludes the possibility of the existence of nonzero net currents at the thermodynamic equilibrium [1]. It has been shown that the directed transport (DT) of Brownian particles, i.e., the emergence of a net flow in phase space, in the absence of any macroscopic gradient of forces, can be observed, provided that the potential exhibits some spatial asymmetry and, furthermore, detailed balance is broken (the nonequilibrium condition). Explorations of such systems (ratchets) [2] may help us to understand the physics of (bio-)molecular motors [3], vortex dynamics in superconductors [4], Josephson-junction lattices [5] , quantum dots [6], and nanotechnology [7].

Much effort has been made on the directed motion of particles in the absence of mutual interactions. On the other hand, collective transport properties of spatially extended systems and groups of interacting elements have been extensively investigated in relating to diffusions, spatiotemporal pattern dynamics and waves, stochastic resonances, and ratchet motions. As for directed transport, the effect of coupling among units is significant in governing the transport properties in many physical cases [8,9]. In studies of bimolecular motors, it was revealed that internal degrees of freedom are essential for net directional motion to occur [10]. These studies indicate that the DT in many cases depends less on the fluctuation (noise) environment, while intrinsic structures (architectures) and symmetry properties of the system may play a more important role in inducing a directed net current (the so-called *deterministic* ratchets [11]). From the viewpoint of physics, it should also be interesting to explore the interconnections between the DT and the intrinsic dynamics in coupled systems. In this case, competitions among various spatiotemporal scales will lead to complicated collective directional transport behaviors.

There have been a number of explorations on DT in coupled systems. Numerous studies on coupled ratchet motions have been focused on ratchet potentials (the spatially asymmetric potential) [12]. However, under many circumstances one may not have a ratchet potential, or one does not care about the spatial asymmetry. Another important issue is the *dynamical* control of transport processes, which received much attention recently [13,14]. It has been shown that asymmetries in the coupling or the drives (deterministic ac forces or stochastic forces) could also lead to directed motion [11,13].

In this Letter we propose a mechanism responsible for the DT by investigating a model composed of a *symmetrically* coupled lattice in a *symmetric* periodic potential field, which is driven by an external wave. We observe the interesting DT behavior induced by the collaboration of the lattice and the wave. The dependence of the transport properties on the parameters of the system and their intrinsic mechanism are investigated. The mode-locking between the lattice and the external wave may lead to resonant steps of the current, which is numerically observed and theoretically predicted. The DT can be optimized and further controlled by adjusting the frequency and phase shifts.

Let us take the 1D N nearest-neighbor coupled oscillators as our working model. The Hamiltonian of the lattice can be written as $H = \sum_{j=1}^{N}[V(\theta_j) + U(\theta_j - \theta_{j-1})]$, where $V(\theta)$ is the symmetric periodic potential satisfying $V(\theta + b) = V(\theta) = V(-\theta)$ (b is the period), and $U(\theta)$ is a function describing the interaction. Now we apply a wave $A\exp[i(\omega t + j\phi)]$ to the lattice, where A, ω, and ϕ are the amplitude, frequency, and spatial wave vector (phase shift), respectively. In the dissipative limit one has the following equations of motion:

$$\dot{\theta}_j = -V'(\theta_j) + \left[\frac{\partial U(\theta_{j+1} - \theta_j)}{\partial \theta_j} - \frac{\partial U(\theta_j - \theta_{j-1})}{\partial \theta_j}\right] + A\cos(\omega t + j\phi). \quad (1)$$

Zero mode (Goldstone mode) excitation due to wave-wave coupling is well known in many hydrodynamic and pattern formation problems [15], while it is nontrivial in lattice systems due to their inherent discreteness and nonlinearity. To make the following discussion more concise, one may choose the simplest forms for $V(\theta)$ and $U(\theta)$. The Frenkel-Kontorova (FK) lattice [16] is one of the simplest forms that is capable of capturing the essential features of the competitive interactions, where $V(\theta) = d[1 - \cos(2\pi\theta/b)]$ and $U(\theta) = \frac{1}{2}K(\theta - a)^2$ with d, K, and a being the height of the potential, the coupling strength, and the static distance between units without the periodic potential, respectively. $\delta = a/b$ measures the frustration (mismatch) between the periodic potential and the spring constant. The equations of motion (1) for the FK case can be written as

$$\dot{\theta}_j = -d\sin\theta_j + K(\theta_{j+1} - 2\theta_j + \theta_{j-1}) + A\cos(\omega t + j\phi). \qquad (2)$$

Because of the lattice-dependent periodic forces, this equation cannot be interpreted as the description of real particle systems. However, Eqs. (1) and (2) have good physical background, which can be experimentally well accomplished in systems such as Josephson-junction arrays and ladders, charge-density waves [9], micro-electro-mechanical-systems (MEMS) resonator array [17], and coupled phase-locked loops. For example, the phase variable comes naturally from the basic Josephson equations for the current and voltage across the junction. The lattice-dependent ac forces can easily be achieved by a phase lag between adjacent junctions, which can be readily adjustable. One may thus study the transport behavior in the Josephson-junction arrays by resorting to the net voltage $V_j(t) = \hbar\dot{\theta}_j(t)/2e$. In the following simulations we use the periodic boundary conditions $\theta_{j+N}(t) = \theta_j(t) + aN$. We further introduce an average current as

$$J = \lim_{T\to\infty} \frac{1}{NT} \sum_{j=1}^{N} \int_0^T \dot{\theta}_j(t)dt. \qquad (3)$$

Throughout the Letter we fix $N = 100$, $\omega/2\pi = 0.1$, and $K = 1$ (unless specifically mentioned). Initial conditions of the system are set to be about the ground state of the FK lattice. This ensures the uniqueness of the current [18].

When $d = 0$ (no periodic potential), $K = 0$ (no coupling), or $A = 0$ (no external wave), no net current can be observed [19]; i.e., any two of these nonzero factors cannot lead to a DT. We are concerned with the possibility of observing a net current of the lattice (periodic potential + coupling) in the presence of the wave. Obviously, there is no net current for the in-phase case $\phi = 2\pi n$ with n an integer. Also, there is no DT when a is multiples of 2π. In the absence of the external wave ($A = 0$), the lattice stays at its ground state (pinned state). A weak wave is not enough to give rise to a directed motion. For stronger wave amplitude, the ground state may lose its stability and give rise to a macroscopic DT of the lattice, as presented in Fig. 1, where the evolutions of the second element (denoted by dashed lines) are shown for $d = 3.0$, $A = 3.5$, $a/2\pi = 0.09$, and $\phi/2\pi = 0.09$. It is interesting that here we observe the DT for *symmetrically* coupled elements by an external wave in a *symmetric* potential, while the average drive on the lattice is zero. In fact, each unit can feel the drive of a periodic force. The wave acting on the lattice destroys the symmetry of both the coupling and the periodic potential and leads to an inhomogeneity (modulation) in space and time. This broken symmetry produces "effective" asymmetric ac forces on each oscillator, i.e., $\dot{\theta}_j + d\sin\theta_j = f_j(t)$. In Fig. 1, the effective force on the second oscillator $f_2(t)$ is shown by the solid line. Obviously, the time average of the effective force $\langle f_2(t)\rangle = 0$, but it is temporally asymmetric; i.e., this effective force experiences a weaker effect on the oscillator in the negative direction, while it has a stronger pulselike action in the positive direction. This broken symmetry then causes a deterministic DT. The energy injected by the wave (ac drive) is transformed into that for DT (dc response) with a collaboration between the wave and the lattice.

In Fig. 2(a), the current J varying with the wave amplitude A is given for $a/2\pi = 0.09$ and $\phi/2\pi = 0.09$. For all heights of the potential, one always finds the oscillating behavior; i.e., there are multiple maximums of the net current when varying the wave amplitude. For very large driving amplitudes, the coupling plays a less significant role, and thus the efficiency of DT decreases. Moreover, we find the interesting barrier-assisted transport when increasing the barrier height d. In 2(a), when $d = 2.0$ and 3.0, the system has a much higher efficiency of transport than the case $d = 1.0$. One may observe some plateaus corresponding to a maximum transport rate $J = \omega$, indicating the mode-locking (resonance) of the lattice to the wave. In Fig. 2(b) the current J against d is plotted for different wave amplitudes. For a

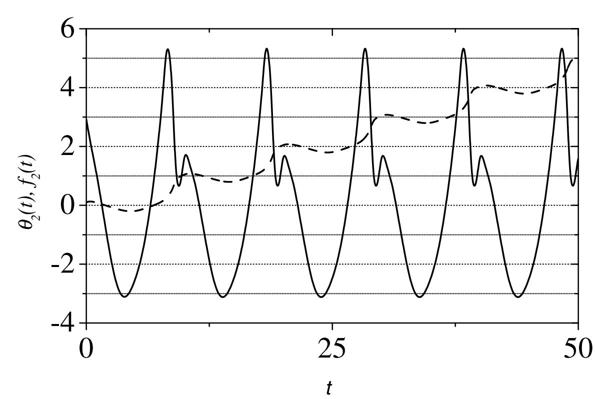

FIG. 1. The evolution of the second oscillator and the effective force for $A = 3.5$, $d = 3.0$, and $a/2\pi = \phi/2\pi = 0.09$. DT can be observed.

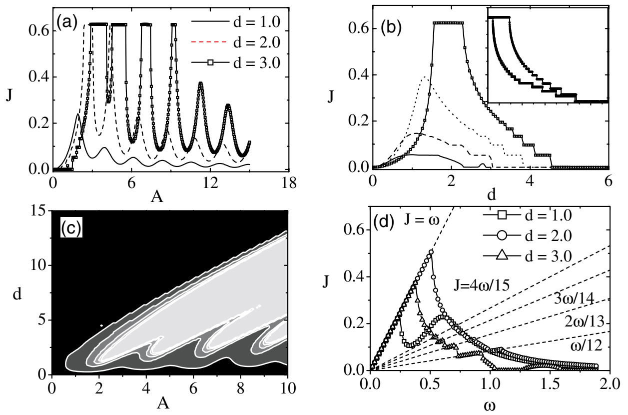

FIG. 2 (color online). (a) The net current J varies against the wave amplitude A for $d = 1.0$ (solid line), 2.0 (dashed line), and 3.0 (dotted line). Here $a/2\pi = \phi/2\pi = 0.09$. (b) The relation between J and the barrier height d for $A = 1.0$ (solid line), 1.5 (dashed line), 2.0 (dotted line), and 2.5 (solid and squares line); inset: the current against d for $a/2\pi = \phi/2\pi = 0.09$ (circles) and 0.07 (squares), $A = 4.0$, and a weaker coupling $K = 0.5$. (c) A contour plot of the current against A and d. Tongues with high-efficiency DT are observed. (d) The dependence of J on ω.

small d, J scales with d as a power law: $J \propto d^2$, similar to the case we found for the asymmetric coupling [13]. One has an optimal current at certain d, and then the DT is decreased when further increasing d. In Fig. 2(c), we give a contour for the current against both wave amplitude and the substrate. One can observe the interesting resonant "tongues" representing the complete mode-locking plateau, where there is the most efficient DT.

Some current steps for higher d (or small A) can be observed in Fig. 2(b) [also Fig. 2(a)]. These steps can be interpreted as the so-called Shapiro steps in coupled Josephson junctions. In traditional studies of Josephson junctions, Shapiro steps are caused by external ac + dc forces, while our results indicate that a combination of lattice coupling and external wave can also lead to Shapiro steps. We attribute these steps to the subharmonic mode lockings between the lattice and the wave. Furthermore, they can be predicted by the following symmetry analysis. An important symmetry of Eqs. (2) is that given a steady-state solution $\{\theta_j(t)\}$ of system (2), the transformation $T_{l,m,n}$,

$$T_{l,m,n}\{\theta_j(t)\} = \{\theta_{j+l}(t - 2\pi m/\omega) + 2\pi n\}, \qquad (4)$$

produces another steady-state solution $\{\theta'_j(t)\}$ [20]. Here l, m, n are arbitrary integers. If the lattice is locked to the wave, the above solution should be invariant under the transformation $T_{l,m,n}$, i.e., $T_{l,m,n}\{\theta_j(t)\} = \{\theta_j(t)\}$. This consequently leads to resonant steps:

$$J = \frac{la + 2\pi n}{l\phi + 2\pi m}\,\omega. \qquad (5)$$

In the inset of Fig. 2(b), we plot the current J against d for $a/2\pi = \phi/2\pi = 0.09$ (circle) and 0.07 (square), $A = 4.0$, and $K = 0.5$. One may find there are a series of steps when d is decreased, and they approach the principal step $J = \omega$ through $J/\omega = 1/12, 2/13, 3/14, \ldots$ for 0.09 and $J/\omega = 1/15, 2/16, 3/17, \ldots$ for 0.07. In fact, this can be well explained by (5) for $a = \phi$, where $n = 0$, $m = 1$. The steps are $J = l\omega/(\mathrm{int}[2\pi/a] + l)$. For $a/2\pi = \phi/2\pi = 0.09\ (0.07)$, we have $\mathrm{int}(2\pi/a) = 11\ (14)$, which agrees very well with the above step cascades. In Fig. 2(d), the dependence of J on ω is plotted for $a/2\pi = \phi/2\pi = 0.09$, $A = 3.0$, and $K = 1.0$ with $d = 1.0$, 2.0, and 3.0. In all cases J first increases with ω linearly and then decreases in a power law (complex dependence can be found for larger d). We also plot theoretical straight lines $J = \omega/12, 2\omega/13, 3\omega/14, 4\omega/15$, and ω, which represent resonances. It can be seen that the agreement with numerical results is quite good (see the big straight segment and some small segments for $d = 2$ and 3).

In the above studies we fixed both a and ϕ, which are very important in governing the DT behavior. In Figs. 3(a) and 3(b), we give the contour of the current varying against a and ϕ for $(A, d) = (1.0, 1.25)$ and $(4.0, 3.0)$. In Fig. 3(a), it can be found that with the collaboration of both parameters, the system possesses an optimal transport at $(a, \phi) \approx (0.816, 0.754)$. For larger A and d, the DT

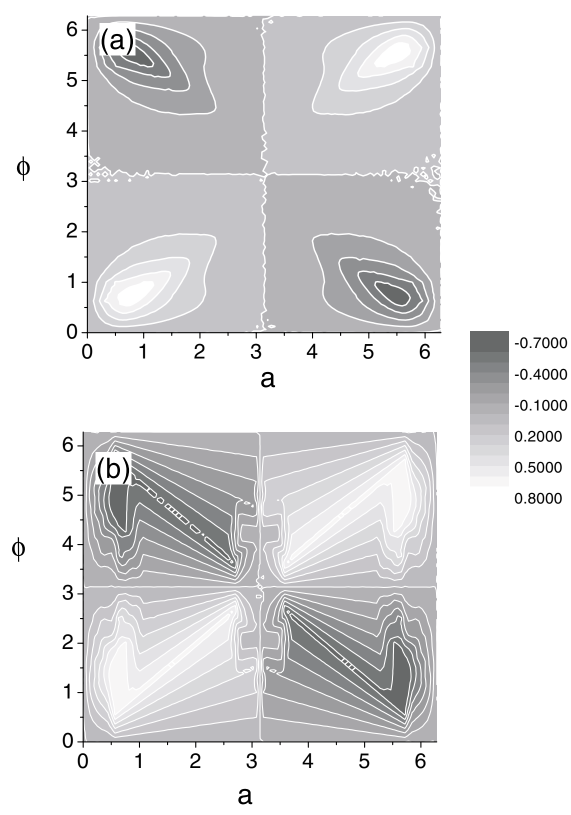

FIG. 3. Contour plots of the current J versus the harmonic static length a and the phase shift ϕ. In (a) $d = 1.25$, $A = 1.0$, and in (b) $d = 3.0$, $A = 4.0$.

can be greatly enhanced in a larger parameter regime of (a, ϕ), as indicated by Fig. 3(b). Equation (2) has several transformation-invariance properties [11]. The invariances of the transformations $\phi \to \phi + 2\pi$ and $a \to a + 2\pi$ immediately give rise to $J(a + 2\pi, \phi) = J(a, \phi + 2\pi) = J(a, \phi)$. In both contours one clearly finds the symmetry property of the current: $J(a, \phi) = J(2\pi - a, 2\pi - \phi) = -J(a, 2\pi - \phi) = -J(2\pi - a, \phi)$. In fact, due to the symmetry of the interaction, Eq. (2) is invariant under the transformation of spatial reflection and time shift $T_s : (\{\theta_j\}, t, a) \to (\{-\theta_j\}, t + T/2, 2\pi - a)$, with $T = 2\pi/\omega$. (Note that *the transformation is also related to the parameter* a.) This invariance indicates that if the system for the parameter a has a net current, there should also be a same net current in the opposite direction for the parameter $2\pi - a$, i.e., $J(a, \phi) = -J(2\pi - a, \phi)$. Under the transformation of time reflection $T_t : (\{\theta_j\}, t, \phi) \to (\{\theta'_j\}, -t, 2\pi - \phi)$, the equations of motion of Eq. (2) change to $-\dot{\theta}'_j = -d \sin\theta'_j + K(\theta'_{j+1} - 2\theta'_j + \theta'_{j-1}) + A \cos(\omega t + j\phi)$. Thus one has $J(a, \phi) = -J(a, 2\pi - \phi)$. It can easily be found that when $a = \pi$ or $\phi = \pi$, one has $J(\pi, \phi) = -J(\pi, \phi)$, and $J(a, \pi) = -J(a, \pi)$, leading to $J(\pi, \phi) = J(a, \pi) = 0$.

The above studies indicate that the dynamical control of the directed transport of spatiotemporal systems is possible by adjusting the parameters of the wave. The interplay between waves and lattices has long been an important subject in condensed matter physics as well as other related fields. For spatiotemporal systems with a large number of degrees of freedom, by applying an external wave, one can control the direction and efficiency of the DT processes. For example, one can appropriately increase the wave amplitude to enhance the transport efficiency. One can also simply adjust the phase shift to alter the transport direction and optimize the transport efficiency. By simply adjusting the external wave, there is no need to change the structural properties of the system (e.g., building the ratchet potential), which sometimes need tactics in technology.

To summarize, in this Letter we investigate the possible DT for a lattice with symmetric couplings in a symmetric periodic potential under the drive of an external wave. We show the existence of the net unidirectional current, and the optimal DT can be achieved by tuning the parameters of the system in a controllable way. We believe that the results and mechanism presented here can be well realized in experiments, such as Josephson-junction lattices, charge-density waves, MEMS resonator arrays, and coupled phase-locked loops.

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[1] M.V. Smoluchowski, Phys. Z. **13**, 1069 (1912); R. P. Feynman *et al.*, *The Feynman Lectures on Physics* (Addison-Wesley, Reading, MA, 1963).
[2] P. Reimann, Phys. Rep. **361**, 57 (2002).
[3] R. D. Astumian, Science **276**, 917 (1997); F. Julicher *et al.*, Rev. Mod. Phys. **69**, 1269 (1997).
[4] C. S. Lee *et al.*, Nature (London) **400**, 337 (1999); C. J. Olson *et al.*, Phys. Rev. Lett. **87**, 177002 (2001).
[5] E.Trias *et al.*, Phys. Rev. E **61**, 2257 (2000).
[6] H. Linke *et al.*, Science **286**, 2314 (1999).
[7] J. Rousselet *et al.*, Nature (London) **370**, 446 (1994); A.V. Oudenaarden *et al.*, Science **285**, 1046 (1999).
[8] I. Derenyi *et al.*, Chaos **8**, 657 (1998).
[9] L. Floria and J. Mazo, Adv. Phys. **45**, 505 (1996).
[10] R. D. Vale *et al.*, Science **288**, 88 (2000); S. A. Endow *et al.*, Nature (London) **406**, 913 (2000).
[11] P. Jung *et al.*, Phys. Rev. Lett. **76**, 3436 (1996); J. L. Mateos, Phys. Rev. Lett. **84**, 258 (2000); S. Flach *et al.*, Phys. Rev. Lett. **84**, 2358 (2000); O. Yevtushenko *et al.*, Europhys. Lett. **54**, 141 (2001).
[12] I. Derenyi and T. Vicsek, Phys. Rev. Lett. **75**, 374 (1995); Z. Csahok *et al.*, Phys. Rev. E **55**, 5179 (1997); A. Igarashi *et al.*, Phys. Rev. E **64**, 051908 (2001).
[13] Z. Zheng *et al.*, Phys. Rev. Lett. **86**, 2273 (2001); Z. Zheng and X. Li, Commun. Theor. Phys. **36**, 151 (2001).
[14] M. Porto *et al.*, Phys. Rev. Lett. **84**, 6058 (2000); B. Hu and J. Zhu, Phys. Rev. E **65**, 016202 (2002); S. Cilla *et al.*, Phys. Rev. E **63**, 031110 (2001).
[15] H. Greenside *et al.*, Phys. Rev. Lett. **60**, 2269 (1988); U. Frisch *et al.*, J. Fluid Mech. **168**, 221 (1986); H. Chang *et al.*, Phys. Rev. E **55**, 2818 (1997).
[16] J. Frenkel and T. Kontorova, Phys. Z. Sowjetunion **13**, 1 (1938); S. Aubry, Phys. Rep. **103**, 12 (1984); Z. Zheng *et al.*, Phys. Rev. B **58**, 5453 (1998); Phys. Rev. E **62**, 4294 (2000); and references therein.
[17] E. Buks and M. Roukes, cond-mat/0008211 (unpublished).
[18] The Middleton's no-passing rule for the coupled lattice asserts that for a lattice with convex coupling in the dissipative case, if initially two configurations satisfy $\{\theta_j(t_0)\} < \{\theta'_j(t_0)\}$, then we always have $\{\theta_j(t)\} < \{\theta'_j(t)\}$ for all $t > t_0$ [see A. Middleton and D. Fisher, Phys. Rev. Lett. **66**, 92 (1991)]. An important consequence of this "no-passing" rule is the uniqueness of the average current J. A proof can be found in [9].
[19] When $d = 0$, the evolution can be analytically solved by inserting the ansatz $\theta_j(t) = ja + C_1 \cos[\omega t + j\phi] + C_2 \sin[\omega t + j\phi]$ into Eqs. (2). The two constants are $C_1 = 4A \sin^2(\phi/2)[\omega^2 + 16 \sin^4(\phi/2)]^{-1}$ and $C_2 = A\omega[\omega^2 + 16 \sin^4(\phi/2)]^{-1}$. This implies a zero net current for $d = 0$.
[20] L. Floria and F. Falo, Phys. Rev. Lett. **68**, 2713 (1992).