

## Is There a Significant Excess in Bottom Hadroproduction at the Tevatron?

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We discuss the excess in the hadroproduction of  $B$  mesons at the Tevatron. We show that an accurate use of up-to-date information on the  $B$  fragmentation function reduces the observed excess to an acceptable level. Possible implications for experimental results reporting bottom quark cross sections, also showing an excess with respect to next-to-leading order theoretical predictions, are discussed.

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For a few years, bottom production has been one of the very few instances in which experimental results and quantum chromodynamics (QCD) predictions have sometimes displayed not very good agreement. Bottom quark hadroproduction cross sections have been measured by the UA1 Collaboration [1,2] at the CERN  $S\bar{p}\bar{p}S$  collider and by both the CDF [3–5] and D0 [6] experiments at the Fermilab Tevatron in  $p\bar{p}$  collisions, and found to be about a factor of 2 or more larger than next-to-leading order (NLO) QCD predictions [7–9]. CDF has recently published data for  $B^+$  meson production [10]. They claim an excess over QCD predictions by about a factor of 3. The H1 and ZEUS experiments at the electron-proton collider HERA have both measured  $D^*$  production cross sections [11,12]. Both measurements are compatible with QCD calculations [13], although the ZEUS data is on the high side of the theoretical uncertainty band. Bottom production has been measured at HERA and from photon-photon collisions at LEP, and found to be larger than predictions, by about a factor of 3 or more [14–17].

By pushing the parameters of the theoretical calculation to somewhat extreme values, it is not impossible to accommodate the bottom spectrum observed at the Tevatron. Alternatively, one can take the discrepancy more seriously, and invoke some “new physics” contribution [18] in order to explain it. It has also been known for a long time that the fixed order, NLO QCD calculation may be insufficient to explain the data, because of the presence of some enhanced contributions, that can be included via resummation of large classes of Feynman diagrams, and that contribute positively to the cross sections. These contributions are threshold effects, small- $x$  effects, and high transverse momentum logarithms, which may be important since most of the cross section is measured at large transverse momentum. A full calculation of next-to-next-to-leading QCD contributions, years ahead in the future, might finally also contribute to explain the apparent discrepancy.

In this Letter, we shall not try to improve on the perturbative aspects of heavy quark production. We shall instead focus our attention on a specific nonperturbative issue, namely, on the implementation of hadronization effects. In fact, we shall argue that a good part of the discrepancy between theory and data arises when one tries to supplement the perturbative prediction for  $b$  quark production with a nonperturbative model for the formation of a  $B$  meson from the  $b$  quark, or, alternatively, to correct the data in an attempt to give a  $b$  quark spectrum rather than a  $B$  meson one. The nonperturbative hadron formation effect is usually introduced by writing the hadron-level cross section for  $B$  mesons as

$$\frac{d\sigma^B}{dp_T} = \int d\hat{p}_T dz \frac{d\sigma^b}{d\hat{p}_T} D(z) \delta(p_T - z\hat{p}_T), \quad (1)$$

the function  $D(z)$  being a phenomenological parametrization of hadronization effects. Traditionally, the Peterson *et al.* [19]  $D(z; \epsilon)$  form of the fragmentation function is used, implemented in conjunction with a quark cross section given by a shower Monte Carlo program. The  $\epsilon$  parameter is obtained from fits to  $e^+e^-$  data [20]. The effect of fragmentation is to reduce the momentum of the  $B$  meson with respect to that of the  $b$  quark. It is roughly a 10% effect, being of the order of  $\bar{\Lambda}/m$ , where  $\bar{\Lambda}$  is a hadronic scale, of the order of a few hundred MeV, and  $m$  is the bottom quark mass. It has, however, an important impact on the value of the cross section, because of the steeply falling transverse momentum spectrum of the  $b$  quark. Since transverse momentum cuts are always applied, the measurable cross section is strongly reduced by this effect. It should be clear from this discussion that, in order to assess the presence of a discrepancy in the  $B$  production data, the effect of fragmentation should be assessed clearly and unambiguously.

In Ref. [10], the CDF Collaboration compares its data to a theoretical prediction obtained by convoluting the NLO cross section for bottom quarks with a Peterson

fragmentation function. They use  $\epsilon = 0.006 \pm 0.002$ , which is the traditional value proposed in Ref. [20]. They claim that their data is a factor of 2.9 higher than the QCD calculation.

The purpose of this Letter is precisely to implement correctly the effect of heavy quark fragmentation in the QCD calculation. Several ingredients are necessary in order to do this: (i) A calculation with resummation of large transverse momentum logarithms at the next-to-leading level (NLL) should be used for heavy quark production [21], in order to correctly account for scaling violation in the fragmentation function. (ii) A formalism for merging the NLL resummed results with the NLO fixed order calculation (FO) should be used, in order to account properly for mass effects [22]. This calculation will be called FONLL in the following. (iii) A NLL formalism should be used to extract the nonperturbative fragmentation effects from  $e^+e^-$  data [23–29].

We begin by pointing out that, as shown in Refs. [27,28], the value  $\epsilon = 0.006$  is appropriate only when a leading-log (LL) calculation of the spectrum is used, as is the case in shower Monte Carlo programs. When NLL calculations are used, smaller values of  $\epsilon$  are needed to fit the data. It must further be pointed out that, as noted in [30,31], it is not the detailed knowledge of the whole spectrum of  $D(z)$  in  $z \in [0, 1]$  to be relevant for the calculation of hadronic cross sections. For the steeply falling differential distributions  $d\sigma/dp_T$ , that have usually a power law behavior, the knowledge of some specific moment of the fragmentation function  $D_N \equiv \int D(z)z^{N-1} dz$  is sufficient to obtain the hadronic cross section. In fact, assuming that  $d\hat{\sigma}/d\hat{p}_T = A\hat{p}_T^{-n}$  in the neighborhood of some  $\hat{p}_T$  value, one immediately finds

$$\frac{d\sigma}{dp_T} = \int dz d\hat{p}_T D(z) \frac{A}{\hat{p}_T^n} \delta(p_T - z\hat{p}_T) = \frac{A}{p_T^n} D_n. \quad (2)$$

Thus, the hadronic cross section is given by the product of the partonic cross section times the  $n$ th moment of the fragmentation function, where  $n$  is the power behavior of the cross section in the neighborhood of the value of  $p_T$  being considered. In Ref. [31], it is also shown that this is an excellent approximation to the exact integral in the cases of interest. The value of  $n$  for the  $p_T$  spectrum in the region of interest ranges from 3 to 5. It is therefore clear that, when fitting  $e^+e^-$  data, getting a good determination of the *moments* of the nonperturbative fragmentation function between 3 and 5 is more important than attempting to describe the whole  $z$  spectrum.

Figure 1 shows the moments calculated from the  $x_E$  (the  $B$  meson energy fraction with respect to the beam energy) distribution data for weakly decaying  $B$  mesons in  $e^+e^-$  collisions published by the ALEPH Collaboration [32]. The experimental error bars shown in the plot have been evaluated by taking into account the full bin-to-bin correlation matrix [33]. Four curves are

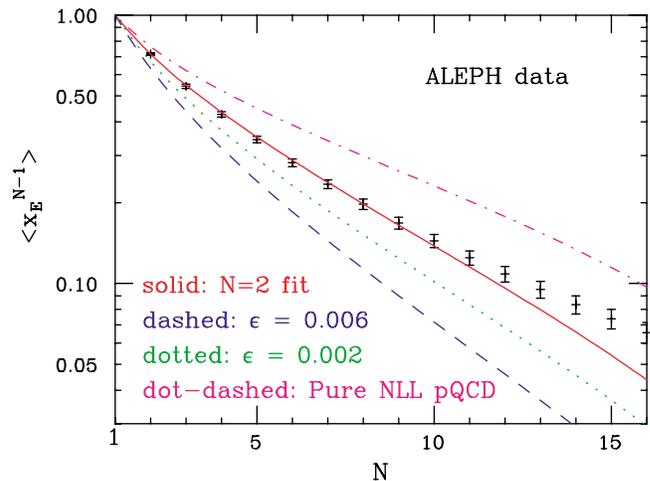


FIG. 1 (color online). Moments of the measured  $B$  meson fragmentation function, compared with the perturbative NLL calculation supplemented with different  $D(z)$  nonperturbative fragmentation forms. The solid line is obtained using a one-parameter form fitted to the second moment.

superimposed to the data. All of them have been obtained with an underlying NLL perturbative description [23,29]. The bottom quark mass  $m$  has been taken equal to 4.75 GeV and the QCD scale has been fixed to  $\Lambda^{(5)} = 0.226$  GeV. Sudakov resummation has not been included, since its effect is negligible in the low-moment region [29]. These are the default values of the parameters that we shall use in this work for the computation of the hadronic cross section.

The dot-dashed line represents the purely perturbative part. The dashed line represents the convolution of the perturbative part described above with a Peterson form with  $\epsilon = 0.006$ . It is evident that this produces a poor description of even the lowest moments. The dotted line is obtained using  $\epsilon = 0.002$ , a value known to produce good fits of the  $x_E$  distribution when used together with a NLL perturbative calculation [27,28]. The description of the moments improves, but the line still cannot fall within the error bars. There is thus a problem in obtaining a good fit of the low moments of the fragmentation function using the Peterson parametrization. The problem can be traced back to the need to fit points with very large  $x_E$  (where most of the  $e^+e^-$  data is) since there the perturbative calculation becomes less reliable. Normally, the very large  $x_E$  region is excluded from the fit because of this reason. The computed cross section is thus allowed to become negative in this region, a fact that leads to an underestimate of the low moments.

It should be clear from the aforementioned arguments that, in order to make accurate predictions for hadronic cross sections, the nonperturbative part of the fragmentation should be fitted in such a way that the low moments are well reproduced. This is shown in the solid line in the figure. A one-parameter form of the nonperturbative

fragmentation function has been used [34], and its free parameter has been fixed by fitting the  $N = 2$  point in moments space, i.e., the average energy fraction  $\langle x_E \rangle$ . In this case, the functional form is good enough to describe well the experimental data up to  $N \approx 10$ . It is therefore a good candidate to be employed in the calculation of the hadronic cross section according to Eq. (1). We shall refer to this fit in the following as the “ $N = 2$  fit.”

We notice that the effect of nonperturbative fragmentation (i.e., the ratio of the dashed, dotted, and solid curves with the dot-dashed curve) is considerably reduced when the moments are fitted. Thus, perturbation theory alone gives a much better description of the low moments of the fragmentation function, rather than of its shape in  $x$  space, requiring less nonperturbative input. This is a consequence of the fact that, at large  $x$ , many enhanced nonperturbative contributions and hadronization effects due to the limited phase space come into play. Indeed, the form of the leading power correction in moment space is well known, and reads [36]

$$D_N = 1 - (N - 1) \frac{\bar{\Lambda}}{m} + \mathcal{O}\left(\frac{\bar{\Lambda}^2}{m^2}\right). \quad (3)$$

It can easily be checked that the form we employed in the  $N = 2$  fit is consistent with this leading power correction, provided one replaces  $\alpha$  with  $2m/\bar{\Lambda}$ . One can also clearly see the nonperturbative correction to be minimal for  $N = 2$ . It is therefore always desirable to study moments, rather than the  $x$  shape of the fragmentation function, also in order to perform QCD studies.

In Figs. 2 and 3, we show the final prediction for  $B$  hadroproduction at the Tevatron, obtained by the procedure outlined above. It is clearly shown how the

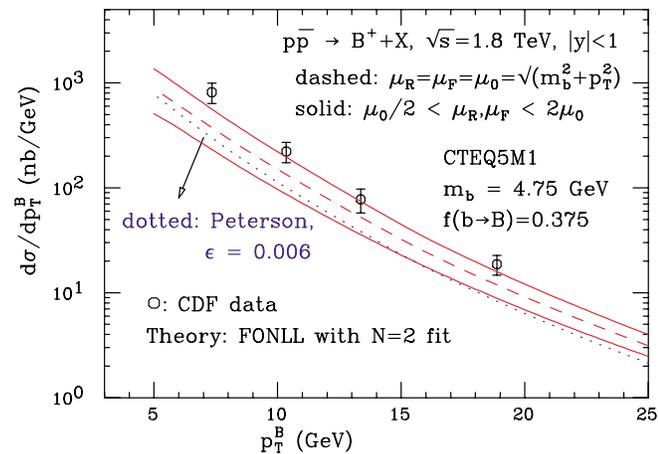


FIG. 2 (color online). Prediction for the  $B$  cross section, obtained using the calculation of Ref. [22] supplemented with the  $N = 2$  fit of the nonperturbative fragmentation function, compared to the CDF data of Ref. [10]. For comparison, the result obtained using a Peterson form with  $\epsilon = 0.006$  is also shown.

“Peterson with  $\epsilon = 0.006$ ” choice underestimates the  $B$  cross section at large values of  $p_T$ . It is also clear that the claimed discrepancy of a factor of 2.9 is now reduced to a factor of 1.7 with respect to the central value prediction. The band obtained by varying the scales gives an idea of the theoretical error involved, and, as one can see, the data are not far above it.

At this point, we wish to quantify to what extent the various ingredients of the present calculation affect the computed cross section, so that it is in fact larger than the one given in Ref. [10]. This is shown in Fig. 4, where we normalize the curves to the FO (fixed order) calculation with  $\epsilon = 0.006$ . Using the more appropriate value  $\epsilon = 0.002$  brings about a 20% increase of the cross section at  $p_T = 20$ . Using the FONLL calculation of Ref. [22] brings about another 20% increase, and so does also the use of the  $N = 2$  fit. The total effect is an increase by a factor of  $1.2^3 \approx 1.7$ , which turns the factor of 2.9 reported in Ref. [10] into the 1.7 observed here.

Our FO,  $\epsilon = 0.006$  cross section is also higher than the one presented in Ref. [10] in the low  $p_T$  region. This difference could be due to the different possible treatments of fragmentation at small transverse momentum. We have applied the fragmentation to the momentum, rather than the energy or the  $+$  component, of the fragmenting particle. We believe that these other choices, although acceptable in the large- $p_T$  region, are not appropriate in the nonrelativistic limit.

The SLD experiment has also published accurate data on the  $b$  fragmentation function [37]. Using their data instead of the ALEPH ones brings about a slight decrease of the cross section, below 4% in the region of interest. On the other hand, using more recent parton distribution function sets [38], we find an increase of the predicted cross section between 4% and 8% in the region of interest.

In summary, we find that an appropriate treatment of the fragmentation properties of the  $b$  quark considerably

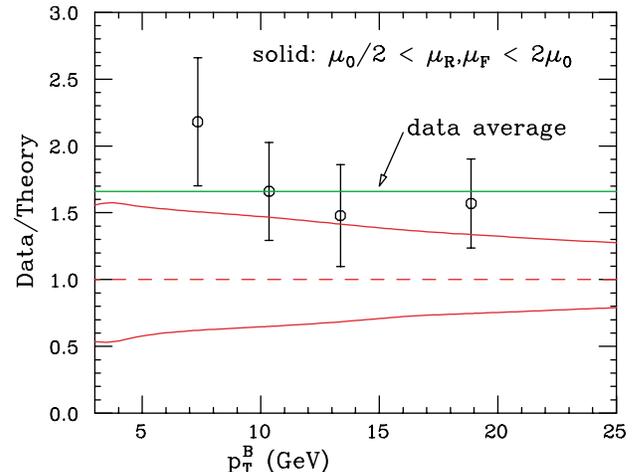


FIG. 3 (color online). Data over theory ratio for  $B$  production. Data points and theoretical curves are as in Fig. 2

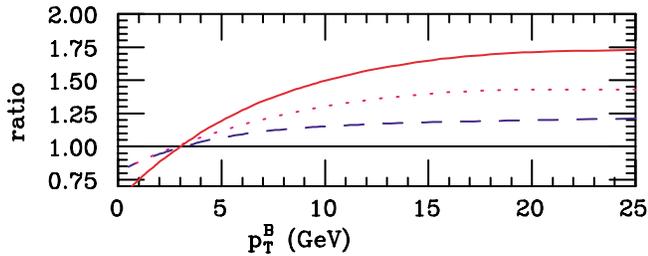


FIG. 4 (color online). The effect of the different ingredients in the calculation presented in this work, normalized to a fixed order calculation with Peterson fragmentation and  $\epsilon = 0.006$ . Dashed line: FO,  $\epsilon = 0.002$ ; dotted line: FONLL,  $\epsilon = 0.002$ ; solid line: FONLL,  $N = 2$  fit.

reduces the discrepancy of the CDF transverse momentum spectrum for the  $B$  mesons and the corresponding QCD calculation. The experimental points are compatible with predictions obtained using the present value of the QCD scale parameter and of the structure functions, and a  $b$  pole mass of 4.75 GeV, and lie near the upper region of the theoretical band obtained by varying the factorization and renormalization scales. Including experimental and theoretical uncertainties, the updated data/theory ratio can be written as  $1.7 \pm 0.5$  (experiment)  $\pm 0.5$  (theory). The calculation we have adopted includes in a consistent way fixed order QCD results and the NLL resummation of transverse momentum logarithms. Furthermore, the “moments” method introduced here avoids the difficult large- $x$  region in the fragmentation function, which would require more complex treatment and introduce further uncertainties.

We also observe that our findings are consistent with the good agreement found in comparing data [39] and theoretical results [40] for jets containing a bottom quark, an observable which is less dependent on the hadronization properties of the heavy quark.

While we have here convoluted a perturbative prediction to get a hadron-level result, the opposite path is followed by experiments when they deconvolute their hadron-level cross sections in order to publish quark level data [1–6]. In light of what was argued in this Letter, and of the apparent excess also shown by those data, it will be advisable to investigate whether a similar bias might have affected those results.

In the meantime, we emphasize the importance of the direct measurements of the moments of the fragmentation function for heavy quarks. Measuring directly moments instead of the  $x$  distribution could be useful for the purpose of QCD studies, and also for the computation of production cross sections in hadronic collisions.

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