Overcoming the Diffraction Limit in Wave Physics Using a Time-Reversal Mirror and a Novel Acoustic Sink

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In recent years, time-reversal (TR) mirrors have been developed that create TR waves for ultrasonic transient fields propagating through complex media. A TR wave back propagates and refocuses exactly at its initial source. However, because of diffraction, even if the source is pointlike the wave refocuses on a spot size that cannot be smaller than half a wavelength. Here, by using a TR interpretation of this limit, we show that this latter limitation can be overcome if the source is replaced by its TR image. This new device acts as an acoustic sink that absorbs the TR wave. Here we report the first experimental result obtained with an acoustic sink where a focal spot size of less than 1/14th of one wavelength is recorded.

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One can verify the time-reversal invariance of the acoustic wave equation in a nondissipative medium by introducing a mirror of the time variable [1,2]. In an image world such a mirror exists by taking a motion picture of the wave propagation and then running the film backwards. In the real world reversing the time variable means generating experimentally a back-propagated replica from a forward propagation field. From Huygens principle, the mirror of the time variable can be replaced by a time-reversal operation performed on a closed surface that surrounds the initial source location [1]. This active surface, called a time-reversal (TR) cavity, is covered with reversible transducers (that act both as microphones and loudspeakers). In the first step, a pointlike source emits a pulsed field that is measured by the transducers. In a second step, the recorded field is time reversed and then retransmitted back into the medium by the same transducers. As the wave is back propagated from all directions, the focusing is perfectly isotropic around the initial source position with a focal spot as narrow as one half wavelength. Time-reversal experiments have been performed through many different complex media including scatterer forests, acoustical waveguides and biological tissues for various purposes such as nondestructive testing, medical acoustic or ocean underwater acoustic. Until now, the initial source was removed during the back propagation. Therefore due to energy flux conservation around the focus, a diverging wave appeared as soon as the converging time-reversed wave had reached the focus. This diverging wave breaks the TR symmetry. Indeed this wave has no time-symmetric counterpart (a converging wave) in the forward propagation: a "complete" time reversal would consist of only a converging wave. It was in 1992 that Cassereau *et al.* [3] first theoretically raised the problem of the behavior of the time-reversed wave near the focus. More precisely, it has been shown that the superimpositon of the converging and the diverging waves limits the focal spot size of the back-propagate field to one half wavelength even if the initial source dimensions are much smaller than the half wavelength. This result is consistent with diffraction theory which states that in an homogeneous medium with no source the shortest spatial wave-field fluctuations are precisely one half wavelength. In fact, the breaking of the time-reversal process near the focus during the back propagation comes from the absence of the time-reversed source. Indeed, the forward propagation inside the TR cavity is not only determined by the field values at the boundary but also by the source emission. In the reversed situation, the complete TR field results in the time reversal of the source and the wave on the boundary. Therefore, the time-reversed source acts as an acoustic sink that absorbs all the converging field at the focus in order to avoid the diverging part.

In this paper, for the first time, we present an experiment where an ultrasonic source is time reversed. We show that the time-reversed source is the initial source excited with the initial pulse taken in the reverse chronological order. Moreover, a focal spot size of less than 1/14th of one wavelength is recorded when the time-reversed source is operating. Hence the usual diffraction limit of focused waves is overcome. The possibility of focusing towards a subwavelength spot underlines the issue of the time reversal of a field containing evanescent waves. This particular topic has been recently introduced by Carminati *et al.* [4] and is experimentally highlighted here.

Basically, it is in accounting for the source term in the wave equation that leads to the method of time reversing the source. A general demonstration for waves inside any kind of media (fluid, solid, anisotropic, etc.) can be performed in terms of the powerful Green's functions formalism [5]. Nevertheless, here, for simplicity we focus only on propagation of longitudinal waves of sound speed (c) inside an infinite homogeneous 3D medium. In such a medium, a pointlike source localized at point O (position $\mathbf{r}_0$) and excited by a waveform $f(t)$ generates a wave field

$\Psi_s(\mathbf{r};t)$ which is the solution of the wave equation with a source term

$$\nabla^2\Psi_s(\mathbf{r};t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Psi_s(\mathbf{r};t) = -\delta(\mathbf{r}-\mathbf{r}_0)f(t). \quad (1)$$

$\mathbf{r}_0$ is the source position. For a monochromatic source with complex amplitude A $[f(t) \propto Ae^{-i\omega t}]$,

$$\Psi_s(\mathbf{r};t) = Ae^{-i\omega t}e^{ik_0R}/(4\pi R), \qquad R = \|\mathbf{r}-\mathbf{r}_0\|. \quad (2)$$

This is the mathematical expression of an isotropic wave that diverges from point O. This wave propagates up to the surface of the time-reversal cavity. The TR cavity then generates a spherical wave field, $\Psi_{\mathrm{TRC}}(\mathbf{r};t)$, that focuses on point O. This field can be written in terms of the well known sinc expression $[\mathrm{sinc}(x) = \sin(x)/x]$:

$$\Psi_{\mathrm{TRC}}(\mathbf{r};t) = \frac{-ik_0A^*}{2\pi}\mathrm{sinc}(k_0R)e^{-i\omega t}. \quad (3)$$

The focal spot size is $\lambda/2$. This is the smallest spot size when one wants to focus a wave in the far-field regime. A useful expansion of Eq. (3) is

$$\Psi_{\mathrm{TRC}}(\mathbf{r};t) = A^*\frac{e^{-ik_0R}}{4\pi R}e^{-i\omega t} - A^*\frac{e^{ik_0R}}{4\pi R}e^{-i\omega t}. \quad (4)$$

The focusing field pattern can thus be interpreted as the superposition of an incoming wave [first term in Eq. (4)] and an outgoing wave (second term). If the incoming wave is the time-reversed image of the emitted wave, $\Psi_s(\mathbf{r};-t)$, the outgoing part has no time-symmetric image and therefore breaks the TR process. Basically, $\Psi_{\mathrm{TRC}}(\mathbf{r};t)$ is not equal to $\Psi_s(\mathbf{r};-t)$ because these fields are not a solution of the same propagation equation. On the one hand, the wave produced by the TR cavity propagates inside a volume with no source. Therefore, Ψ_{TRC} is the solution of the propagation equation with no right-hand side:

$$\nabla^2\Psi_{\mathrm{TRC}}(\mathbf{r};t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Psi_{\mathrm{TRC}}(\mathbf{r};t) = 0.$$

On the other hand, it is easy to show that $\Psi_s(\mathbf{r};-t)$ is the solution of a propagation equation with a source term

$$\nabla^2\Psi_s(\mathbf{r};-t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Psi_s(\mathbf{r};-t) = -\delta(\mathbf{r}-\mathbf{r}_0)f(-t). \quad (5)$$

Furthermore, Eq. (5) gives the trick to generate $\Psi_s(\mathbf{r};-t)$ instead of $\Psi_{\mathrm{TRC}}(\mathbf{r};t)$: not only the wave but also the source has to be time reversed. Indeed, the right-hand side of Eq. (5) represents the time-reversed image of a source excited by a $f(t)$ waveform at location $\mathbf{r}_0$. This result makes more sense if one thinks in terms of diverging and converging monochromatic waves. The TR source term generates a field Ψ_{TRS} which is equal to

$$\Psi_{\mathrm{TRS}}(\mathbf{r};t) = \frac{A^*}{4\pi R}e^{ik_0R}e^{-i\omega t}. \quad (6)$$

Hence the total field consists of adding up the fields generated by the TR cavity $[\Psi_{\mathrm{TRC}}(\mathbf{r};t)]$ and the TR source $[\Psi_{\mathrm{TRS}}(\mathbf{r};t)]$. The diverging wave produced by the TR source is opposite in sign to the diverging part of $\Psi_{\mathrm{TRC}}(\mathbf{r};t)$ [cf. Equations (4) and (6)]. Therefore they interfere destructively and only a converging wave equal to $\Psi(\mathbf{r};-t)$ remains.

In this section we have shown mathematically, in a simple situation, that in order to completely define the time-reversed field during the second step of a time-reversal experiment, both the source and the field should be time reversed. The proof inside a 2D medium [6] is not more difficult.

From a practical point of view, to realize such an acoustical sink we need first a time-reversal cavity. However, a time-reversal cavity is difficult to realize and, to limit the number of transducers, the time-reversal operation is usually performed over a limited angular aperture. This time-reversal device is called a time-reversal mirror. Of course, such a TR device spoils the focusing of the back-propagated wave. Another solution is to add reflecting boundaries which redirect the incident wave inside the TR mirror aperture. In closed reflecting cavities with ergodic and mixing properties, all the information emanating from a pointlike source will reverberate throughout the whole volume and thus can be collected at only a single detector location. Therefore the time-reversal operation can be performed using only one pointlike transducer. Draeger *et al.* have carried out the first time-reversal experiments in such a configuration [7] using chaotic silicon plate cavities. In particular, it has been shown experimentally that TR waves focus isotropically. Here, to perform time reversal inside a reverberant cavity, we use a slightly different experimental setup than the one used by Draeger *et al.* that allows us to record the field close to the time-reversed source. The cavity is a 1.9 mm thick transparent glass plate whose shape is a 80 mm by 80 mm quarter stadium (Fig. 1). This chaotic geometry is chosen to obtain quasi-isotropic focusing.

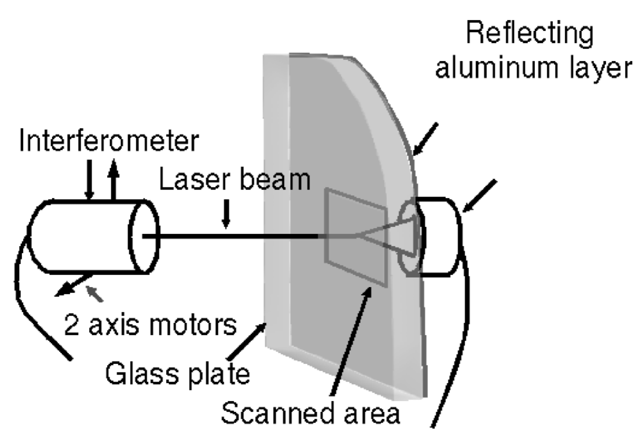

FIG. 1. Schematic of experimental setup.

Elastic waves that propagate in such a plate are Lamb waves. A brass cone coupled to a longitudinal transducer generates the field at one point of the cavity. The dimension of the contact zone is less than 100 μm, which corresponds to $\lambda/14$ at the 500 kHz central working frequency. The -3 dB bandwidth of this transducer device is equal to 200 kHz. In the original experiment, a second identical transducer was used as the TR device. In the present setup the same transducer is used to generate both the forward field and the time-reversed field. A heterodyne laser interferometer records the field time dependence. The optical beam that scans the field has a lateral dimension of 6 μm. A thin aluminum layer (1 μm thick) is evaporated on the far face of plate. The laser beam is thus reflected back from the face of the plate that is in contact with the source (Fig. 1). During the first step of the experiment, a 5 μs pulse is emitted by the point-like transducer. This 5 μs initial pulse generates a field that reverberates for more than 2 ms inside the cavity. With the laser spot directed towards the position of the source, the normal displacement is recorded and sampled until the field completely vanishes. In the second step, a 1.5 ms long window (between times 250 and 1750 μs) is time reversed and reemitted by the transducer (Fig. 2). The interferometer is placed on a two axis stepping motor bench which allows us to record the time evolution of the time-reversed wave over a 20 mm by 20 mm square centered on the source [Fig. 3(a)]. The TR process account for all reversible effects such as dispersion, anisotropy, reflections. However, the superimposed background field adds to the time-reversed one. It comes from the fact that this one-channel time-reversal experiment is not able to completely replace a time-reversal cavity [7]. On Fig. 3(a), we clearly observe that a diverging wave follows the converging one. In a second experiment, a short pulse equal to the time-reversed initial waveform [function $f(-t)$] is added at time 1750 μs to the signal emitted by the transducer (Fig. 2). Now, the time-reversed source cancels the diverging wave [Fig. 3(b)] and only a converging wave remains, i.e. TR image of the source emission. This effect is most visible when Figs. 3(a) and 3(b) are compared at times 1751 and 1752 μs [8].

A more striking difference lies in the two focus patterns (Fig. 4). Indeed, when the time-reversed source is off, the focusing pattern is smooth with a focal spot size equal to 2 mm, corresponding to one half wavelength. When the time-reversed source is switched on, a sharp and strong peak appears. In this case the focal spot is narrower and equal to 0.3 mm ($\lambda/14$). Therefore, it seems that a perfect time-reversal process yields a converging wave with a focal spot size much smaller than $\lambda/2$.

The subwavelength focus implies that evanescent waves (near field) are involved in the complete time reversal. The formalism of evanescent waves is important

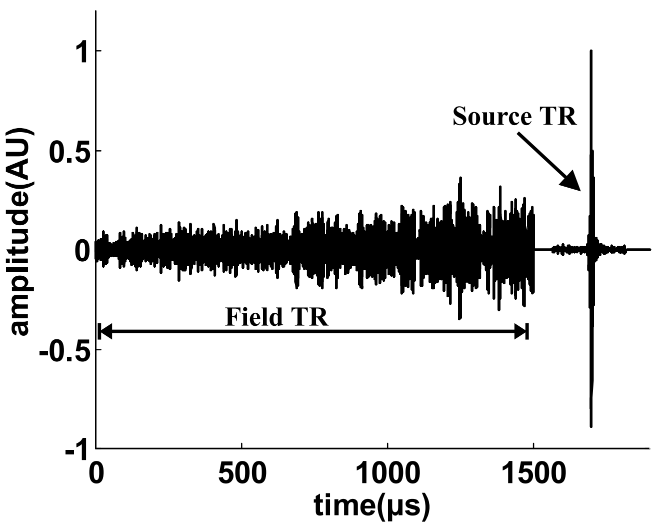

FIG. 2. Signal emitted by the transducer device. The long part (between times 0 and 1.5 ms) is the TR window in order to TR the field inside the reverberating chaotic cavity. When only this part of signal is emitted, the focusing shown on Fig. 3(a) is recorded. The short part (around time 1750 ms) corresponds to the TR initial waveform $f(-t)$ (cf. text) in order to obtain the TR source. When the long and the short parts are emitted, the acoustic sink is obtained [Fig. 3(b)].

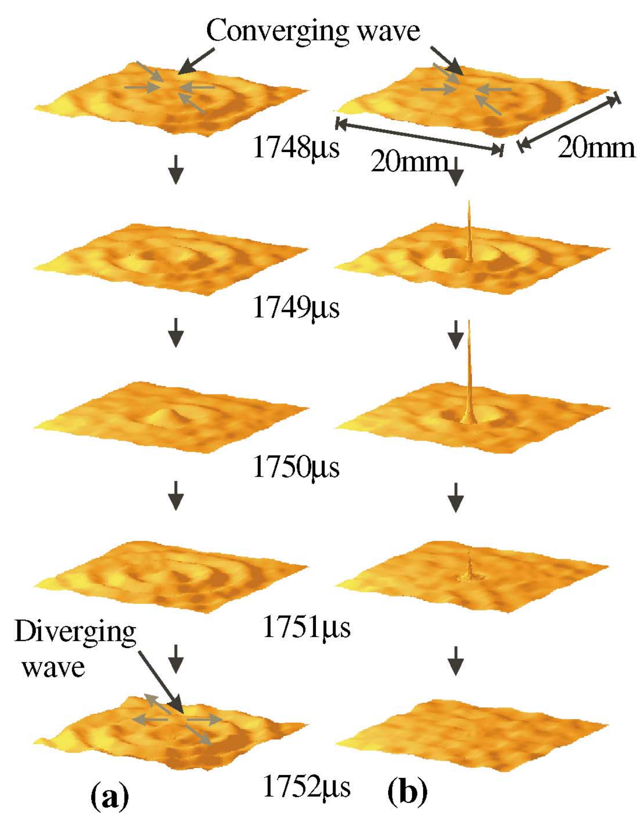

FIG. 3 (color). Time evolution recorded by the interferometer over a 20 mm by 20 mm square around the initial source during time-reversal propagation (1 ms separates each snapshot): (a) without the TR source, (b) with the TR source. On sequence (b), an acoustic sink is obtained.

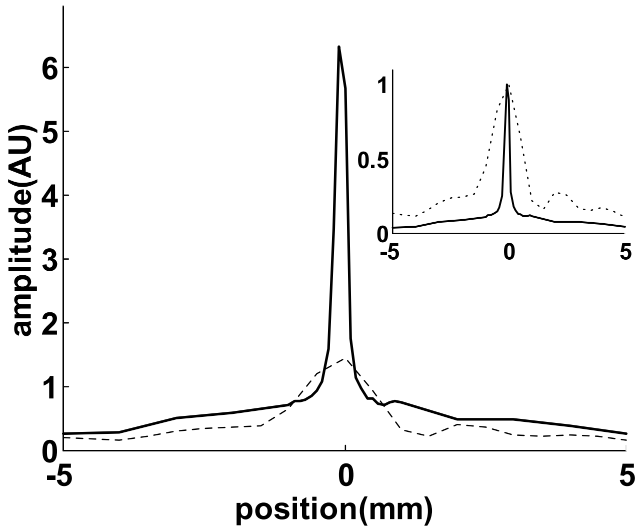

FIG. 4. Focal spot sections without the TR source (dashed line), with TR source (continuous line). Inside the graphic box, the focal spots normalized with respect to their maximums are plotted.

in many different fields: tunneling effect, near field optics, acoustical microscopy, etc. Reference [4] deals with the issue of time reversal in terms of evanescent waves. In the following, the analysis is limited to homogeneous 3D media. In the monochromatic regime, the evanescent waves appear explicitly when the angular spectrum or plane-wave expansion of the field is used. Table I compares the Fourier transforms of the different fields generated by the source, the TR cavity alone, the TR source alone, and the TR source plus the TR cavity. These Fourier transforms are performed on an arbitrary plane (Π) containing point O. $k_{\|}$ is the norm of the projection of the wave vector in the plane Π. Therefore, due to the dispersion relation, the absolute value of the wave-vector component perpendicular to Π, $k_{\perp}$, is equal to $\sqrt{k_0^2 - k_{\|}^2}$. When $k_{\|}$ is lower than k_0, $k_{\perp}$ is real and corresponds to propagative components. When $k_{\|}$ is higher than k_0, $k_{\perp}$ is fully imaginary. This means that these components are evanescent. Evanescent components correspond to high frequency spatial fluctuation fields. They are nonpropagative and decrease exponentially over roughly one wavelength. The spectrum of the field Ψ_{TRC} produced by the time-reversal cavity alone contains no evanescent components (see Table I). This result is in agreement with the diffraction theory. When the time-reversed source is switched on, the spectrum of Ψ_{TRS} adds to the one of Ψ_{TRC} . Then the spectrum of the complete TR appears to be equal to the conjugate of the spectrum of the forward emission (see Table I). Indeed, phase conjugation is the monochromatic (narrow band) equivalent of the time-reversal operation on transient signals (wideband). Beyond this result, it is shown clearly that during the

TABLE I. Angular spectrum or plane-wave expansion of fields Ψ_s, Ψ_{TRC}, Ψ_{TRS}, and $\Psi_{\mathrm{TRC}} + \Psi_{\mathrm{TRS}}$ on an arbitrary plane Π containing point O. $k_{\|}$ is the norm of the projection of the wave vector in the plane Π.

Emitter Field	Source Ψ_s	TR cavity Ψ_{TRC}	TR source Ψ_{TRS}	TRC + TRS $\Psi_{\mathrm{TRC}} + \Psi_{\mathrm{TRS}}$
$k_{\|} < k_0$ (propagative)	$\frac{iA}{2\sqrt{k_0^2 - k_{\|}^2}}$	$\frac{-iA^*}{\sqrt{k_0^2 - k_{\|}^2}}$	$\frac{iA^*}{2\sqrt{k_0^2 - k_{\|}^2}}$	$\frac{-iA^*}{2\sqrt{k_0^2 - k_{\|}^2}}$
$k_{\|} \geq k_0$ (evanescent)	$\frac{A}{2\sqrt{k_{\|}^2 - k_0^2}}$	0	$\frac{A^*}{2\sqrt{k_{\|}^2 - k_0^2}}$	$\frac{A^*}{2\sqrt{k_{\|}^2 - k_0^2}}$

complete time-reversal process, the evanescent components originate only from the TR source. Thus the evanescent wave emission cannot be separated from the emission of the propagative diverging wave by the TR source in order to cancel the diverging wave due to the TR cavity. Finally, this evanescent approach shows that focusing towards a subwavelength spot with a TR source is not in contradiction with the diffraction limit.

Here, we have presented the first experimental demonstration of an "acoustic sink" that produces a subwavelength focusing wave. In this experiment we have shown how to time reverse not only a wave field but also its source. This experiment can have a strong impact in wave physics because it opens the way to work beyond diffraction limits: wave fields focusing in focal region much smaller than initial wavelengths can be obtained. Feasibility of subwavelength focusing is currently under study for electromagnetic waves.

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[6] In 2D free space, $\Psi_s(\mathbf{r};t) = iAH_0^{(1)}(k_0R)e^{-i\omega t}/4$; $\Psi_{\mathrm{TRC}}(\mathbf{r};t) = iA^* J_0(k_0R)e^{-i\omega t}/2$; $\Psi_{\mathrm{TRS}}(\mathbf{r};t) = iA^* \times H_0^{(1)}(k_0R)e^{-i\omega t}$, and $\Psi_s(\mathbf{r};-t) = -iA^*[H_0^{(1)}(k_0R)]^* e^{-i\omega t}/4$.
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[8] Video sequences of the focusing with and without acoustic sink are available as supplemental material on our lab Web site at URL: http://www.loa.espci.fr/~julien/sink.