

Comment on “Phase Diagram of an Asymmetric Spin Ladder”

In a recent Letter Chen *et al.* [1] investigated the so-called asymmetric spin ladder, a spin-half Heisenberg chain with alternation in the next-nearest-neighbor (nnn) interaction [2]. Based on bosonization and renormalization group analysis, they claimed that in the limit of small frustration (J_2/J_1) the asymmetry in the nnn integrals destabilizes the isotropic Heisenberg fixed point leading to a new phase with gapless excitations and vanishing spin-wave velocity.

In this Comment, using Bethe *ansatz* and conformal field theory we show instead that the nnn spin-Peierls operator

$$\hat{O}_{\text{nnn}} = \sum_l (-1)^l \hat{\mathbf{S}}_l \cdot \hat{\mathbf{S}}_{l+2} \quad (1)$$

represents an *irrelevant* perturbation for the Heisenberg chain in the regime of weak frustration. Since the latter operator is the one associated with the alternation in the nnn exchange, this clearly invalidates the mentioned claim and the conclusions of their Letter.

For the sake of generality, we refer to the anisotropic XXZ chain. In order to study the relevance of the operator (1) we have to identify the quantum numbers $\{j\}$ (referenced to the ground state with energy E_0) of the intermediate states appearing in the Lehmann representation of the associated susceptibility. These are total spin $S^z = 0$, momentum $k = \pi$, even parity under spatial reflection ($l \rightarrow -l$) $R = 1$, and even parity under spin reflection [$(S^x, S^y, S^z) \rightarrow (-S^x, S^y, -S^z)$]. The only difference with the *nearest-neighbor* spin-Peierls operator, a well-known relevant perturbation, is the spatial-reflection quantum number, $R = -1$. As is known by conformal field theory, the scaling dimension X of a given operator (where $X < 1$ characterizes a relevant operator) is related to the finite-size corrections of the energy of the lowest intermediate eigenstate j by the relation [3]

$$\Delta E(L) = E_j(L) - E_0(L) = 2\pi v_s X/L, \quad (2)$$

where v_s is the spin-wave velocity and L is the number of sites of the ring. The finite-size corrections can be computed either by Bethe *ansatz* (for $J_2/J_1 = 0$) or by bosonization in the Luttinger regime ($J_2/J_1 \lesssim 0.241$). For the nearest-neighbor spin-Peierls operator, due to the existence of a low-lying eigenstate with the correct quantum numbers, this procedure gives $X = K$, where $1/2 \leq K \leq 1$ (with $K = 1/2$ at the isotropic point) is the dimensionless coupling constant of the model. In contrast, the finite-size spectrum of the Heisenberg model does not contain low-lying states simultaneously even under both spatial and spin reflection, yielding $X = 9K > 1$, showing the irrelevance of the operator (1). This is illustrated in Fig. 1 using the Lanczos exact diagonalization technique. However, we stress that our conclusions follow directly

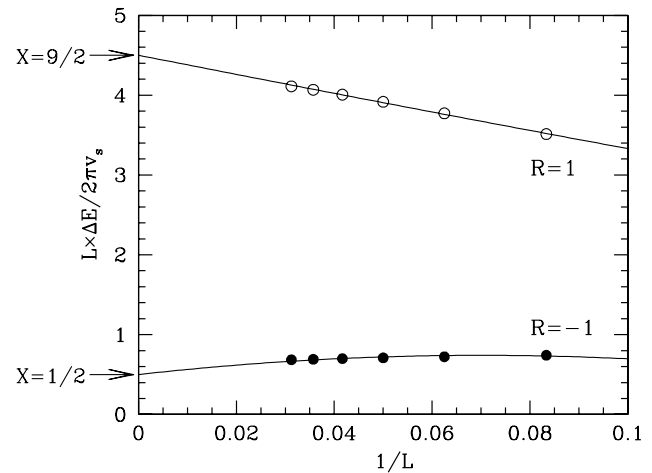


FIG. 1. Size scaling of the gap of the lowest eigenvalues of the isotropic spin-half Heisenberg chain ($J_2/J_1 = 0$) with $S^z = 0$, $k = \pi$, even parity under spin reflections, and $R = \pm 1$ under lattice reflections. Lines are guides for the eye.

from the exact analytical solution of the spin-half Heisenberg chain.

The recent bosonization analysis of Sarkar and Sen [4] is consistent with our conclusion.

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[2] According to the standard notation J_1 and J_2 indicate the nearest and next-nearest-neighbor antiferromagnetic couplings, respectively.

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