

Ab initio Approach to *s*-Shell Hypernuclei ${}^3_{\Lambda}\text{H}$, ${}^4_{\Lambda}\text{H}$, ${}^4_{\Lambda}\text{He}$, and ${}^5_{\Lambda}\text{He}$ with a ΛN - ΣN Interaction

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Variational calculations for *s*-shell hypernuclei are performed by explicitly including Σ degrees of freedom. Four sets of *YN* interactions [SC97d(S), SC97e(S), SC97f(S), and SC89(S)] are used. The bound-state solution of ${}^5_{\Lambda}\text{He}$ is obtained and a large energy expectation value of the tensor ΛN - ΣN transition part is found. The internal energy of the ${}^4\text{He}$ subsystem is strongly affected by the presence of a Λ particle with the strong tensor ΛN - ΣN transition potential.

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Few-body calculations for *s*-shell hypernuclei with mass number $A = 3$ – 5 are important not only to explore exotic nuclear structure, including the strangeness degrees of freedom, but also to clarify the characteristic features of the hyperon-nucleon (*YN*) interaction. Although several interaction models have been proposed [1–3], the detailed properties (e.g., 1S_0 or 3S_1 – 3D_1 phase shift, strength of ΛN - ΣN coupling term) of the *YN* interaction are different among the models. The observed separation energies (B_{Λ}) of light Λ hypernuclei are expected to provide important information on the *YN* interaction, because the relative strength of the spin-dependent term or of the ΛN - ΣN coupling term is affected from system to system.

Recently, few-body studies for $A = 3, 4$ hypernuclei have been conducted using modern *YN* interactions [4–6]. According to these developments, the Nijmegen soft core (NSC) model 97f (or 97e) seems to be compatible with the experimental B_{Λ} , though the calculated B_{Λ} for ${}^4_{\Lambda}\text{H}^*$ or ${}^4_{\Lambda}\text{He}^*$ is actually slightly smaller than the experimental value. These few-body calculations, however, have not yet reached a stage to calculate $B_{\Lambda}({}^5_{\Lambda}\text{He})$.

If one constructs a phenomenological central ΛN potential, which is consistent with the experimental $B_{\Lambda}({}^3_{\Lambda}\text{H})$, $B_{\Lambda}({}^4_{\Lambda}\text{H})$, $B_{\Lambda}({}^4_{\Lambda}\text{He})$, $B_{\Lambda}({}^4_{\Lambda}\text{H}^*)$, and $B_{\Lambda}({}^4_{\Lambda}\text{He}^*)$ values as well as the Λp total cross section, that kind of potential would overestimate the $B_{\Lambda}({}^5_{\Lambda}\text{He})$ value [7,8]. This is known as an anomalously small binding of ${}^5_{\Lambda}\text{He}$. Though a suppression of the tensor forces [9,10] or of the ΛN - ΣN coupling [11,12] was discussed to be a possible mechanism to resolve the anomaly, the problem still remains an enigma [13] due to the difficulty of performing a complete five-body treatment. Only one attempt was made, using a variational Monte Carlo calculation [14] with the NSC89 *YN* interaction. Though NSC89 well reproduces both the experimental $B_{\Lambda}({}^3_{\Lambda}\text{H})$ [4] and $B_{\Lambda}({}^4_{\Lambda}\text{H})$ [6,14] values as well as the experimental Λp total cross section, a bound-state solution of ${}^5_{\Lambda}\text{He}$ was not found. In view of the aim to pin down a reliable *YN* interaction, a systematic study for *all s*-shell hypernuclei is desirable.

The *NN* tensor interaction due to a one-pion-exchange mechanism is the most important ingredient for the binding

mechanisms of light nuclei. More than a third, or about one-half, of the interaction energy comes from the tensor force for the ${}^4\text{He}$ [15–17]. Since the pion (or kaon) exchange also induces the ΛN - ΣN transition for the *YN* sector, both the *NN* and ΛN - ΣN tensor interactions may also play important roles for light hypernuclei. If this is the case, the structure of the core nucleus (e.g., ${}^4\text{He}$) in the hypernucleus (${}^5_{\Lambda}\text{He}$) would be strongly influenced by the presence of a Λ particle.

The purpose of this Letter is twofold: First is to perform an *ab initio* calculation for ${}^5_{\Lambda}\text{He}$ as well as $A = 3, 4$ hypernuclei explicitly including Σ degrees of freedom. Second is to discuss the structural aspects of ${}^5_{\Lambda}\text{He}$ with an appropriate *YN* interaction which is consistent with all of the *s*-shell hypernuclear data.

The Hamiltonian (H) of a system comprising nucleons and a hyperon (Λ or Σ) is given by 2×2 components as

$$H = \begin{pmatrix} H_{\Lambda} & V_{\Sigma-\Lambda} \\ V_{\Lambda-\Sigma} & H_{\Sigma} \end{pmatrix}, \quad (1)$$

where $H_{\Lambda}(H_{\Sigma})$ operates on the Λ (Σ) component and

$$V_{\Lambda-\Sigma} = \sum_{i=1}^{A-1} v_{iY}^{(N\Lambda-N\Sigma)}. \quad (2)$$

We employ the G3RS potential [18] for the *NN* interaction and the SC97d(S), SC97e(S), SC97f(S), or SC89(S) potential [19] for the *YN* interaction, where all interactions have tensor and spin-orbit components in addition to the central one. We omit small nonstatic correction terms [$(\mathbf{L} \cdot \mathbf{S})^2$ and \mathbf{L}^2 terms] in the G3RS *NN* interaction and odd partial-wave components in each interaction in order to focus on the main part of the interaction in the even parity state. The calculated binding energies for light nuclei (${}^2\text{H}$, ${}^3\text{H}$, ${}^3\text{He}$, and ${}^4\text{He}$) are 2.28, 7.63, 6.98, and 24.57 MeV, respectively. The *YN* interactions have Gaussian form factors whose parameters are set to reproduce the low-energy *S* matrix of the corresponding original Nijmegen *YN* interactions [20]. These Gaussian form factors help to save significant computer time.

TABLE I. Λ separation energies, given in units of MeV, of $A = 3-5$ Λ hypernuclei for different YN interactions. The scattering lengths, given in units of fm, of $^1S_0(a_s)$ and $^3S_1(a_t)$ states are also listed.

YN	a_s	a_t	$B_\Lambda(^3\Lambda\text{H})$	$B_\Lambda(^4\Lambda\text{H})$	$B_\Lambda(^4\Lambda\text{H}^*)$	$B_\Lambda(^4\Lambda\text{He})$	$B_\Lambda(^4\Lambda\text{He}^*)$	$B_\Lambda(^5\Lambda\text{He})$
SC97d(S)	-1.92	-1.96	0.01	1.67	1.20	1.62	1.17	3.17
SC97e(S)	-2.37	-1.83	0.10	2.06	0.92	2.02	0.90	2.75
SC97f(S)	-2.82	-1.72	0.18	2.16	0.63	2.11	0.62	2.10
SC89(S)	-3.39	-1.38	0.37	2.55	Unbound	2.47	Unbound	0.35
Experiment			0.13 ± 0.05	2.04 ± 0.04	1.00 ± 0.04	2.39 ± 0.03	1.24 ± 0.04	3.12 ± 0.02

The binding energies of various systems are calculated in a complete A -body treatment. The variational trial function must be flexible enough to incorporate both the explicit Σ degrees of freedom and higher orbital angular momenta. The trial function is given by a combination of basis functions:

$$\Psi_{JMTM_T} = \sum_{k=1}^N c_k \varphi_k, \quad (3)$$

with $\varphi_k = \mathcal{A}\{G(\mathbf{x}; A_k)[\theta_{L_k}(\mathbf{x}; u_k, K_k)\chi_{S_k}]_{JM}\eta_{kTM_T}\}$.

Here, \mathcal{A} is an antisymmetrizer acting on nucleons and χ_{S_k} (η_{kTM_T}) is the spin (isospin) function. η_{kTM_T} has two components: upper (lower) component refers to the Λ (Σ) component. The abbreviation $\mathbf{x} = (\mathbf{x}_1, \dots, \mathbf{x}_{A-1})$ is a set of relative coordinates. A set of linear variational parameters (c_1, \dots, c_N) is determined by the Ritz variational principle.

A spatial part of the basis function is constructed by the correlated Gaussian (CG) multiplied by the orbital angular momentum part $\theta_L(\mathbf{x})$, expressed by the global vector representation (GVR) [21]. CG is defined by

$$\begin{aligned} G(\mathbf{x}; A_k) &= \exp\left\{-\frac{1}{2}\sum_{i<j}^A \alpha_{kij}(\mathbf{r}_i - \mathbf{r}_j)^2\right\} \\ &= \exp\left\{-\frac{1}{2}\sum_{i,j=1}^{A-1} (A_k)_{ij}\mathbf{x}_i \cdot \mathbf{x}_j\right\}. \end{aligned} \quad (4)$$

The $(A-1) \times (A-1)$ symmetric matrix (A_k) is uniquely determined in terms of the interparticle correlation parameter (α_{kij}) . The GVR of $\theta_{L_k}(\mathbf{x}; u_k, K_k)$ takes the form

$$\begin{aligned} \theta_{L_k}(\mathbf{x}; u_k, K_k) &= v_k^{2K_k+L_k} Y_{L_k}(\hat{\mathbf{v}}_k), \\ \text{with } \mathbf{v}_k &= \sum_{i=1}^{A-1} (u_k)_i \mathbf{x}_i. \end{aligned} \quad (5)$$

The A_k and u_k are sets of nonlinear parameters which characterize the spatial part of the basis function. Allowing the factor $v_k^{2K_k}$ ($K_k \neq 0$) is useful to improve the short-range behavior of the trial function. The value of K_k is assumed to take 0 or 1. The variational parameters are optimized by a stochastic procedure. The above form of the trial function gives accurate solutions. The reader is referred to Refs. [16,21] for details and recent applications.

For the spin and isospin parts, all possible configurations are taken into account.

Table I lists the results of the Λ separation energies. The scattering lengths of the $^1S_0(a_s)$ and $^3S_1(a_t)$ states for each YN interaction are also listed in Table I, where the interactions are given in increasing order of $|a_s|$ (and in decreasing order of $|a_t|$). The SC89(S) interaction produces no or very weakly bound state for $^4\Lambda\text{H}^*$, $^4\Lambda\text{He}^*$, or $^5\Lambda\text{He}$. For the SC97d-f(S), the $B_\Lambda(^5\Lambda\text{He})$ value is about 2–3 MeV. This is a *first ab initio* calculation to produce the bound state of $^5\Lambda\text{He}$ with explicit Σ degrees of freedom.

The order of the spin doublet structure of the $A = 4$ system is correctly reproduced for all YN interactions; the ground (excited) state has spin parity, $J^\pi = 0^+(1^+)$ for both isodoublet hypernuclei $^4\Lambda\text{H}$ and $^4\Lambda\text{He}$. Although the strengths of the 1S_0 and 3S_1 interactions of the SC97d(S) are almost the same as each other, the energy level of the 0^+ state is clearly lower than that of the 1^+ state. All of the $A = 3$ bound states given in Table I have $J^\pi = \frac{1}{2}^+$, in agreement with experiment. No other bound-state has been obtained for all of the YN interactions. For the SC97e(S), the differences between the calculated and experimental B_Λ values are the smallest among the YN interactions employed in the present study.

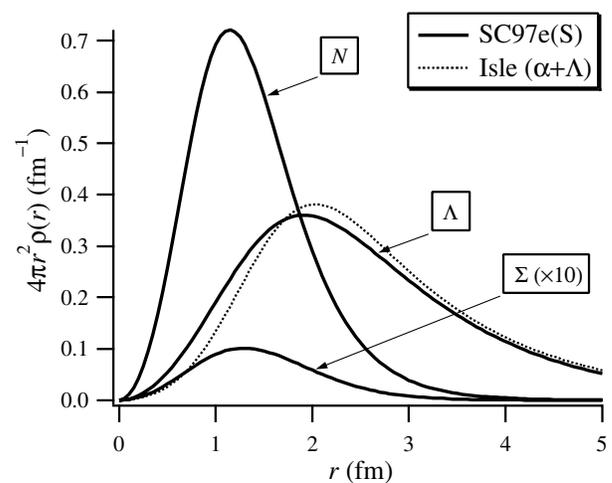


FIG. 1. Density distributions of N , Λ , and Σ for ^5He as a function of r , the distance from the center-of-mass of ^4He . The SC97e(S) YN interaction is used. Note that the Σ distribution has been multiplied by a factor of 10 to clarify the behavior. The dotted line is taken from Ref. [22].

TABLE II. Probabilities, given in percentage, of finding a Σ particle in $A = 3-5$ Λ hyper-nuclei for different YN interactions.

YN	$P_{\Sigma}({}^3_{\Lambda}\text{H})$	$P_{\Sigma}({}^4_{\Lambda}\text{H})$	$P_{\Sigma}({}^4_{\Lambda}\text{H}^*)$	$P_{\Sigma}({}^4_{\Lambda}\text{He})$	$P_{\Sigma}({}^4_{\Lambda}\text{He}^*)$	$P_{\Sigma}({}^5_{\Lambda}\text{He})$
SC97d(S)	0.06	1.27	1.37	1.24	1.35	2.04
SC97e(S)	0.15	1.49	0.98	1.45	0.96	1.55
SC97f(S)	0.23	1.88	1.09	1.83	1.08	1.87
SC89(S)	0.65	3.73	Unbound	3.59	Unbound	1.33

Table II lists the probability, P_{Σ} (in percentage), of finding a Σ particle in the system. The sizable amount of $P_{\Sigma}({}^5_{\Lambda}\text{He})$ is obtained. This implies that the $\Lambda - \Sigma$ coupling plays an important role, even for the ${}^5_{\Lambda}\text{He}$, despite a large excitation energy of the core nucleus, ${}^4\text{He}$ (with the isospin 1), in the Σ component. For the $A = 4$ system, the P_{Σ} of the 0^+ state are about 1–2%, except for the SC89(S), while the P_{Σ} of the 1^+ state are nearly equal to or smaller than that of the 0^+ state.

Figure 1 displays the density distributions for ${}^5_{\Lambda}\text{He}$ using SC97e(S), and of N , Λ , and Σ from the center-of-mass (c.m.) of ${}^4\text{He}$. Figure 1 also shows the Λ distribution obtained from the Isle $\Lambda - \alpha$ potential [22]. The experimental pionic decay width of ${}^5_{\Lambda}\text{He}$ suggests that the Λ distribution should spread over a rather outer region compared to the distribution of the α , as was discussed in Ref. [22]. The present curve of the Λ distribution is similar to that obtained by the Isle potential. The Σ distribution has a shape similar to the N distribution. The root-mean-square (rms) radii of N , Λ , and Σ from the c.m. of the ${}^4\text{He}$ are 1.5, 2.9, and 1.6 fm, respectively.

Table III lists the energy expectation values of the kinetic and potential energy terms for ${}^5_{\Lambda}\text{He}$. The contributions from the spin-orbit and the Coulomb potentials are not shown in the table, though the calculations include them. Here, T_c is the kinetic energy of the core nucleus (c) subtracted by the c.m. energy of c :

$$T_c = \sum_{i=1}^{A-1} \frac{\mathbf{p}_i^2}{2m_N} - \frac{(\sum_{i=1}^{A-1} \mathbf{p}_i)^2}{2(A-1)m_N}. \quad (6)$$

The kinetic energy of the relative motion between the Y and the c.m. of c is given by

$$T_{Y-c} = \frac{\boldsymbol{\pi}_{Y-c}^2}{2\mu_Y} + (m_Y - m_{\Lambda})c^2, \quad (7)$$

TABLE III. Energy expectation values of the kinetic and potential energy terms for ${}^5_{\Lambda}\text{He}$, given in units of MeV. The SC97e(S) YN interaction is used. For each potential part, a summation over appropriate particle pairs is taken into account [see Eq. (2), for example] and two central (1E and 3E) and a tensor (3E) components are listed separately. The first (second) term of each element $\langle T_c \rangle$, $\langle T_{Y-c} \rangle$, or $\langle V_{NN} \rangle$ represents the first (second) term of Eq. (8) ($\mathcal{O} = T_c, T_{Y-c}$, or V_{NN}). The energy expectation values of $\langle T_c \rangle$ and three $\langle V_{NN} \rangle$ for isolated ${}^4\text{He}$ are 84.86, -33.22 , -33.05 , and -43.93 MeV, respectively.

$\langle T_c \rangle$	$\langle T_{Y-c} \rangle$	$\langle V_{NN} \rangle$	$\langle V_{N\Lambda} \rangle$	$2\langle V_{\Lambda-\Sigma} \rangle$	$\langle V_{N\Sigma} \rangle$	
83.43 + 2.74	9.11 + 3.88	$-33.14 - 0.35$	-3.97	-0.02	0.07	(Central, 1E)
		$-32.03 - 0.27$	2.98	-1.02	1.56	(Central, 3E)
		$-40.91 - 0.12$	-2.24	-19.51	0.87	(Tensor, 3E)

where $\mu_Y = [(A-1)m_N m_Y]/[(A-1)m_N + m_Y]$ is the reduced mass for the $Y + c$ system, and $\boldsymbol{\pi}_{Y-c}$ is the canonical momentum of the relative coordinate between Y and c ($Y = \Lambda, \Sigma$). T_{Y-c} also counts the difference in the rest-mass energy between Λ and Σ . Each potential part $\langle V \rangle$ takes account of a summation over appropriate particle pairs [see Eq. (2), for example]. The energy expectation values of the first three columns in Table III are written as

$$\langle \mathcal{O} \rangle = \langle \Psi_{\Lambda} | \mathcal{O} | \Psi_{\Lambda} \rangle + \langle \Psi_{\Sigma} | \mathcal{O} | \Psi_{\Sigma} \rangle, \quad (8)$$

where the upper (lower) component of the Ψ_{JMTM_T} is denoted by Ψ_{Λ} (Ψ_{Σ}). The first (second) term of each element ($\langle T_c \rangle$, $\langle T_{Y-c} \rangle$, or $\langle V_{NN} \rangle$) in Table III represents the first (second) term of Eq. (8). The energy of the ${}^4\text{He}$ subsystem changes a lot from that of the isolated one,

$$\Delta E_c = (\langle T_c \rangle + \langle V_{NN} \rangle)_{{}^5_{\Lambda}\text{He}} - (\langle T_c \rangle + \langle V_{NN} \rangle)_{{}^4\text{He}} \approx 4.7 \text{ MeV}. \quad (9)$$

This difference is considerably large despite the fact that the rms radius of N from the c.m. of ${}^4\text{He}$ for the ${}^5_{\Lambda}\text{He}$ hardly changes from that for ${}^4\text{He}$. (Both radii are 1.5 fm.) Most of the change is due to a reduction of the energy expectation value of the tensor NN interaction,

$$[\langle V_{NN}(\text{tensor}) \rangle]_{{}^5_{\Lambda}\text{He}} - [\langle V_{NN}(\text{tensor}) \rangle]_{{}^4\text{He}} \approx 2.9 \text{ MeV}.$$

On the other hand, the tensor $\Lambda N - \Sigma N$ transition part has a surprisingly large energy expectation value (about -20 MeV). This large coupling energy makes ${}^5_{\Lambda}\text{He}$ bound in spite of both the energy loss of ΔE_c and the extremely high energy of the Σ component ($\frac{\langle H_{\Sigma} \rangle}{P_{\Sigma}} \sim 600$ MeV).

The calculated wave function is divided into orthogonal components according to the total orbital angular momentum (L), the total spin (S), the core nucleus spin (S_c),

TABLE IV. Probability, given in percentage, of each component with the total orbital angular momentum (L), total spin (S), core nucleus spin (S_c), and core nucleus isospin (T_c) in the Λ or in the Σ component for ${}^5_\Lambda\text{He}$. The SC97e(S) YN interaction is used. The probability in the S or in the D state for ${}^4\text{He}$ is also listed.

	$L = 0$		$L = 2$		
	$S = \frac{1}{2}$		$S = \frac{3}{2}$		$S = \frac{5}{2}$
	$S_c = 0$	$S_c = 1$	$S_c = 1$	$S_c = 2$	$S_c = 2$
${}^5_\Lambda\text{He}$					
$(T_c = 0) \otimes \Lambda$	89.14	0.03	0.19	3.74	5.36
$(T_c = 1) \otimes \Sigma$	0.10	0.09	1.34	~ 0	0.01
${}^4\text{He}$	89.56		10.44		

and the core nucleus isospin (T_c). Table IV displays the probability of each component for ${}^5_\Lambda\text{He}$. The table also lists the probability of the S state or of the D state for ${}^4\text{He}$. The sizable amount of probability of the Σ component is found in the D state while the sum of D -state probabilities in the Λ component is slightly smaller than that for ${}^4\text{He}$. Moreover, though the presence of a Λ in ${}^4\text{He}$ with the strong tensor ΛN - ΣN transition potential influences the structure of the D -state component and reduces the energy expectation value of the tensor NN interaction, the large coupling energy $\langle V_{\Lambda-\Sigma} \rangle$ of the tensor part bears the bound state of ${}^5_\Lambda\text{He}$ instead.

In summary, we have made a systematic study of all s -shell hypernuclei based on *ab initio* calculations using YN interactions with an explicit Σ admixture. The bound-state solution of ${}^5_\Lambda\text{He}$ was obtained. As Ref. [6] claimed, though there are none of the interaction models to describe very precisely the experimental B_Λ , the five-body calculation convinced us that the anomalous binding problem would be resolved by taking account of the explicit Σ admixture. The B_Λ values, obtained by using SC97e(S), are the closest to the experimental values, among the YN interactions employed in this study. A sizable amount of P_Σ was obtained, even for the ${}^5_\Lambda\text{He}$, in spite of the large excitation energy of ${}^4\text{He}$. The contribution of the energy from the tensor ΛN - ΣN coupling is quite large, and this coupling is considerably important to make ${}^5_\Lambda\text{He}$ bound. This is a novel finding in contrast with the Brueckner-Hartree-Fock calculation [19]. The present study for ${}^5_\Lambda\text{He}$ is a first step toward a detailed description of light strange nuclear systems. The core nucleus, ${}^4\text{He}$, is no longer rigid in interacting with a Λ particle. A similar situation can occur for strangeness $S = -2$ systems. Investigations into the strength of the $\Lambda\Lambda$ interaction based on the experimental data of the binding energy for double Λ hypernuclei (e.g., ${}^6_{\Lambda\Lambda}\text{He}$ [23]) should take account of the energy reduc-

tion of the core nucleus (ΔE_c for ${}^6_{\Lambda\Lambda}\text{He}$ is expected to be larger than the present ΔE_c for ${}^5_\Lambda\text{He}$).

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