High Energy Field Theory in Truncated Anti–de Sitter Space

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In this Letter we show that in five-dimensional anti–de Sitter (AdS) space truncated by boundary branes, effective field theory techniques are reliable at high energy (much higher than the scale suggested by the Kaluza-Klein mass gap), provided one computes suitable observables. We argue that in the model of Randall and Sundrum for generating the weak scale from the AdS warp factor, the high energy behavior of gauge fields can be calculated in a *cutoff independent manner,* provided one restricts Green's functions to external points on the Planck brane. Using the AdS/CFT (conformal field theory) correspondence, we calculate the one-loop correction to the Planck brane gauge propagator due to charged bulk fields. These effects give rise to nonuniversal logarithmic energy dependence for a range of scales above the Kaluza-Klein gap.

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It is widely believed that there exists a large energy desert separating the weak scale from the scale of grand unification, or perhaps the Planck scale. A primary reason for this belief is the observation that high-scale perturbative unification seems plausible in supersymmetric extensions of the standard model. On the other hand, it appears that in models in which the weak scale is fundamental, such as scenarios based on extra dimensions, one has to abandon the idea of gauge unification at high energies.

Several authors have argued that in scenarios where the weak/Planck hierarchy is generated by a single warped extra dimension [1] [the Randall-Sundrum (RS) model], it is possible for bulk gauge couplings to evolve logarithmically over a large range of scales [2,3]. This is in sharp contrast to the behavior of a gauge theory propagating on compactified 5D flat space (for instance, flat $R^4 \times S^1$), where vacuum polarization effects give rise to power-law corrections that become dominant at mass scales comparable to the compactification scale. At the scales where these power corrections dominate, effective field theory techniques break down. In order to make predictions beyond such energies, local field theory must be embedded into a suitable UV completion that resolves its short distance singularities.

While we agree that high energy logarithmic behavior is possible in gauge theories propagating in compactified anti–de Sitter (AdS), we find that this is physically realized in a manner that is somewhat different from the proposal of [2]. We will show that as in flat space, the one-loop correction to the gauge field zero mode propagator in compactified AdS contains a power-law term that is saturated at the Kaluza-Klein (KK) mass scale (of order TeV). This power correction is a finite, nonanalytic function of the 4D momentum. Because of this, it cannot be removed by any procedure for regulating the ultraviolet divergences that arise in loop calculations. However, while in a flat space theory the emergence of power-law behavior at the KK mass scale signifies a breakdown of the higher dimensional field theory description, the same is not true for theories propagating in background AdS spaces. Rather, in AdS, large loop corrections at the TeV scale imply a breakdown of the field theoretic description of the zero mode observable, but not necessarily of other correlators. Thus, high energy gauge theory effects may be accessible via local field theory, provided one calculates the appropriate observable.

In the RS proposal for obtaining the weak scale from the AdS warp factor [1], a set of such calculable quantities is the 5D gauge theory correlators with external points restricted to the Planck brane. On the Planck brane the local cutoff for correlators is of order the AdS curvature scale (which is taken to be of order the Planck scale and is much larger than the KK scale). This implies that it is this scale, and not the TeV scale, which suppresses power corrections to these observables. Thus the logarithms due to KK zero modes in loops give the dominant nonanalytic corrections for a large range of scales.

To illustrate these claims, in this Letter we will work in the context of 5D massless scalar electrodynamics as a model of gauge dynamics. In principle, calculations in the model can be done in 5D, in a manner that is independent of any specific regulator. However, in order to compute the correlators on the Planck brane, it is easier to employ the AdS/CFT (conformal field theory) correspondence [4,5] as it applies to models with boundary branes [6–8]. The Planck brane vacuum polarization of our toy model can be computed in a dual 4D field theory that consists of a massless scalar and a gauge field which are both weakly coupled to a (broken) CFT. The one-loop logarithm of the 5D theory can be calculated in this framework without any detailed knowledge of the CFT.

At energies of order the KK mass gap, the Planck brane correlators match on to the zero mode Green's functions. Therefore, there is a calculable relation between high

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energy couplings and the parameters measured in low-energy experiments. In particular, symmetry constraints on the dynamics near the curvature scale (for instance, high energy unification of gauge forces) could have meaningful implications for low-energy physics.

Our setup is as follows. We will work in the context of field theory propagating in a background five-dimensional (Euclidean) AdS spacetime with the coordination

$$ds^2 = G_{MN}dX^M dX^N = \frac{1}{(kz)^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2), \quad (1)$$

where z parametrizes the location in the AdS bulk. The AdS boundary is cut off by a Planck brane at $z = 1/k$, and the AdS horizon is removed by the presence of a TeV brane at $z = 1/T$. Here k is the AdS curvature parameter, and T is an energy scale that sets the masses of KK excitations of bulk fields. For applications to the hierarchy problem all parameters are taken to scale as appropriate powers of the Planck scale, except $T \sim$ TeV.

We begin by addressing which AdS observables can be studied in the framework of effective field theory (EFT) at a given energy scale. Consider a simple model consisting of a bulk scalar with higher derivative "interaction" terms

$$S = \frac{1}{2}\int d^5X\sqrt{G}(\partial\Phi)^2 + \frac{\lambda_2}{2}\int d^5X\sqrt{G}(\Box\Phi)^2 + \sum_{n=3}^{\infty}\frac{\lambda_n}{2}\int d^5X\sqrt{G}\Phi\Box^n\Phi, \quad (2)$$

where $\Box$ is the scalar Laplacian on AdS. First we examine an $\mathcal{O}(\lambda_2)$ correction to the two-point function of Φ:

$$\langle\Phi(X)\Phi(X')\rangle \sim D(X, X') + \lambda_2\frac{\delta^5(X - X')}{\sqrt{G}}.$$

In this equation, $D(X, X')$ is the scalar propagator derived from the free scalar action. Performing a Fourier transform along the 4D coordinates x^μ, this becomes (with p^μ the coordinate momentum, $p \cdot x = \eta_{\mu\nu}p^\mu x^\nu$, and $p = \sqrt{\eta_{\mu\nu}p^\mu p^\nu}$)

$$G_p(z, z') \equiv \int d^4x e^{ip\cdot x}\langle\Phi(x, z)\Phi(0, z')\rangle \sim D_p(z, z') + \lambda_2(kz)^5\delta(z - z').$$

Projecting onto the mth KK wave function, $\psi_m(z)$, we find

$$\int_{1/k}^{1/T}\frac{dz'}{(kz')^3}\psi_m(z')G_p(z, z') \sim \psi_m(z)\left[\frac{1}{p^2 + m_n^2} + \lambda_2(kz)^2\right],$$

It is natural to take λ_2 to scale as $\lambda_2 \sim M_5^{-2}$, where M_5 is the fundamental scale in 5D. For fixed n and $p \gg m_n$, the λ_2 correction to the propagator becomes leading when $p \sim M_5/kz$. This is simply the well known result [1] that the local cutoff scales with z. Here we have made clear the relevance of the sliding cutoff for particular modes, and the importance of the localization of the mode to the breakdown of the EFT becomes apparent. From this estimate, it is clear that the loss of predictability has nothing to do with loop effects, or with the nature of a specific choice of regulator.

From this analysis, we conclude that we should not expect to be able to calculate zero mode observables [and, in particular, renormalization group (RG) flows] for momenta much larger than a TeV, contrary to what was proposed in [2]. Instead, if we are interested in calculating at such large momenta we must restrict ourselves to observables localized to the Planck brane.

Let us apply this intuition to gauge theory in AdS backgrounds. We will study scalar QED propagating in the curved background of Eq. (1). First, consider the one-loop gauge field zero mode vacuum polarization. Performing the computation explicitly in 5D, we find using dimensional regularization and working in $A_5 = 0$ gauge (irrelevant constants have been dropped)

$$\Pi_{\mathrm{AdS}}(q^2) = \frac{1}{g_4^2} + \frac{1}{48\pi^2}\left[\frac{1}{\epsilon} - \ln\left(\frac{q^2}{kT}\right) + \frac{1}{2}\ln\left(\frac{\mu^2}{kT}\right)\right] - \frac{1}{16\pi^2}\int_0^1 dxx\sqrt{1 - x^2}\ln N\left(\frac{\sqrt{q^2}x}{2}\right), \quad (3)$$

where $N(p) = I_1(p/T)K_1(p/k) - I_1(p/k)K_1(p/T)$, and $1/g_4^2 = R/g_5^2$, with R the proper distance between the boundaries. As in the case of gauge fields in a flat orbifold [9], it can be shown that the $1/\epsilon$ pole represents logarithmic divergences that renormalize gauge kinetic terms localized on the boundary branes at $z = 1/k$ and $z = 1/T$ [10]. The coefficient of the pole even matches that found in flat space, leading to the same RG equations for the boundary terms. Physically, this is to be expected, since the UV divergences of field theory in curved space arise from distances shorter than the curvature scale and are therefore identical to those in flat space. In particular, the RG flows for couplings to operators that are present in both flat and curved space should be the same. (However, it is possible for new operators composed from powers of the background curvature to play a role in the RG flows.) It follows that as in flat space, the 5D bulk gauge coupling does not run.

For $\sqrt{q^2} \ll T$, the zero mode of the scalar dominates Eq. (3), giving rise to low-energy logarithmic energy dependence. For $\sqrt{q^2} \gg T$ the behavior is power-law, $\Pi_{\mathrm{AdS}}(q^2) \sim \sqrt{q^2}/T$, and the zero mode observable becomes strongly coupled, in accord with our previous discussion. In fact, one does not have to consider loop effects to see power-law behavior. The nonrenormalizable operator

$$S = \cdots + \frac{\lambda}{2}\int d^5X\sqrt{G}F_{MN}\Box F^{MN} \tag{4}$$

will give rise to an analytic power-law contribution to Eq. (3) of the form $\Pi_{\mathrm{AdS}}(q^2) \sim \cdots + k\lambda q^2/T^2$. In [2], it was argued that the zero mode gauge propagator can be studied at energies much higher than the KK scale, provided a suitable momentum cutoff is employed. Because the contribution from Eq. (4) is a tree-level effect, it is clear the breakdown of the zero mode observable cannot be avoided by a mere choice of regulator.

Although we cannot use the zero mode observables to study high energy gauge theory, the Planck brane correlator is still perturbative at energies much larger than the KK scale. To see that loop corrections to this quantity are logarithmic for a large range of scales, consider the gauge propagator (in $A_5 = 0$ gauge) with one point on the Planck brane for external momenta larger than T:

$$D_p(z, 1/k) \simeq \frac{kz}{p}\frac{K_1(pz)}{K_0(p/k)}\eta_{\mu\nu} + \text{pure gauge}. \tag{5}$$

For z near $1/T$, $pz \gg 1$, and $K_1(pz) \sim \sqrt{\pi/(2pz)}\exp(-pz)$. We then see that this quantity has almost no overlap with the excited KK states of the bulk scalar, which are localized towards the TeV brane. It follows that the contribution from these states to the one-loop Planck correlator, which could potentially give rise to a power-law of the form $\sqrt{q^2}/T$ is practically zero (it is suppressed by powers of $T/k \ll 1$). However, because the scalar zero mode profile is flat, it does not suffer from the exponential suppression, and gives the dominant contribution to the correlator. We thus expect the zero mode logarithm to be the leading energy dependence of the one-loop Green's function. As the external momentum reaches the curvature scale k, the heavy KK modes which have support near the Planck brane start to contribute. These modes give rise to power-law behavior that becomes dominant near the scale k.

Instead of performing the 5D calculation just described, we will show how to use AdS/CFT duality as it applies to the RS model to obtain quantitative information about the one-loop Planck correlator. Our toy gauge theory in AdS corresponds to the 4D theory of a U(1) gauge field, a charged scalar, and a CFT explicitly broken by a UV cutoff (the dual of the Planck brane) and spontaneously broken in the IR (the TeV brane in the 4D context) [6,7]. The gauge field couples minimally to the scalar, and couples weakly to an anomaly free U(1) subgroup of the global symmetries of the CFT. By similar arguments as those of [11] for the graviton, the presence of the Planck brane in the 5D theory renders the KK zero mode of the bulk scalar normalizable, and therefore implies the existence of a 4D massless scalar which couples to the CFT through a dimension four operator. (This can be verified by checking that the KK corrections to scalar exchange on the Planck brane in the 5D theory match the contributions to the force law from the CFT in the 4D dual). Thus our 5D theory has a 4D dual description (ignoring couplings to 4D gravity) given by

$$\mathcal{L}_{4D} = \mathcal{L}_{\mathrm{CFT}} + \frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu} + A_\mu J^\mu_{\mathrm{CFT}} + |D_\mu\phi|^2 + c(\phi\mathcal{O}_4 + \text{H.c.}),$$

where c is a coupling of order $1/k$. While we do not know precisely what CFT is dual to the RS model, it is still possible to use this Lagrangian to compute Planck brane correlators of bulk fields. For instance, the Planck brane vacuum polarization is given by

$$\Pi^{\mu\nu}(q^2) = (q^\mu q^\nu - q^2\eta^{\mu\nu})\left[\frac{1}{g^2} - \frac{1}{48\pi^2}\ln\left(\frac{q^2}{\mu^2}\right)\right] + \int d^4x e^{iq\cdot x}\langle J^\mu_{\mathrm{CFT}}(x)J^\nu_{\mathrm{CFT}}(0)\rangle_{\mathrm{CFT}} + \mathcal{O}(|c|^2). \tag{6}$$

The first line is simply the contribution of a 4D scalar to the vacuum polarization. The second line is the correction to the correlator due to pure CFT effects. Writing

$$\int d^4x e^{iq\cdot x}\langle J^\mu_{\mathrm{CFT}}(x)J^\nu_{\mathrm{CFT}}(0)\rangle_{\mathrm{CFT}} = (q^\mu q^\nu - q^2\eta^{\mu\nu})\Pi(q^2), \tag{7}$$

it can be shown [6] that for $\sqrt{q^2} \gg T$ (note that the effects of being in a vacuum state with broken conformal symmetry are suppressed exponentially at energies much larger than T)

$$\Pi(q^2) \simeq \frac{1}{2g_5^2 k}\ln\left(\frac{q^2}{k^2}\right).$$

In the AdS description, this term corresponds to the contribution of the KK modes of the gauge field to the tree-level Planck correlator. Because in the bulk theory Eq. (7) represents a tree-level effect, it gives the leading behavior of the correlator. In settings where the AdS/CFT correspondence has been well tested, the CFT is a large N gauge theory, so Eq. (7) is expected to be larger than the scalar correction by powers of N. However, the coefficient of this classical logarithm is universal, so pure CFT effects will cancel in predictions for the difference of low-energy couplings in terms of high energy parameters.

Besides the terms explicitly shown in Eq. (6), there are also corrections which involve insertions of the operator $\mathcal{O}_4$. These terms are suppressed by powers of k, and correspond to the exponentially damped scalar KK corrections to the 5D correlator. They can be calculated in terms of CFT correlators (in the vacuum with broken conformal symmetry) of $\mathcal{O}_4$ and $J^{\mu}_{\rm CFT}$. Such correlators can be obtained [10] using the rules of [5] for relating CFT correlators to solutions of classical AdS field equations.

So far we have considered only the evolution of the Planck brane vacuum polarization due to one-loop effects of bulk matter. It is easy to see that brane localized fields will also contribute logarithms with the usual 4D beta function coefficients. The effects of Planck localized fields persist for all energies up to the curvature scale. However, for $\sqrt{q^2} \gg T$, the contribution due to TeV brane fields on the Planck brane correlator is suppressed by an exponential of $\sqrt{q^2}/T$, since for high energies this is a highly nonlocal effect (explicitly, the suppression is by two powers of Eq. (5) evaluated at $z = 1/T$). This can be understood also in the 4D dual description [6,7]. There, Planck brane fields are spectators to the CFT dynamics, so it is clear that they give rise to the usual 4D vacuum polarization effects. On the other hand, TeV brane localized fields arise in the 4D dual as condensates of CFT states in the IR. These bound states do not contribute to the running of the couplings above TeV scale energies.

Although we explicitly considered only a toy scalar model, we expect that loops of bulk non-Abelian gauge and fermion fields will also generate logarithmic corrections to the Planck correlator with the usual 4D coefficient. Because at energies below the KK scale the Planck correlators match the Green's functions of zero mode fields, gauge coupling evolution in the RS model with standard model gauge fields in the bulk is generically similar to the situation in the minimal (4D) standard model with an energy desert. It is therefore possible to make perturbative predictions for the low-energy couplings of the model in terms of the parameters in the underlying 5D Lagrangian.

This leads to the intriguing possibility, first noted by [2], that a version of the RS proposal for addressing the hierarchy problem with bulk standard model gauge fields could also predict coupling constant unification within a grand unified context. In order to maintain the hierarchy in the presence of bulk gauge fields, the Higgs scalar responsible for electroweak symmetry breaking must be confined to the TeV brane. It contributes to gauge coupling evolution only up to the TeV scale. Standard model fermions may propagate either in the bulk or on the TeV brane. However, since the fermions come in complete SU(5) multiplets, the RS prediction for low-energy coupling constant relations is model independent, and qualitatively similar to that of the standard model. Achieving unification, though, could necessitate additional, perhaps *ad hoc,* physics since the running above the TeV scale excludes the Higgs. As in the standard model, there are potential threshold corrections to logarithmic evolution near the GUT scale. There are also TeV threshold corrections that arise from the matching of the Planck to the zero mode Green's functions. Although these corrections are not accessible to our 4D CFT calculation, they can be unambiguously calculated in 5D. The details for plausible models have yet to be worked out.

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