Collisions between Optical Spatial Solitons Propagating in Opposite Directions

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(Received 12 April 2002; published 9 September 2002; corrected 7 November 2002)

We formulate the theory describing the evolution and interactions between optical spatial solitons that propagate in opposite directions. We show that coherent collisions between counterpropagating solitons give rise to a new focusing mechanism resulting from the interference between the beams, and that interactions between such solitons are insensitive to the relative phase between the beams.

DOI: 10.1103/PhysRevLett.89.133901 PACS numbers: 42.65.Tg, 05.45.Yv, 42.65.Jx, 42.40.Eq

Collisions between solitons are perhaps the most fascinating features of soliton phenomena because the interacting self-trapped wave packets exhibit many particle-like features [1]. Solitons collisions have been extensively studied theoretically, both for the integrable $(1+1)D$ Kerr case [2] and for the more general case in saturable nonlinearities (see [1] and references therein). Experimental studies include elastic collisions between Kerr solitons [3], almost-elastic collisions between solitons in saturable media at collision angles above the critical angle for total internal reflection [4], and inelastic collisions that yield fusion [4–6], fission [7], annihilation [7], and spiraling [8]. Soliton collisions can be classified into two categories: coherent and incoherent interactions. Coherent interactions occur when the nonlinear medium responds to interference effects taking place where the beams overlap. Such collisions occur for optical nonlinearities with an extremely fast response (Kerr [3] and quadratic nonlinearities). In materials with a long response time τ (e.g., photorefractive and thermal), coherent collisions occur only if the relative phase between the beams is stationary for a time longer than τ [6,7]. In such media, if the relative phase between the beams varies much faster than τ, then the contribution of the interference terms is averaged out and the surviving terms (in the nonlinear change of the refractive index Δn), depends only on the sum of the intensities of the beams [4]. This latter case is referred to as incoherent collisions [1,4]. The interaction between two solitons can be described through the "forces" they exert on one another. For coherent interactions, this force depends on the relative phase between the solitons. For example, two bright solitons launched in parallel attract (repel) each other if the relative phase between them is zero (π) [1,2,9]. For incoherent interactions, the interference terms do not contribute to Δn (as the relative phase between solitons varies much faster than τ). Thus, the incoherent force between bright solitons is always attractive, and is weaker than the force in a coherent interaction [4,10]. Thus far, all studies on optical soliton collisions have dealt with solitons propagating in the same general direction. That is, previous research on soliton interactions assumed that the collision angle in dimensional units is very small, so that the entire interaction falls into the framework of the paraxial wave equation.

Here, we study theoretically the interactions between solitons that propagate in opposite directions (Fig. 1). In this geometry, coherent interactions give rise to a new focusing mechanism, resulting from the interference between the beams. Such collisions display new features, among them (i) the interactions are insensitive to the relative phase between the solitons, and (ii) the collision involves radiation loss even in ideal Kerr media.

The basic difference between this new scheme and the traditional "co-propagation configuration" is the relative propagation directions of the carrier waves. Consider a coherent interaction between two solitons in both schemes. The solitons interfere and give rise to a grating in Δn. For copropagating solitons, the grating is periodic in the transverse direction (x) with a period much greater the optical wavelength λ, thus the interacting solitons go through very few (~ 3) grating periods. On the other hand, for counterpropagating collision the grating is in the propagation direction (z) and its period is $\sim\lambda/2$; hence the interacting solitons go through many ($\sim 10^6$) periods. Consequently, the interaction in the counterpropagation scheme is strongly affected by the grating: Bragg scattering plays a dominant role. Second, in the new scheme, the relative phase between the solitons, and hence the dominant term in the soliton-soliton force varies periodically along the propagation axis on a scale much shorter than the soliton propagation scale (the soliton period). Thus, all the effects depending on the

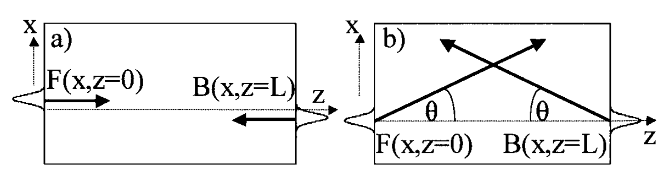

FIG. 1. Interactions between spatial solitons that propagate in opposite directions.

relative phase between solitons oscillate many ($\sim 10^3$) times over one soliton period. Hence, the effective force between the solitons is independent of their relative phase.

Another important difference between the two schemes has to do with the boundary conditions. In the copropagation case, the boundary conditions are at a single input face, whereas in the counterpropagation scheme, the boundary conditions are at two different (opposite) planes. To see the difficulty, consider the total field at the $z = 0$ plane, which consists of one input term (the forward beam) and one "output" term from the (backward) beam launched at the $z = L$ plane. However, the backward field is affected (via the interaction throughout propagation) by the forward field, whose only known quantity is its value at $z = 0$ (the input term). Because the problem is time harmonic, there are no causality difficulties associated with the fact that the field at each boundary includes both an input term and an output term. However, the collision process is not a Cauchy problem (as in the copropagating scheme) but a conceptually more subtle one. The problem is not even a standard boundary condition problem, because only partial information is known at each boundary (only the forward beam at $z = 0$ and the backward beam at $z = L$ are given, but the backward beam at $z = 0$ and the forward beam at $z = L$ are unknown).

We first derive the equations governing the evolution of counterpropagating mutually coherent beams in a medium whose refractive index is $n(x, z) = n_0 + \Delta n[I(x, z)]$ where n_0 is the linear index of refraction, and I is the intensity. Consider Fig. 1. Two $(1 + 1)D$ mutually coherent beams enter the medium from the opposite faces. The beams propagate at tiny angles, $0 \leq \theta \ll 1$, with respect to the $\pm z$ directions, so that the paraxial approximation can be used for each wave separately. The scalar optical field, E, is a sum of forward and backward waves: $E = F(x, z)e^{i(kz-\omega t)} + B(x, z)e^{-i(kz+\omega t)} + \text{c.c}$ where F and B are the forward and backward envelopes. The wave vector is $k = \omega n_0/c$, ω the temporal frequency, and c the vacuum speed of light. To within a proportionality factor, the time-averaged intensity is $I \propto |E|^2 = |F|^2 + |B|^2 + F^* B e^{-2ikz} + B^* F e^{2ikz}$. Within the slowly varying amplitude approximation, I is periodic in z with a period $\Lambda = \pi/k$. Since Δn depends only on I, it is also periodic in z and can be Fourier expanded:

$$\Delta n(x, z) = \Delta n_0 \sum_{m=-\infty}^{m=\infty} C_m(F, B)e^{2imkz}. \tag{1}$$

In a local self-focusing medium Δn is real, hence Δn_0 is a real constant, and $C_m = (C_{-m})^*$. We substitute E in the nonlinear wave equation [11], assume $|\Delta n| \ll n_0$, so that $n^2 = (n_0 + \Delta n)^2 \cong n_0^2 + 2n_0\Delta n$, apply the paraxial approximation, and find

$$e^{ikz}\left[\frac{\partial^2 F}{\partial x^2} + 2ik\frac{\partial F}{\partial z}\right] + e^{-ikz}\left[\frac{\partial^2 B}{\partial x^2} - 2ik\frac{\partial B}{\partial z}\right] = -\frac{2k^2\Delta n}{n_0}(Fe^{ikz} + Be^{-ikz}). \tag{2}$$

Substituting Eq. (1) into Eq. (2) and selecting synchronous terms yields

$$\begin{aligned}\frac{\partial^2 F}{\partial x^2} + 2ik\frac{\partial F}{\partial z} &= -\frac{2k^2\Delta n_0}{n_0}[C_0 \cdot F + C_1 \cdot B],\\ \frac{\partial^2 B}{\partial x^2} - 2ik\frac{\partial B}{\partial z} &= -\frac{2k^2\Delta n_0}{n_0}[C_0 \cdot B + C_{-1} \cdot F].\end{aligned} \tag{3}$$

The nonlinear terms can also be written as $[C_0 \cdot F + C_1 \cdot B] = [C_0 + |B|^2 g]F$ and $[C_0 \cdot B + C_{-1} \cdot F] = [C_0 + |F|^2 g]B$, where C_0 and g are both real functions of $|F|$ and $|B|$. The fields F and B do not exchange power, that is, $dP_F/dz = dP_B/dz = 0$, where $P_F \equiv \int_{-\infty}^{\infty} |F|^2 dx$, $P_B \equiv \int_{-\infty}^{\infty} |B|^2 dx$. Note that, if the interaction between the beams is incoherent, i.e., if the grating is washed out, then the above procedure leads to the same Eqs. (3) but with different C_0 and with $C_{\pm 1} = 0$. Equations (3) also describe vector solitons composed of two exactly counterpropagating beams ($\theta = 0$), which were studied theoretically in Kerr media in [12], and recently demonstrated in photorefractives [13].

In Eqs. (3), there are two nonlinear terms for each beam. To elucidate their physical origin, consider two bright beams propagating exactly counter to one another in a focusing medium, together forming a joint entity: a vector soliton. The first terms, C_0, reflect an average value of Δn (averaged along z). These C_0 terms represent the waveguide induced by both beams. This waveguide can guide other beams at a frequency that may be different than that of F and B, just like any soliton-induced waveguide [14]. The second terms in the right-hand-side (rhs) of Eqs. (3) result from the periodic modulation in Δn along z, which couples the two beams through Bragg reflections. A portion of the forward propagating F beam is Bragg-reflected backwards and is added coherently to the B beam propagating in the $-z$ direction. At the same time, a portion of B is Bragg-reflected to propagate in the $+z$ direction and is added coherently to F. These Bragg-reflected beams are $\pi/2$ phase retarded relative to the primary beams into which they are reflected [15]. As the Bragg-reflected beams are added to the primary beams, they effectively slow down the phase velocities of the "total" beams, which is equivalent to increasing the refractive index. Since the effect is more intense at the center of the beams than at the beams' margins, it reduces the natural beam divergence. We address this new focusing mechanism as holographic focusing [16]. In contrast to the "conventional" focusing represented by the first terms on the rhs of Eqs. (3) which are insensitive to ω, the holographic focusing occurs only for those beams that induced the hologram. A second difference between the two focusing mechanisms has to do with their response times. The nonlinear response time, τ, is the time it takes Δn to respond to intensity variations, and it can range from 100 fmsec in semiconductors (the dephasing time) to many seconds in photorefractive, thermal, and other nonlinearities associated with transport (of charge, temperature, etc.). The time τ is also the response time of the "conventional" focusing mechanism. Holographic focusing has two different characteristic response times. The first is the formation time of the grating, which is τ. The second is the response time for holographic focusing of one of the beams due to blocking of the second beam, once the Δn grating is set. This response is always (irrespective of τ) extremely fast ($\approx$ the dephasing time), because the holographic focusing on the first beam results from the phase-delayed Bragg-reflected portion of the second beam. Thus, once Δn is set, one can instantaneously turn off (or on) the holographic focusing effect on the forward beam by blocking (or unblocking) the backward beam, and vice versa.

Equations (3) can describe many schemes. Here, we consider interactions between scalar solitons in self-focusing Kerr media, that is, $\Delta n = n_2 I$ where n_2 is a positive constant. Substituting $\Delta n = n_2 I$, into Eq. (1) we identify $C_0 = |F|^2 + |B|^2$, $C_1 = B^* F$, and $C_{-1} = F^* B$. Substituting these into Eqs. (3) and transforming into dimensionless units ξ and ζ, yields

$$\begin{aligned} &\frac{\partial^2 f}{\partial \xi^2} + i\frac{\partial f}{\partial \zeta} + [|f|^2 + |b|^2(1+h)]f = 0, \\ &\frac{\partial^2 b}{\partial \xi^2} - i\frac{\partial b}{\partial \zeta} + [|b|^2 + |f|^2(1+h)]b = 0, \end{aligned} \tag{4}$$

where f and b are the normalized amplitudes, and $h = 1$ ($h = 0$) for coherent (incoherent) interactions. We solve Eq. (4) numerically. As input conditions, we use the single soliton hyperbolic-secant solution for $f(\xi, \zeta = 0)$ and $b(\xi, \zeta = L)$, with unity amplitudes. We note that for $h \neq 0$, system (4) is known to be nonintegrable [12,17–19].

Consider first the interactions in a configuration where two such beams are launched from two opposite planes $\zeta = 0$ and $\zeta = 18$ [Fig. 1(a)]. The beams are launched parallel to each other with a transverse spacing (between peaks) of $\xi = 7.5$. The coherent interaction between these parallel counterpropagating beams is shown in Figs. 2(a)–2(c). For clarity, we present the forward beam, $|f|$ [Figs. 2(a) and 2(c)], and the backward beam, $|b|$ [Fig. 2(b)] in separate plots. Figures 2(d)–2(f) show an incoherent interaction between the same beams. For comparison, we simulate the same beams in copropagating scheme [Figs. 2(g)–2(i)] [20]. Figure 2(g) [2(h)] shows a coherent interaction in which the relative phase between the launched beam is 0 [π]. Figure 2(i) shows an incoherent interaction. Clearly, the outcome of the interaction between the beams in the counterpropagating scheme is very different than that in the copropagating scheme, in both the coherent and incoherent cases. First, in the copropagating scheme, the mutual force between the solitons is in proportion to $-\cos(\Delta\Phi)$ [9], where $\Delta\Phi$ is the relative phase between the solitons, hence the interaction can be attractive [Fig. 2(g)] or repulsive [Fig. 2(h)]. In contrast, in the counterpropagating case the relative phase oscillates on a scale much shorter than the soliton period, thus the relative phase does not play any role. Specifically, varying the initial relative phase between the launched solitons does not affect the interaction (except for very tiny oscillations occurring within $z \approx \Lambda$, in which asynchronous terms contribute). Moreover, since the dominant coherent term in the mutual force, which is in proportion to $\cos(\Delta\Phi)$, is averaged out, the force between the counterpropagating beams is dominantly the incoherent term, which is always attractive and weaker than the coherent term. The second major difference between the counter- and copropagating cases has to do with radiation. The coherent interaction in the counterpropagating scheme radiates [Figs. 2(a) and 2(b)], which again proves that this system is nonintegrable. On the other hand, the incoherent interaction between the counterpropagating solitons does not radiate [Figs. 2(d)–2(f)]. Finally, we notice that a portion of the forward beam couples into the region where the backward beam is propagating. In the incoherent interaction, the forward beam gradually tunnels into the backward soliton region, hence the forward intensity at the backward soliton region increases monotonically [Fig. 2(f)]. This behavior represents directional coupling (resonant tunneling). For coherent interactions the dynamics are more complex, as

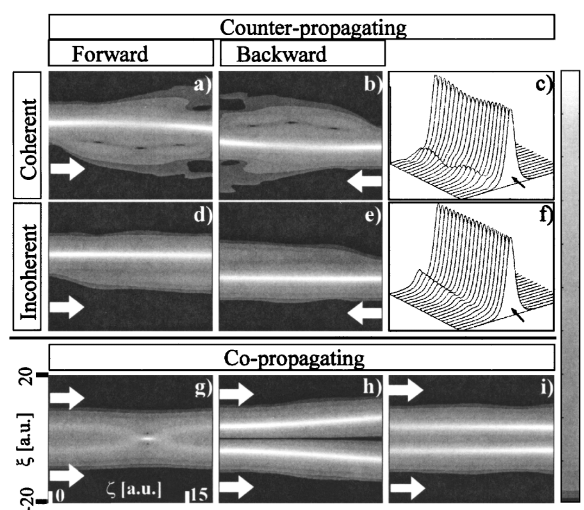

FIG. 2. (a)–(c) Coherent interactions between the counterpropagating (a),(c) forward and (b) backward solitons. (d)–(f) Incoherent interactions between the counterpropagating (d),(f) forward and (e) backward solitons. For comparison, interactions between coherent in-phase (g) and π out of phase (h), and incoherent (i) copropagating solitons. The plots show absolute values of the field amplitudes. The arrow indicates the propagation direction of each beam. The column to the right is a contrast bar indicating normalized intensities between 0 and 1.

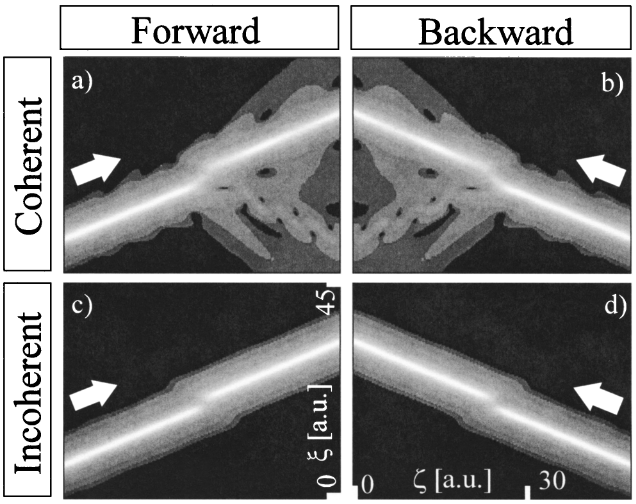

FIG. 3. Coherent (a),(b) and incoherent (c),(d) interactions between almost-counterpropagating solitons. The plots show absolute values of the field amplitudes, and the arrows indicate the propagation directions.

the intensity coupled from the forward beam to the region "under" the backward beam oscillates [see the sidebands in Fig. 2(c)], and, in contradistinction to the incoherent case, light does not accumulate in the “sidebands.” The explanation is as follows. The Bragg reflections of the backward beam serves as an extra source to the forward beam propagating at the backward soliton region. But, the forward beam under the backward soliton region is propagating slower than the original forward beam (due to the holographic effects). Hence, the relative phase between the original beam and its sideband is alternating, and subsequently, the energy transfer (through tunneling) between these beams is alternating as well.

Finally, we investigate collisions at angles close (but not equal) to 180° [Fig. 1(b)]. We launch two beams from planes $\zeta = 0$ and $\zeta = 48$ that initially propagate at a (dimensionless) angle of $\varphi = 26.5°$ [21]. The coherent and incoherent interactions are shown in Fig. 3. The incoherent collision is fully elastic, and merely leads to a lateral displacement, resembling collisions of copropagating solitons. Coherent collisions [Figs. 3(a) and 3(b)], on the other hand, lead to radiation. Moreover, because the forward radiation is partly localized under the waveguide induced by the backward propagating beam, the backward beam is not completely stationary: there are small oscillations in the backward beam just before the collision [Fig. 3(a)]. The same process applies to the backward radiation [Fig. 3(b)] [22].

In conclusion, we have formulated the theory of coherent and incoherent interactions between counterpropagating solitons. We have shown that coherent interactions in this scheme exhibit “holographic interaction” arising from interference between the counterpropagating beams. The collisions between such solitons display several key new features, among them the fact that the interactions are insensitive to the relative phase between the solitons, and that radiation exists even in ideal Kerr media.

This work was supported by the Israeli Science Foundation.

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