Chen, Büttner, and Voit Reply In the preceding Comment [1], Capriotti *et al.* argue that the next-nearest-neighbor spin-Peierls operator $\hat{O}_{\mathrm{nnn}} = \sum_l (-1)^l \mathbf{S}_l \cdot \mathbf{S}_{l+2}$ is an irrelevant perturbation for the Heisenberg chain in the regime of weak frustration and hence the claim of our Letter [2] of the existence of an intermediate fixed point is unreasonable. In Ref. [2], we investigated the physical effect of the operator $\hat{O}_{\mathrm{nnn}}$ by renormalization group (RG) analysis. Recently, a related work was carried out by Sarkar and Sen [3] and the same bosonized operator of $\hat{O}_{\mathrm{nnn}}$ is obtained. However, the main discrepancy between our work and Refs. [1,3] is that we kept the bosonized operator of $\hat{O}_{\mathrm{nnn}}$ and analyzed it by RG, while they just discarded it by giving an argument of the irrelevance of $\hat{O}_{\mathrm{nnn}}$. For an anisotropic XXZ chain, our RG result indeed indicates that the operator $\hat{O}_{\mathrm{nnn}}$ is irrelevant [2] in the meaning that it does not drive the system to a new phase, and this is consistent with that of Refs. [1,3]. But the main difference lies in the issue of whether an intermediate fixed point exists and whether this fixed point corresponds to a phase different from a Luttinger liquid.

Despite the discussions in Ref. [1], we think that the scheme of discarding the operator $\hat{O}_{\mathrm{nnn}}$ based on its irrelevance seems to be oversimplified. We notice first that the magnetization curve of the Heisenberg model with an additional operator $\hat{O}_{\mathrm{nnn}}$ [4] gives an obvious different magnetization susceptibility from the one without $\hat{O}_{\mathrm{nnn}}$, which is a signal of the renormalization of the spin velocity. A complete scheme to deal with $\hat{O}_{\mathrm{nnn}}$ should give a correct description of the induced effect not only on the weak frustration region but also on the strong frustration region. Furthermore, the omission of $\hat{O}_{\mathrm{nnn}}$ could not give any explanation why the operator $\hat{O}_{\mathrm{nnn}}$ shrinks the spin gap sizes in the regime of strong frustration [2,5]. In contrast to Ref. [1], our RG analysis gives a qualitative explanation of the influence of $\hat{O}_{\mathrm{nnn}}$ on spin gap sizes, which is consistent with the known result of the difference in gap sizes between the Majumdar-Ghosh and sawtooth chains [5,6].

Based on the RG analysis we gave an argument for a vanishing spin-wave velocity and thus explained the intermediate fixed point as a phase with gapless excitations different from a spin-liquid phase of the Heisenberg model. We admit that the argument of a vanishing spin-wave velocity plays a crucial role in the existence of such an unusual phase. If the spin velocity does not vanish, we must explain the fixed point as a spin-liquid phase as in the model without $\hat{O}_{\mathrm{nnn}}$. However, we still think that the spin velocity is renormalized, even though it may not renormalize to zero. In addition, our work suggests that the quantum phase transition parameter ($J_2/J_1 \simeq 0.241$) to the dimer phase is changed by $\hat{O}_{\mathrm{nnn}}$, which should not be difficult to be verified by numerical simulations. We hope that more numerical simulations will eventually be able to resolve this disagreement.

Shu Chen,* H. Büttner, and J. Voit†
Theoretische Physik 1,
Universität Bayreuth,
D-95440 Bayreuth, Germany

Received 5 June 2002; published 12 September 2002
DOI: 10.1103/PhysRevLett.89.149702
PACS numbers: 75.10.Jm, 71.10.Hf, 75.10.–b

*Present address: Institute for Theoretical Physics IV, Heinrich-Heine-Universität Düsseldorf, 40225 Düsseldorf, Germany.
†Present address: Deutscher Sparkassen- und Giroverband, Abteilung C IV, Simrockstrasse 4, 53113 Bonn, Germany.

[1] L. Capriotti, F. Becca, S. Sorella, and A. Parola, preceding Comment, Phys. Rev. Lett. **89**, 149701 (2002).
[2] S. Chen, H. Büttner, and J. Voit, Phys. Rev. Lett. **87**, 087205 (2001).
[3] S. Sarkar and D. Sen, Phys. Rev. B **65**, 172408 (2002).
[4] R. M. Wiessner, A. Fledderjohann, K. H. Mütter, and M. Karbach, Eur. Phys. J. B **15**, 475 (2000).
[5] S. Chen, H. Büttner, and J. Voit (unpublished).
[6] D. Sen, S. Sharstry, R. E. Walstedt, and R. Cava, Phys. Rev. B **53**, 6401 (1996); T. Nakamura and K. Kubo, Phys. Rev. B **53**, 6393 (1996).

