

Tuning the Dipolar Interaction in Quantum Gases

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We have studied the tunability of the interaction between permanent dipoles in Bose-Einstein condensates. Based on time-dependent control of the anisotropy of the dipolar interaction, we show that even the very weak magnetic dipole coupling in alkali gases can be used to excite collective modes. Furthermore, we discuss how the effective dipolar coupling in a Bose-Einstein condensate can be tuned from positive to negative values and even switched off completely by fast rotation of the orientation of the dipoles.

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In the Bose-Einstein condensates (BECs) created thus far [1], atoms interact essentially only at very short distances. At low temperatures, the interaction between neutral atoms can be typically characterized by a single parameter, the s -wave scattering length. The magnitude and sign of the s -wave scattering length can be changed by tuning the external magnetic fields in the vicinity of a so-called Feshbach resonance [2]. This technique has opened new possibilities for the study and manipulation of BECs. In particular, the value of the s -wave scattering length can be varied in real time, allowing, for example, the observation of collapsing and exploding BECs [3].

Recently, the dipole-dipole interaction in ultracold gases has attracted increasing interest [4–12]. Dipolar interactions would largely enrich the variety of phenomena to be observed due to their long-range and anisotropic character. If atomic dipole moments are sufficiently large, the resulting dipole-dipole forces may influence, or even completely change, the properties of Bose gases [4–6,11], the conditions for a BCS transition in Fermi gases [7], or the phase diagram for quantum phase transitions in ultracold dipolar gases confined in optical lattices [8]. The interplay of short-range scattering and long-range interaction may give rise to phenomena like ferromagnetic order and spin waves [9]. Moreover, the dipolar particles are considered to be promising candidates for the implementation of fast and robust quantum-computing schemes [10].

In this Letter, we consider the case of permanent (magnetic) dipoles. For alkali atoms with a magnetic moment m of μ_B , where μ_B is a Bohr magneton, the long-range part of the magnetic dipole-dipole interaction is generally neglected since it is very small compared to the s -wave pseudopotential. Here we show that even in alkali BECs this dipolar interaction can be made visible if the dipole-dipole interaction is modulated by rotating the atomic dipoles. For appropriate rotation frequencies, a coupling to elementary excitations can be achieved leading to a dynamic growth of the amplitude of the excitation. In particular, we demonstrate how the dipolar coupling in an alkali condensate can be used to excite the quadrupole mode and how it is possible to extract

quantitative information about the ratio of dipolar and s -wave coupling.

In ensembles of atoms with a larger magnetic moment, such as chromium where $m = 6\mu_B$, the magnetic dipole-dipole interaction energy may become comparable to the s -wave contribution to the mean-field energy. Therefore, even the static properties of a condensate such as the aspect ratio are significantly altered. Here we show how the effective strength of the dipolar interaction and thus its effect on the static properties of a condensate can be varied by fast rotation of the atomic dipoles. Tuning of the relative strength of dipolar and contact interaction will allow a detailed study of the effects of the dipolar interaction on a condensate. In contrast to tuning the contact interaction with a Feshbach resonance [2], which is not possible in all atomic systems, our proposed method of tuning the dipolar interaction is applicable to all dipolar systems and requires only small magnetic fields on the order of 1 G.

In the case of a dipolar interaction which is stronger than the s -wave interaction, only certain combinations of trap geometry and orientation of the dipoles lead to a stable (or metastable) condensate [4,6]. By tuning the dipolar interaction, unstable condensates can thus be transformed into stable ones and vice versa, similar to tuning the s -wave interaction from attractive to repulsive by using a Feshbach resonance [13]. The methods we propose here are not only valid for magnetic dipoles but also can be adapted for particles with electric dipole moments like heteronuclear molecules. In this case, the dipolar interaction is expected to dominate over the s -wave interaction, and thus tuning of the dipolar interaction might be necessary to reach stability.

Manipulating the magnetic dipolar coupling in a BEC.—The long-range part of the interaction between two atomic magnetic dipole moments \mathbf{m}_1 and \mathbf{m}_2 located at \mathbf{r}_1 and \mathbf{r}_2 takes the familiar dependence

$$U_{\text{dd}}(\mathbf{r}) = -\frac{\mu_0}{4\pi} \frac{3(\mathbf{m}_1 \cdot \hat{\mathbf{r}})(\mathbf{m}_2 \cdot \hat{\mathbf{r}}) - (\mathbf{m}_1 \cdot \mathbf{m}_2)}{r^3} \quad (1)$$

as a function of the interatomic distance $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$,

where $\hat{\mathbf{r}} = \mathbf{r}/r$ and μ_0 is the magnetic permeability of the vacuum. We consider the case of a general *time-dependent* homogeneous magnetic field

$$\mathbf{B}(t) = B\mathbf{e}(t), \quad (2)$$

slowly varying with respect to the Larmor frequency $\omega_{\text{Larmor}} = mB/\hbar$, where $m = |\mathbf{m}_1| = |\mathbf{m}_2|$ for identical atoms. A spin-polarized atomic ensemble will adiabatically follow the external magnetic field $B(t)$, and the resulting interatomic energy becomes

$$U_{\text{dd}}(\mathbf{r}, t) = -\frac{\mu_0 m^2}{4\pi} \frac{3[\mathbf{e}(t) \cdot \hat{\mathbf{r}}]^2 - 1}{r^3} \quad (3)$$

and is thus *time dependent*.

Our analysis is based on the mean-field approach in the Thomas-Fermi limit already used in the context of dipolar Bose gases [4–12]. Such a description can be accomplished through the hydrodynamic equations for superfluids [1]:

$$\frac{\partial n}{\partial t} = -\nabla \cdot (n\mathbf{v}), \quad (4)$$

$$\frac{\partial \mathbf{v}}{\partial t} = -\nabla \cdot \left(\frac{v^2}{2} + \frac{\delta\mu}{M} \right), \quad (5)$$

where n is the density, \mathbf{v} the velocity field, and $\delta\mu$ the difference between the mean-field plus external fields and the chemical potential μ given by

$$\delta\mu = gn + \Phi_{\text{dd}} + \frac{M\omega_0^2 r^2}{2} - \mu. \quad (6)$$

Here M is the atomic mass, $M\omega_0^2 r^2/2$ is an isotropic harmonic potential which could be created using optical dipole forces, $g = 4\pi\hbar^2 a/M$, and a is the s -wave scattering length which we assume to be positive. $\Phi_{\text{dd}}(\mathbf{r}, t)$ is the mean-field potential generated by the (time-dependent) dipolar interaction

$$\Phi_{\text{dd}}(\mathbf{r}, t) = \int d\mathbf{r}' U_{\text{dd}}(\mathbf{r} - \mathbf{r}', t) n(\mathbf{r}'). \quad (7)$$

In the Thomas-Fermi limit it is useful to define a dimensionless quantity

$$\varepsilon_{\text{dd}} = \frac{\mu_0 m^2 M}{12\pi\hbar^2 a}, \quad (8)$$

which is a measure of the strength of the dipole-dipole interaction relative to the s -wave scattering energy.

For chromium the s -wave scattering length a_{Cr} is not known. If we assume that a_{Cr} is equal to the sodium scattering length ($a = 2.8$ nm) the dipolar strength parameter becomes $\varepsilon_{\text{dd}}^{\text{Cr}} = 0.29$ ($m = 6\mu_B$). Therefore, the dipolar force is expected to lead to a visible modification in the density of a chromium BEC. Differently, in both rubidium and sodium the magnetic dipolar energy is rather small compared to the mean-field s -wave scattering energy. The values of ε_{dd} are $\varepsilon_{\text{dd}}^{\text{Rb}^{87}} = 0.0064$ and $\varepsilon_{\text{dd}}^{\text{Na}} =$

0.0035, in the doubly spin-polarized ground state, where $m = \mu_B$. For heteronuclear molecules with an electric dipole moment of one Debye the corresponding dipolar coupling will be increased by approximately the inverse square of the fine structure constant. This implies that ε_{dd} [14] is on the order of 10^2 .

Probing the quadrupole modes in sodium and rubidium.—The time modulation of the dipolar coupling (3) can be used to resonantly excite a quadrupole mode even for sodium and rubidium condensates. For simplicity we restrict ourselves to the case of a condensate in the Thomas-Fermi regime confined in an isotropic harmonic trap with frequency ω_0 (for example, $\omega_0/2\pi = 100$ Hz). Because ε_{dd} is small, the dipole-dipole mean-field potential Φ_{dd} can be calculated using the symmetric equilibrium density distribution $n_0 = M\omega_0^2(R^2 - r^2)/2g$, where R is the radius, resulting in

$$\Phi_{\text{dd}}(\mathbf{r}, t) = -\frac{\varepsilon_{\text{dd}} M \omega_0^2}{5} \{3[\mathbf{e}(t) \cdot \hat{\mathbf{r}}]^2 - 1\} r^2, \quad r < R, \quad (9)$$

$$= -\frac{\varepsilon_{\text{dd}} M \omega_0^2}{5} \{3[\mathbf{e}(t) \cdot \hat{\mathbf{r}}]^2 - 1\} \frac{R^5}{r^3}, \quad r > R. \quad (10)$$

The potential outside the condensate region ($r > R$) corresponds exactly to the field generated by N dipoles located in the center of the condensate. Differently, inside the condensate ($r < R$) the mean-field potential has an anisotropic harmonic dependence.

If the interaction is driven on a resonance frequency, the evolution of the condensate can be modified even for very small dipolar coupling. Consider a magnetic field rotating in the plane $x - y$ as $\mathbf{B}(t) = B\mathbf{e}(t)$, where

$$\mathbf{e}(t) = \cos(\Omega t)\hat{\mathbf{x}} + \sin(\Omega t)\hat{\mathbf{y}}. \quad (11)$$

The resulting dipolar interatomic energy (3) and its corresponding contribution to the mean-field potential (9) and (10) will have a “rotating” part. For small dipolar coupling within alkali gases the nonoscillating part of (9) can be neglected and the time-dependent part will resonantly couple to the “transversal” quadrupole mode when $\Omega \approx \omega_0/\sqrt{2}$ (see Ref. [15]). After linearization of the equations of motion (4) and (5) we obtain

$$\frac{\partial^2 \delta n}{\partial t^2} = \nabla \cdot \left(\frac{gn_0}{M} \nabla(\delta n) \right) + \nabla \cdot \left(\frac{n_0}{M} \nabla(\Phi_{\text{dd}}) \right), \quad (12)$$

where $\delta n = n - n_0$ is the density displacement around the equilibrium density n_0 . The time-dependent part of Φ_{dd} corresponds in Eq. (12) to the rotation of an anisotropic harmonic trap with deformation

$$\varepsilon = \frac{\omega_X^2 - \omega_Y^2}{\omega_X^2 + \omega_Y^2} = \frac{3}{10} \varepsilon_{\text{dd}}, \quad (13)$$

where ω_X and ω_Y are the maximum and minimum values of the rotating trap frequencies. Neglecting the time-independent part of (9) the density will evolve as

$$\delta n = -\frac{3\varepsilon_{\text{dd}}M\omega_0^2}{10g}r^2\sin^2\theta\sin(2\phi-2\Omega t)\Omega t, \quad (14)$$

where ϕ is the azimuthal angle ($r < R$). It is noteworthy that the rotating part of (9) couples *only* to the first quadrupole mode in the Thomas-Fermi limit [15]. From the study of the evolution of the condensate deformation

$$\alpha = \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle} = -\frac{3}{5}\varepsilon_{\text{dd}}\Omega t\sin(2\Omega t), \quad (15)$$

we can extract information about the ratio between dipolar coupling and standard mean-field energy. If the quality factor $Q = \sqrt{2}\omega_0\tau$ of such a resonance, where τ is the corresponding decay time, is assumed to be 50 [16], the amplitude of the oscillation of the condensate anisotropy α is expected to be $|\alpha| = 3Q\varepsilon_{\text{dd}}/10 \sim 9.5 \times 10^{-2}$ for rubidium (see Fig. 1). We have performed our calculation in the linear regime and for an isotropic trap. The non-linearity of the mean field is expected to give a significant correction for larger Q . The extension of the calculation to an axially symmetric cigar-shaped trap is expected to yield similar results.

Tunability of the magnetic dipolar interaction.—We consider a condensate in an axially symmetric optical trap, with radial and axial trap frequencies ω_r and ω_z . The magnetic dipoles are aligned along an external magnetic field

$$\mathbf{B}(t) = B\{\cos\varphi\hat{\mathbf{z}} + \sin\varphi[\cos(\Omega t)\hat{\mathbf{x}} + \sin(\Omega t)\hat{\mathbf{y}}]\}, \quad (16)$$

which is a combination of a static magnetic field B_z directed along the z direction and a fast rotating field B_ρ in the radial plane. The frequency is chosen such that the atoms are not significantly moving during the time Ω^{-1} , while the magnetic moments will follow adiabatically the external field $\mathbf{B}(t)$; this corresponds to $\omega_{\text{Larmor}} \gg \Omega \gg \omega_r, \omega_z$. In this limit we can consider the average of the interaction (3) in the period $2\pi/\Omega$, resulting in the cylindrically symmetric interatomic potential

$$\langle U_{\text{dd}}(r, \theta, \varphi) \rangle = -\frac{\mu_0 m^2}{4\pi} \left(\frac{3\cos^2\theta - 1}{r^3} \right) \left(\frac{3\cos^2\varphi - 1}{2} \right). \quad (17)$$

The averaged energy [Eq. (17)] equals the interaction energy of aligned dipoles [Eq. (3)] times a factor $(3\cos^2\varphi - 1)/2$. By varying the angle φ (see Fig. 2), the factor can be changed continuously from $-1/2$ to 1 , providing the possibility to change the dipolar interaction from attractive to repulsive. At a particular angle $\varphi_M = 54.7^\circ$ the dipolar interaction averages to zero. This angle, the so-called magic angle, is well known in solid-state nuclear magnetic resonance technology [17]. The tunability of the dipolar interaction will allow one to observe changes in the condensate eccentricity which should be a 10% effect in Cr assuming $a_{\text{Cr}} = a_{\text{Na}}$ or in the collective excitation frequencies [4,18].

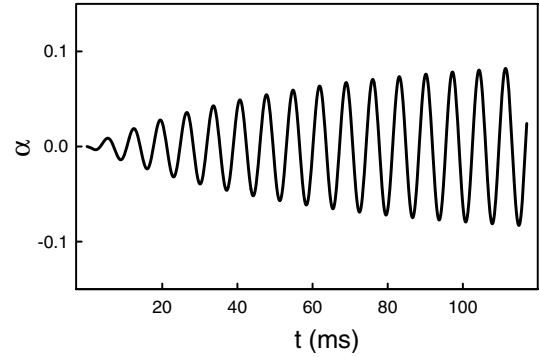


FIG. 1. Excitation of the transverse quadrupole mode in a rubidium condensate mediated by the dipole-dipole interaction. The eccentricity α [Eq. (15)] of a rubidium condensate is growing in time when the orientation of the magnetic dipole moments is rotated with half the quadrupole frequency. Here we assume an isotropic trap with $\omega_0 = 2\pi 100$ Hz and a quality factor $Q = 50$ of the resonance.

Stabilizing strongly interacting dipolar gases.—Let us now apply this tunability to the stability diagram of dipolar BECs. We consider the case in which the dipolar coupling is larger than the s -wave pseudopotential ($\varepsilon_{\text{dd}} \geq 1$), and the cloud may become unstable against collapse in the Thomas-Fermi limit. Here the tunability can stabilize strongly interacting dipolar gases like heteronuclear molecules which otherwise exhibit instability over a wide range of parameters. This regime, considered also in Refs. [4,6], may be achieved in a cloud of magnetic dipolar atoms, by making use of a Feshbach resonance to reduce the s -wave scattering length.

Within the Thomas-Fermi approximation, the stability analysis can be made using a scaling variational approach

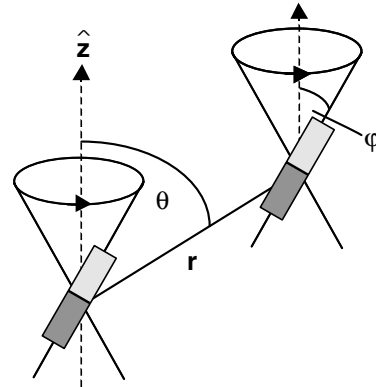


FIG. 2. Tunability of the magnetic dipole interaction. Using time-varying magnetic fields (16), the dipoles are rapidly rotated around the z axis. The angle φ between the dipole orientation and the z axis determines the strength and sign of the effective interaction. $\varphi = 0$: the magnetic dipoles are polarized along the z direction. $\varphi = \pi/2$: the sign of the effective interaction is inverted and the absolute value is only half of the polarized case. $\varphi = 54.7^\circ$ (magic angle): the dipolar interaction averages to zero.

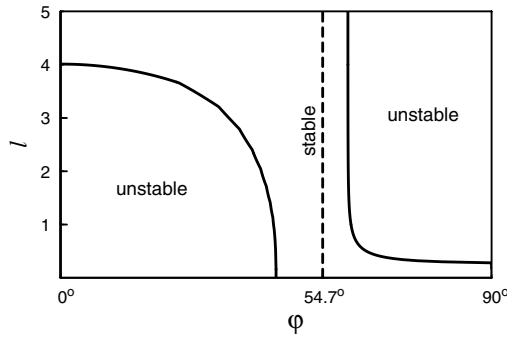


FIG. 3. Stability diagram of a condensate with dipolar interaction in the φ - l plane ($l = \omega_z/\omega_r$) obtained in the Thomas-Fermi limit ($N \rightarrow \infty$) and with $\epsilon_{dd} = 4$. Condensates are always stable in the vicinity of the magic angle $\varphi_M = 54.7^\circ$.

[19] in analogy to Ref. [4]. Assuming a density for the condensate of the form

$$n(x, y, z) = \frac{1}{\lambda_r^2 \lambda_z} n_0 \left(\frac{x}{\lambda_r}, \frac{y}{\lambda_r}, \frac{z}{\lambda_z} \right), \quad (18)$$

where λ_r and λ_z are scaling variational parameters and n_0 is a spherically symmetric density distribution (for instance, the usual Thomas-Fermi inverted parabola). The condensate anisotropy λ_r/λ_z can be found by minimizing the expectation value of the total energy $H_{\text{tot}} = H_{\text{ho}} + H_{\text{int}}$ where

$$H_{\text{ho}} = c_1 N m (2\omega_r^2 \lambda_r^2 + \omega_z^2 \lambda_z^2) \quad (19)$$

is the harmonic potential energy and

$$H_{\text{int}} = c_2 \frac{N^2 g}{\lambda_r^2 \lambda_z} \left[1 - \epsilon_{dd} \left(\frac{3 \cos^2 \varphi - 1}{2} \right) f \left(\frac{\lambda_r}{\lambda_z} \right) \right] \quad (20)$$

the mean-field interaction energy, where c_1 and c_2 are numerical constants and

$$f(\kappa) = \frac{1 + 2\kappa^2}{1 - \kappa^2} - \frac{3\kappa^2 \tanh^{-1} \sqrt{1 - \kappa^2}}{(1 - \kappa^2)^{3/2}} \quad (21)$$

derives from an angular integration.

The stability can be varied for fixed ϵ_{dd} and trap anisotropy l by varying just the angle φ . In Fig. 3 we show one example of how the stability phase diagram for $\epsilon_{dd} = 4$ can be varied as a function of the angle φ . For very large ϵ_{dd} the stable region in the φ - l plane becomes a thin region around the magic-angle line.

Conclusion.—We have shown that the anisotropy of the dipole-dipole interaction of a polarized gas can be used to tune the strength of the dipolar coupling and to excite collective excitations. Tuning the dipolar coupling will become an important tool for designing atomic quantum gases with novel properties and dynamical aspects. This approach in optically trapped Bose condensed gases should allow one to make even very weak dipolar cou-

pling visible with experimentally feasible parameters (like B fields in the Gauss region and spinning frequencies up to several 10 kHz). The tuning of the interaction might become a relevant technique for stabilizing strongly interacting dipolar systems like heteronuclear molecular gases.

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