Capture and Release of a Conditional State of a Cavity QED System by Quantum Feedback

W. P. Smith,[1] J. E. Reiner,[1] L. A. Orozco,[1] S. Kuhr,[2] and H. M. Wiseman[3]

[1]*Department of Physics and Astronomy, SUNY Stony Brook, Stony Brook, New York 11794-3800*

[2]*Institut für Angewandte Physik, Universität Bonn, Wegelerstrasse 8, D-53115 Bonn, Germany*

[3]*Centre for Quantum Dynamics, School of Science, Griffith University, Brisbane, Queensland 4111, Australia*

(Received 8 February 2002; published 4 September 2002)

Detection of a single photon escaping an optical cavity QED system prepares a nonclassical state of the electromagnetic field. The evolution of the state can be modified by changing the drive of the cavity. For the appropriate feedback, the conditional state can be captured (stabilized) and then released. This is observed by a conditional intensity measurement that shows suppression of vacuum Rabi oscillations for the length of the feedback pulse and their subsequent return.

DOI: 10.1103/PhysRevLett.89.133601 PACS numbers: 42.50.Lc, 03.65.Ta, 03.67.–a, 42.50.Dv

Feedback control of quantum systems was first studied about 15 years ago [1–3] in quantum optics. In these approaches, the feedback could be understood in an essentially classical way, with quantum field theory entering only to dictate the magnitude of the fluctuations. This is possible if fluctuations are small compared to the mean fields being detected. More recently, a different approach to quantum optical feedback has been developed [4,5], based on quantum trajectories [6–8], which specify the stochastic evolution of a quantum state conditioned on continuous monitoring (such as by photodetection). This theory allows the treatment of feedback in the deep quantum regime, where quantum fluctuations are not small compared to the mean. It is also arguably the best way to approach feedback, as the conditioned state by definition comprises all of the knowledge of the experimenter on which feedback could be based [9,10].

Thus far, experiments in quantum feedback, such as Refs. [1,11–15], have all been in the regime of small fluctuations [16]. Cavity quantum electrodynamics (QED) is able to explore the opposite regime, where fluctuations in the conditional state are large. In this Letter, we present experimental results for the application of feedback in this regime. Following a photodetection, the conditioned quantum state of the system is $|\psi_c(\tau)\rangle$ [17,18]. Given our knowledge of this evolution, we can, for certain times τ, change the parameters of the system dynamics so as to capture the system in that conditioned state. When the parameters are later restored to their usual values, the released system state resumes its interrupted evolution. This directly demonstrates both the reality of the conditioned state and its usefulness for quantum feedback.

A cavity QED system consists of a single mode of the electromagnetic field of a cavity interacting with one or a collection of N two-level atoms [19]. Microwave cavity QED systems have been used recently to produce multiparticle entanglement [20], and to prepare photon number states of the electromagnetic field [21]. Optical cavity QED systems can now trap single atoms in the electric field of the cavity when its average occupation is about one photon [22,23]. They also show the conditional evolution of the electromagnetic field [24,25]. The system size and dynamics are characterized by two ratios: the saturation photon number n_0 and the single atom cooperativity C_1. They scale the influence of a photon and the influence of an atom in cavity QED. These two numbers relate the reversible dipole coupling of a single atom to the cavity mode (g) with the irreversible coupling to the reservoirs through cavity (κ) and atomic radiative (γ) decays: $C_1 = g^2/\kappa\gamma$ and $n_0 = \gamma^2/3g^2$. The strong coupling regime of cavity QED requires $n_0 \leq 1$, and $C_1 \geq 1$. Strictly speaking, the coupling constant g is spatially dependent and together with the N moving atoms may be described by effective constants g_{eff} and N_{eff}.

With weak driving, the system can be accurately modeled with a basis that includes up to two excitations of the coupled normal modes of the field and the atoms [17,18]. In this regime, photodetections are very infrequent, and the state before a detection can be taken to be the steady state, which is almost pure:

$$|\psi_{\mathrm{ss}}\rangle = |0, G\rangle + \lambda\left(|1, G\rangle - \frac{2g\sqrt{N}}{\gamma}|0, E\rangle\right) + \lambda^2\left(\zeta_0 \frac{1}{\sqrt{2}}|2, G\rangle - \theta_0 \frac{2g\sqrt{N}}{\gamma}|1, E\rangle\right) + \ldots \tag{1}$$

Here $|n, G\rangle$ represents n photons with all (N) atoms in their ground state; $|n, E\rangle$ represents n photons with one atom in the excited state with the rest ($N-1$) in their ground state, symmetrized over all atoms. The small parameter is $\lambda = \langle\hat{a}\rangle = \epsilon/[\kappa(1 + C_1 N)]$, which depends on the input driving field ϵ, while ζ_0 and θ_0 are coefficients of order unity for the two-excitation components that have nonzero photon number, and depend on g, κ, and γ [17,18]. After a photodetection occurs $|\psi_{\mathrm{ss}}\rangle$ collapses to $\hat{a}|\psi_{\mathrm{ss}}\rangle/\sqrt{\langle\hat{a}^\dagger\hat{a}\rangle_{\mathrm{ss}}}$, which evolves as the conditioned state:

$$|\psi_c(\tau)\rangle = |0, G\rangle + \lambda\left(\zeta(\tau)|1, G\rangle - \theta(\tau)\frac{2g\sqrt{N}}{\gamma}|0, E\rangle\right) + O(\lambda^2). \tag{2}$$

This is different from the initial state because ζ (the "field" evolution) and θ (the "atomic polarization" evolution) oscillate coherently at the vacuum Rabi frequency ($\approx g\sqrt{N}$) over time as the system reequilibrates exchanging energy between the atomic polarization and the cavity field [17,18].

If we choose a time $\tau = T$ for Eq. (2) such that $\zeta(T) = \theta(T)$ then, to order λ, we obtain

$$|\psi_c(T)\rangle \simeq |0, G\rangle + \lambda'\bigg(|1, G\rangle - \frac{2g\sqrt{N}}{\gamma}|0, E\rangle\bigg), \quad (3)$$

This is of the form of $|\psi_{ss}\rangle$ in Eq. (1) but with a different mean field $\lambda' = \zeta(T)\lambda$. This conditional state can be stabilized if, at time T, we change the driving amplitude by a factor $\zeta(T)$. Despite the almost 90° out of phase oscillations between the field (ζ) and the atomic polarization (θ) [18], the time T is close to the time when the field fluctuation crosses the mean. This way of stabilizing the conditional state is possible because it is a pure quantum state with two real parameters (ζ and θ) and two control parameters (the change in the drive $\lambda' - \lambda$ and the timing of the change T). A classical system of two coupled harmonic oscillators with added classical noise that has $g^{(2)}(0) \geq 1$ would require more than two parameters to describe it, and the above feedback method would almost certainly provide insufficient control to freeze the dynamics.

Conditional quantum states such as Eq. (2) can be measured using the intensity correlation function $g^{(2)}(\tau)$ [25]. The normalized correlation function of the intensity is the time-ordered and normally ordered average:

$$g^{(2)}(\tau) = \frac{\langle \hat{a}^\dagger(t)\hat{a}^\dagger(t+\tau)\hat{a}(t+\tau)\hat{a}(t)\rangle_{ss}}{\langle \hat{a}^\dagger(t)\hat{a}(t)\rangle_{ss}^2} = \frac{\langle \hat{n}(t+\tau)\rangle_c}{\langle \hat{n}(t)\rangle_{ss}}, \quad (4)$$

where $\hat{n} = \hat{a}^\dagger\hat{a}$, and c means "conditioned on a detection at time t in steady state."

Ordinarily, $g^{(2)}(\tau)$ would always be symmetrical in τ, as the $\tau = 0$ point is randomly determined by the detection of a photon at one detector [26]. However, if this detection is used to trigger a feedback pulse on the system, the correlation function will no longer be time symmetric. Nevertheless, for $\tau > 0$ the expression (4) still measures the conditional state in the presence of feedback (fb):

$$g^{(2)}(\tau) \simeq \frac{|\langle 1, G|\psi_{c+fb}(\tau)\rangle|^2}{|\langle 1, G|\psi_{ss}\rangle|^2} = [\zeta_{fb}(\tau)]^2. \quad (5)$$

Figure 1 shows the conditional evolution of the state of the cavity QED system, as given by Eq. (5). We start with the quantum theory valid for N two-level atoms identically coupled to the cavity in the weak field regime [17]. We find $g_{eff} < g$ and N_{eff} [27,28] using our experimentally determined values for $g^{(2)}(0)$ such that $g\sqrt{N} = g_{eff}\sqrt{N_{eff}}$. All broadening effects are incorporated by the modification of the atomic decay rate, $\gamma \rightarrow \gamma'$. We

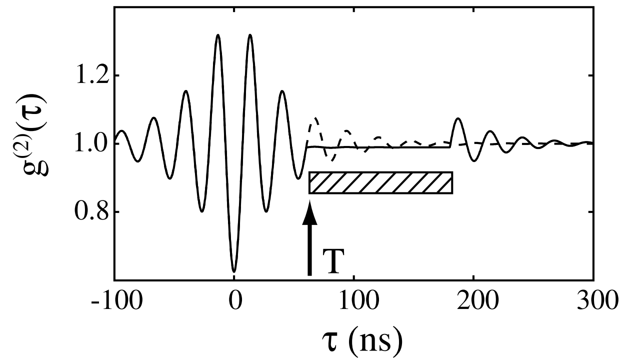

FIG. 1. Conditional evolution of cavity QED system, $(g\sqrt{N}, \gamma', \kappa)/2\pi = (37.3, 9.1, 3.7)$ MHz, with (continuous line) and without (dashed line) feedback. The shaded rectangle region corresponds to an instantaneous step down of $\Delta I = -0.2\%$ of the driving intensity applied at time T.

numerically solve the time evolution with the driving step. This simplified approach agrees with our previous work for $g^{(2)}(\tau)$ [29]. The dashed line is the free evolution of the system, and shows the time symmetry of the correlation function. The application of a feedback pulse at time T alters the evolution of the system. The continuous line shows the evolution when the step change in the driving intensity ϵ satisfies the conditions necessary to reach a new steady state described by Eq. (3). The parameters of the calculation are close to those of our experiment. We assume that the rise and fall of the change in the intensity (ΔI) are instantaneous. The new state reached by the system after the change of driving intensity no longer shows the vacuum Rabi oscillations and instead has a new value for the steady state slightly lower than the original one. The duration of the pulse that changes the steady state is finite and our model shows the reappearance of the oscillation delayed by the length of the pulse.

Our cavity QED apparatus and intensity correlator, described in detail in Ref. [29], consist of the cavity, the atomic beam, an excitation laser, and the detector system. Two high reflectivity curved mirrors (input transmission mirror 10 ppm, output transmission mirror 285 ppm, and separation $l = 880$ μm) form the optical cavity (waist of the TEM_{00} mode $w_0 = 34$ μm). A Pound-Drever-Hall stabilization technique keeps the cavity locked to the appropriate atomic resonance. An effusive oven produces a thermal (440 K) beam of Rb atoms with an angular spread of 2.8 mrad at the cavity mode. A laser beam intersects the atomic beam before the atoms enter the cavity in a region with 2.5 G uniform magnetic field. It optically pumps all the ^{85}Rb atoms of the $F = 3$ ground state into the magnetic sublevel $F = 3$, $m_F = 3$. The three rates that characterize our cavity QED system are $(g, \kappa, \gamma/2)/2\pi = (5.1, 3.7, 3.0)$ MHz.

Figure 2 shows a schematic of our apparatus. Light from a Ti:sapphire, locked to the $5S_{1/2}$, $F = 3 \rightarrow 5P_{3/2}$, $F = 4$ transition of ^{85}Rb at 780 nm, drives the cavity

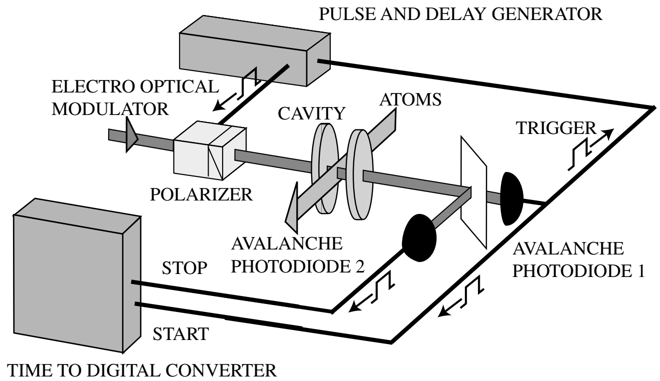

FIG. 2. Simplified diagram of the experimental setup.

QED system. The signal escaping the cavity creates photodetections at the "start"and "stop" avalanche photodiodes (APD). The output pulse of the start detector is split and one part sent to the start channel of the correlator, which measures $g^{(2)}(\tau)$ and consists of a time to digital converter (TDC) with 0.5 ns per bin, histogramming memory, and a computer. The other goes to a variable time delay, and after pulse shaping and lengthening, drives an electro-optical modulator in front of a polarizer to produce a change, ΔI, in the driving intensity of the cavity. This pulse has an 8 ns rise time and is 120 ns long. The delay between the detection of a photon at APD1 and the arrival of the pulse at the cavity can be as short as 45 ns. The other APD sends its pulses to the correlator to stop the TDC that measures the time interval between the two events. A histogram of the delays between the start and the stop gives the conditional evolution of the intensity.

We operate the cavity QED system in a nonclassical regime where the size of the vacuum Rabi oscillations is large enough to permit their rapid identification during data taking. We begin by measuring the antibunched second order correlation function of the intensity escaping our cavity QED system. We then apply the step change in the driving intensity at time T to fulfill the conditions of Eq. (3) and obtain a new steady state.

Figure 3 shows measurements of the correlation function in the absence 3(i) and presence [3(ii), 3(iii), and 3(iv)] of feedback. Traces 3(i) and 3(ii) have the same oscillating frequency while for traces 3(iii) and 3(iv) we have smaller numbers of atoms. τ^* marks the position where the oscillation we want to suppress reaches a maximum. The steady state corresponds to an intracavity intensity of $n/n_0 \leq 0.2$. Figure 3(ii) shows the correlation function with step-down feedback ($\Delta I = -2.6\%$) for 120 ns, beginning at $\tau = T = 57$ ns, when the oscillation crosses unity and is growing. The oscillation that has a maximum in trace 3(i) at the point marked by τ^* has disappeared, the steady state is lower than that marked by the dashed line, and the oscillation reappears after the pulse is turned off with approximately the same amplitude and phase as the suppressed one. Trace 3(iii) shows feedback applied at $T = 56$ ns with a step ($\Delta I = 3.0\%$) to enhance the oscillations. Trace 3(iv) shows step-up feedback ($\Delta I = 3.9\%$) to suppress oscillations at $T =$ 46 ns when the phase is opposite from that in trace 3(ii).

The time T for the application of the pulse is critical to achieve good suppression [Fig. 3(ii)] or enhancement [Fig. 3(iii)] of the oscillation. The slow decay seen in

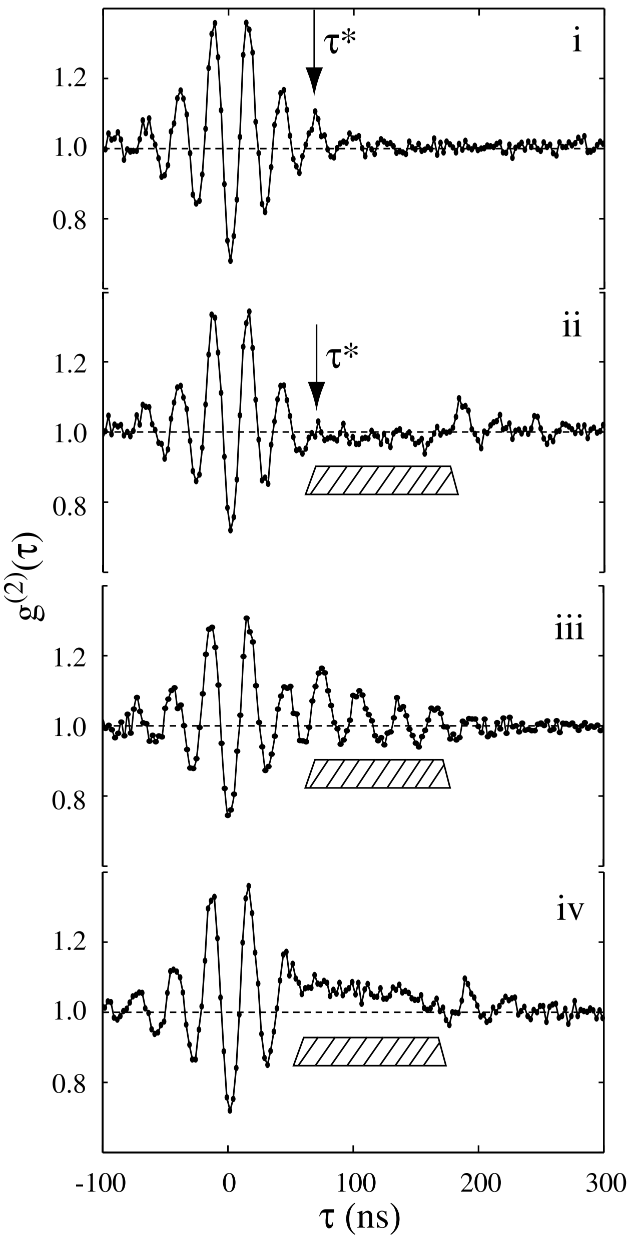

FIG. 3. Measured intensity correlation function with the feedback step applied during the shaded region: (i) no feedback ($g\sqrt{N}/2\pi = 37$ MHz), (ii) suppression ($g\sqrt{N}/2\pi = 37$ MHz, $\Delta I = -2.6\%$), (iii) enhancement ($g\sqrt{N}/2\pi = 34$ MHz, $\Delta I =$ 3%), (iv) suppression ($g\sqrt{N}/2\pi = 31$ MHz, $\Delta I = 3.9\%$). The time T for the beginning of the feedback and its duration are indicated by the position and size of the shaded region. The data has been binned in 2.5 ns intervals joined with a line.

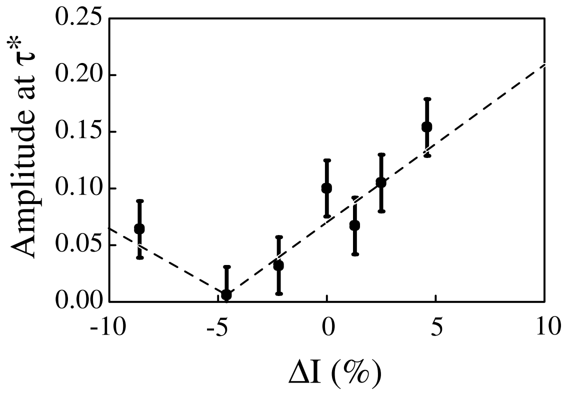

FIG. 4. Amplitude of the normalized intensity response at time $\tau*$ [first oscillation extreme of $g^{(2)}(\tau)$ after the application of the feedback pulse] as a function of the size of the feedback step ΔI. The dashed line is a theoretical prediction.

Fig. 3(iv) is related to distortion of the electrical pulse for higher steps, which also prevents the clear restoration of the oscillation at the end of the pulse. Our electrical pulse generator also limits the available length of the pulse. There is qualitative agreement between the traces i and ii with those of the theory in Fig. 1. They show the suppression and the delayed return of the oscillation by the application of a feedback pulse to the driving intensity. Although the theoretical model does not include all the experimental details that give rise to broadening the main features of the response are clearly explained.

We have followed the size of the amplitude of the oscillation immediately after we apply the feedback pulse, at the time τ^* defined in Fig. 3, as a function of ΔI, to make a quantitative comparison with theory. Figure 4 shows the results for a series of measurements under similar conditions as those for traces 3(i) and 3(ii) for ΔI positive and negative. The theory (dashed line) incorporates the shape of the experimental pulse, and uses $(g\sqrt{N}, \gamma', \kappa)/2\pi = (37, 9.1, 3.7)$ MHz. The plot shows both enhancement and suppression with quantitative agreement.

The quantum feedback in this system is triggered by a fluctuation (detection of a photon) that is large enough, because of the strong coupling, to significantly modify the system. This detection prepares the system in an evolving conditional state. We then change the drive of the system and are able to freeze its evolution to a new time-independent steady state. The suppressed oscillations return once the pulse turns off, with the same phase and amplitude information. This sort of quantum feedback is a novel way to manipulate the fragile conditional states that come from strongly coupled systems.

We thank J. Gripp and J. Wang for their interest and help with this project. This work has been supported by the NSF, NIST, DAAD, and the Australian Research Council.

[1] Y. Yamamoto, N. Imoto, and S. Machida, Phys. Rev. A **33**, 3243 (1986).
[2] H. A. Haus and Y. Yamamoto, Phys. Rev. A **34**, 270 (1986).
[3] J. H. Shapiro, G. Saplakoglu, S.-T. Ho, P. Kumar, B. E. A. Saleh, and M. C. Teich, J. Opt. Soc. Am. B **4**, 1604 (1987).
[4] H. M. Wiseman and G. J. Milburn, Phys. Rev. Lett. **70**, 548 (1993).
[5] H. M. Wiseman, Phys. Rev. A **49**, 2133 (1994).
[6] H. J. Carmichael, *An Open Systems Approach to Quantum Optics* (Springer-Verlag, Berlin, 1993).
[7] R. Dum, P. Zoller, and H. Ritsch, Phys. Rev. A **45**, 4879 (1992).
[8] K. Mølmer, Y. Castin, and J. Dalibard, J. Opt. Soc. Am. B **10**, 524 (1993).
[9] A. C. Doherty and K. Jacobs, Phys. Rev. A **60**, 2700 (1999).
[10] A. C. Doherty, S. Habib, K. Jacobs, H. Mabuchi, and S. M. Tan, Phys. Rev. A **62**, 012105 (2000).
[11] J. G. Walker and E. Jakeman, Opt. Acta **32**, 1303 (1985).
[12] P. R. Tapster, J. G. Rarity, and J. S. Satchell, Phys. Rev. A **37**, 2963 (1988).
[13] J. Mertz, A. Heidmann, and C. Fabre, Phys. Rev. A **44**, 3229 (1991).
[14] M. S. Taubman, H. Wiseman, D. E. McClelland, and H.-A. Bachor, J. Opt. Soc. Am. B **12**, 1792 (1995).
[15] B. C. Buchler, M. B. Gray, D. A. Shaddock, T. C. Ralph, and D. E. McClelland, Opt. Lett. **24**, 259 (1999).
[16] An exception is the following, which is an experiment of a quite different nature: R. J. Nelson, Y. Weinstein, D. Cory, and S. Lloyd, Phys. Rev. Lett. **85**, 3045 (2000).
[17] H. J. Carmichael, R. J. Brecha, and P. R. Rice, Opt. Commun. **82**, 73 (1991).
[18] J. E. Reiner, W. P. Smith, L. A. Orozco, H. J. Carmichael, and P. R. Rice, J. Opt. Soc. Am. B **18**, 1911 (2001).
[19] *Cavity Quantum Electrodynamics*, edited by P. Berman, Advances in Atomic, Molecular and Optical Physics Series (Academic Press, Boston, 1994), Suppl. 2.
[20] A. Rauschenbeutel, G. Nogues, S. Osnaghi, P. Bertet, M. Brune, J. M. Raimond, and S. Haroche, Science **288**, 2024 (2000).
[21] S. Brattke, B. T. H. Varcoe, and H. Walther, Phys. Rev. Lett. **86**, 3534 (2001).
[22] C. J. Hood, R. W. Lynn, A. C. Doherty, A. S. Parkins, and H. J. Kimble, Science **287**, 1447 (2000); A. C. Doherty, T. W. Lynn, C. J. Hood, and H. J. Kimble, Phys. Rev. A **63**, 013401 (2001).
[23] P. W. H. Pinkse, T. Fischer, P. Maunz, and G. Rempe, Nature (London) **404**, 365 (2000).
[24] H. J. Carmichael, H. Castro-Beltran, G. T. Foster, and L. A. Orozco, Phys. Rev. Lett. **85**, 1855 (2000).
[25] G. T. Foster, L. A. Orozco, H. M. Castro-Beltran, and H. J. Carmichael, Phys. Rev. Lett. **85**, 3149 (2000).
[26] A. Denisov, H. M. Castro-Beltran, and H. J. Carmichael, Phys. Rev. Lett. **88**, 243601 (2002).
[27] G. Rempe, R. J. Thompson, R. J. Brecha, W. D. Lee, and H. J. Kimble, Phys. Rev. Lett. **67**, 1727 (1991).
[28] H. J. Carmichael and B. C. Sanders, Phys. Rev. A **60**, 2497 (1999).
[29] G. T. Foster, S. L. Mielke, and L. A. Orozco, Phys. Rev. A **61**, 53821 (2000).