Additional Stringy Sources for Electric Dipole Moments

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We show that string models with low energy supersymmetry which accommodate the fermion mass hierarchy generally give nonuniversal soft trilinear couplings (A terms). In conjunction with the apparently large Cabibbo-Kobayashi-Maskawa (CKM) phase, this results in large fermion electric dipole moments (EDMs) even in the absence of CP violating phases in the supersymmetry breaking auxiliary fields and the μ term. Nonobservation of the EDMs therefore implies that strings select special flavor and/or supersymmetry breaking patterns.

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Recent measurements of the CP asymmetry in the $B \to \Psi K_s$ decay [1] imply that CP is significantly violated in nature and is not an approximate symmetry. Supersymmetric models with large CP phases generally predict fermion electric dipole moments (EDMs) that far exceed the experimental limits [2], and this presents a major challenge for low energy supersymmetry, the so-called SUSY CP problem.

This problem arises because there is no *a priori* reason for the SUSY breaking dynamics to conserve CP. In particular, in supergravity models the auxiliary fields that break supersymmetry (F_S,F_T, etc.) are in general complex, which leads to complex soft SUSY breaking parameters. These new CP phases, which are absent in the nonsupersymmetric case, induce large EDMs. Conventionally, this is believed to be the source of the CP problem in this class of models.

In this Letter, we point out that there is another source of EDMs inherent in more fundamental models such as supersymmetric models derived from strings. It is present even if the SUSY breaking (F_S, F_T, etc.) *conserves CP*, and has its origin in the flavor structures. String theory, as a fundamental theory, has to explain the observed fermion mass hierarchy and mixings. This generally leaves an imprint on the flavor structure of the soft SUSY breaking terms and, when combined with the phenomenological requirement that the Yukawa matrices contain $\mathcal{O}(1)$ CP phases, results in unacceptably large EDMs. The source of the EDMs lies in the CP phases present already in the nonsupersymmetric case, i.e., those in the Yukawa matrices, which are rendered observable by supersymmetry.

Nonobservation of EDMs provides a strong constraint on the fundamental flavor structure of the Yukawas and/or supersymmetry breaking, *in addition* to constraints on any *new CP* phases that may occur in the latter, and indicates that the fundamental theory selects quite special patterns of the flavor structures or supersymmetry breaking.

Let us start with the general arguments that imply nonuniversality in the soft trilinear terms relevant to the EDM calculations. In all supergravity models, the soft SUSY breaking parameters are given in terms of the Kähler potential K and the superpotential W. In particular, the trilinear parameters are written as [3]

$$A_{\alpha\beta\gamma} = F^m[\hat{K}_m + \partial_m \log Y_{\alpha\beta\gamma} - \partial_m \log(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma)]. \tag{1}$$

Here the Latin indices refer to the hidden sector fields while the Greek indices refer to the observable fields; the Kähler potential is expanded in observable fields as $K = \hat{K} + \tilde{K}_\alpha |C^\alpha|^2 + \ldots$ and $\hat{K}_m \equiv \partial_m \hat{K}$. The sum in m runs over all of the SUSY breaking fields. We note that the supergravity notation for the A terms and Yukawas can be connected to the "usual" one by fixing the order of the indices as follows: the first index is to refer to the Higgs fields, the second to the quark doublets, and the third to the quark singlets, e.g., $Y_{H_1 Q_i D_j} \equiv Y^d_{ij}$.

Since string theory is a fundamental theory, the Yukawa matrices must be generated dynamically. That is, the Yukawa couplings are functions of fields that dynamically get vacuum expectation values leading to the observed fermion masses and mixings (in the same way that the VEV of the dilaton field produces the gauge coupling). Also, since CP is a gauge symmetry in strings [4] and can be broken only spontaneously, the fields responsible for generating the Yukawa couplings must attain CP-violating VEVs in order to produce the Cabibbo-Kobayashi-Maskawa (CKM) phase. Therefore, the derivatives of the Yukawa couplings in Eq. (1) will generally be nonzero and should be taken into account. As we will see, this contribution is often significant even if the corresponding SUSY breaking component F^m is small.

To estimate how large these contributions are expected to be, we will consider two possible ways to generate the Yukawa hierarchies: through exponential factors, as in the heterotic string, and through nonrenormalizable operators, as, for example, in type I or other models.

In weakly coupled heterotic orbifolds, the Yukawa couplings are calculable and given in terms of the T-moduli fields. The fermion mass hierarchy has a geometrical origin and appears due to the fact that interaction of the states placed at different orbifold fixed points is exponentially suppressed [5]. The matter fields must belong to the twisted sectors since otherwise the couplings are 1 or 0 and the mass hierarchy cannot be generated at the renormalizable level. The Yukawa coupling of the states at the fixed points $f_{1,2,3}$ belonging to the twisted sectors $\theta_{1,2,3}$ is given by [6]

$$Y_{f_1 f_2 f_3} = N \sum_{u \in Z^n} \exp[-4\pi T(f_{23} + u)^T M(f_{23} + u)], \quad (2)$$

where $f_{23} \equiv f_2 - f_3$, N is a normalization factor, and the matrix M (with fractional entries) is related to the internal metric of the orbifold. Here T is normalized such that $T \to T + i$ is a symmetry of the Lagrangian [7]. For a realistic case, $\mathrm{Re}T = \mathcal{O}(1)$, the sum is dominated by one term and

$$\partial_T \ln Y_{f_1 f_2 f_3} \simeq -4\pi f_{23}^T M f_{23} \quad (3)$$

for some (typically fractional) f_{23} depending on the positions of the fixed points. This expession is independent of T and can be evaluated for various orbifolds. It vanishes if the fixed points coincide, i.e., there is no suppression of the Yukawa interaction. In other cases it is of order 1 (or larger, see, e.g., [8]). This creates a significant nonuniversality in the A terms unless $F_T \simeq 0$. Indeed, to produce a fermion mass hierarchy we are bound to place different quark generations at different orbifold fixed points. Thus, the expression (3) will be generation dependent. The other contributing terms in Eq. (1), $\hat{K}_T = -3/(T + \bar{T})$ and $\partial_T \log(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma) = (n_\alpha + n_\beta + n_\gamma)/(T + \bar{T})$ are generation independent since the sum of the modular weights n_i is fixed by the point group selection rule for the Yukawa interactions. Unless we encounter a special case $F_T \simeq 0$, the nonuniversality will be present. We thus conclude that the A term nonuniversality is a generic feature of realistic heterotic models.

In general, it might be too strict a requirement to demand that all of the required features appear at the renormalizable level. In many string models the renormalizable Yukawa couplings are either unknown or are 0,1 (e.g., type I models). Thus, to reproduce the observed fermion masses, nonrenormalizable operators must be taken into account. The mass hierarchy is then created via powers of a small (in Planck units) VEV of a certain field ϕ [9,10], i.e.,

$$Y_{\alpha\beta\gamma} \sim \phi^{q_{\alpha\beta\gamma}}. \quad (4)$$

Here $q_{\alpha\beta\gamma}$ is an integer and can sometimes be associated with a U(1) charge (as in the Froggatt-Nielsen mechanism). Clearly, if this field breaks supersymmetry, $F_\phi \neq 0$, the generated A terms will be nonuniversal:

$$\partial_\phi \ln Y_{\alpha\beta\gamma} = \frac{q_{\alpha\beta\gamma}}{\phi}. \quad (5)$$

Generally, ϕ is expected to give a (small) contribution to supersymmetry breaking. To estimate its natural size, let us recall that the Kähler potential for untwisted fields is given by $K = -\ln(S + \bar{S}) - 3\ln(T + \bar{T} - \phi\bar{\phi})$. The resulting auxiliary field is then

$$F_\phi = m_{3/2} \frac{T + \bar{T} - \phi\bar{\phi}}{3} \left[\bar{\phi} \frac{W_T}{W} + \frac{W_\phi}{W} \right]. \quad (6)$$

A similar result holds for a twisted ϕ. For small ϕ, F_ϕ is typically of the order of $\phi m_{3/2}$ (see also, e.g., [10]). As a result, the Yukawa-induced contribution to the A terms is

$$\Delta A_{\alpha\beta\gamma} \sim q_{\alpha\beta\gamma} m_{3/2}. \quad (7)$$

The "charges" $q_{\alpha\beta\gamma}$ are generation dependent and order 1, so the resulting nonuniversality is very significant. Again, we come to the conclusion that realistic models of the fermion masses entail nonuniversal A terms. We note that additional nonuniversal contributions may come from the Kähler potential. However, these are not direcly related to the Yukawa structures, so we do not discuss them here.

On general grounds, it is natural to expect that once nonuniversality is created in the Yukawa interactions, it will also appear in the analogous soft-breaking terms. This leads to large EDMs even if the SUSY breaking dynamics do not generate new CP phases, i.e., F_S, F_T, etc., and the μ term are real. To see this let us suppose that initially there are no CP-violating phases in any of the soft-breaking parameters. At first sight this seems to avoid overproduction of the EDMs. However, this is not the case. Indeed, the relevant trilinear interactions are

$$\Delta \mathcal{L}_{\mathrm{soft}} = -\frac{1}{6} A_{\alpha\beta\gamma} Y_{\alpha\beta\gamma} C^\alpha C^\beta C^\gamma. \quad (8)$$

Denoting $\hat{A}_{\alpha\beta\gamma} \equiv A_{\alpha\beta\gamma} Y_{\alpha\beta\gamma}$ and identifying $\hat{A}_{H_1 Q_i D_j} \equiv \hat{A}^d_{ij}$, it is easy to see that generally $\hat{A}$ and Y matrices are "misaligned." In the basis where the Yukawas are diagonal and real (the super-CKM basis), the $\hat{A}$ matrices are not diagonal, while their diagonal elements contain complex phases. Under the superfield rotation $\hat{U}_{L,R} \to V^u_{L,R} \hat{U}_{L,R}$, $\hat{D}_{L,R} \to V^d_{L,R} \hat{D}_{L,R}$ to the super-CKM basis, i.e., $Y^u \to V_L^{u\dagger} Y^u V_R^u = \mathrm{diag}(h_u, h_c, h_t)$, $Y^d \to V_L^{d\dagger} Y^d V_R^d = \mathrm{diag}(h_d, h_s, h_b)$, the $\hat{A}$ matrices transform as

$$\hat{A}^u \to V_L^{u\dagger} \hat{A}^u V_R^u, \qquad \hat{A}^d \to V_L^{d\dagger} \hat{A}^d V_R^d. \quad (9)$$

Since the Yukawa matrices contain $\mathcal{O}(1)$ CP phases, so do the rotation matrices $V_{L,R}$. Therefore, the rotation by itself can induce diagonal CP phases even if the A or $\hat{A}$ matrices initially are completely real. Furthermore, due to nonuniversality the diagonal elements of $\hat{A}$ will have contributions from all three generations, e.g.,

$$\hat{A}^u_{11} \propto m_u + \epsilon m_c + \epsilon' m_t. \tag{10}$$

This is to be contrasted with the universal case when the Yukawas and $\hat{A}$ matrices are diagonalized simultaneously and $\hat{A}_{ii}$ are proportional to m_i. The imaginary parts of the diagonal entries of the $\hat{A}$ matrix in the super-CKM basis induce quark EDMs via the gluino-squark loops with LR mass insertions. Equation (10) implies that the magnitude of the mass insertions contributing to the EDMs, $(\delta^{u,d}_{LR})_{ii} \sim \hat{A}^{u,d}_{ii}\langle H_{u,d}\rangle/\tilde{m}^2$, is significantly enhanced and even a small *CP* phase can result in large EDMs.

To illustrate the strength of this effect, we present the scatter plots (Figs. 1 and 2) of the neutron and mercury EDMs versus *real A* terms. We take representative Yukawa textures (similar to those of Ref. [11] but possessing no symmetry) with *conservative* flavor mixing, i.e., $(V_{L,R})_{13} \sim V^{\rm CKM}_{td} \sim \mathcal{O}(10^{-2})$, $(V_{L,R})_{12} \sim \sin\theta_C$, and containing order 1 complex phases. We fix the universal scalar mass m_0 and $m_{1/2}$ to be 200 GeV (i.e., $m_{\tilde{q}} \sim m_{\tilde{g}} \sim$ 500 GeV at M_Z) and $\tan\beta = 3$. We write $A^{u,d}_{ij} = m_0 + \delta A x_{ij}$ and vary δA from 0 to $3m_0$ with x_{ij} randomly selected in the range $[-1, 1]$. To calculate the mercury EDM we use the results of Ref. [12] and the bounds on the mass insertions of Ref. [13]. The neutron EDM requires $\mathrm{Im}(\delta^{u,d}_{LR})_{11} \leq \mathcal{O}(10^{-6})$, whereas the bound from the mercury EDM is $\mathrm{Im}(\delta^{u,d}_{LR})_{11} \leq \mathcal{O}(10^{-7})$.

We see that the EDMs exceed the experimental bounds by up to 3 orders of magnitude. In order sufficiently to suppress this effect, the sfermion masses must be pushed up above 10 TeV. This would, however, create a large hierarchy between the electroweak and the SUSY scales leading to a large fine-tuning [13], so one of the primary motivations for supersymmetry would be lost.

The effect described above seems quite close in spirit to the notorious FCNC problem. Yet, it is different from the latter since even if the FCNC are suppressed sufficiently, the EDM problem will still persist. Indeed, the strongest FCNC constraints come from the kaon physics and, for instance, ϵ_K requires $\sqrt{|\mathrm{Im}(\delta^d_{LR})^2_{12}|} \leq 3.5 \times 10^{-4}$ for the above parameters [14]. This bound is satisfied automatically in most supergravity models with $\mathcal{O}(1)$ nonuniversality in the *A* terms because these mass insertions are suppressed by the quark masses. In contrast, the level of sensitivity of the EDMs is $\mathcal{O}(10^{-7})$ and even a 10% nonuniversality can overproduce EDMs by an order of magnitude (Figs. 1 and 2).

To be specific, let us consider an example. In the Z_6-I orbifold model of Refs. [7,8], the Higgs fields belong to the θ twisted sector, quark doublets—to the θ^2 sector, and quark singlets—to the θ^3 sector (for the fixed point assignment, see [7]). The squark masses are generation independent since different generations belong to the same twisted sector. The resulting *A* terms depend on the Goldstino angle Θ which measures the balance between the dilaton and moduli contributions to SUSY breaking. For $|T| \sim 1$ and a conservative value $\tan\Theta \sim -2$ which corresponds to mostly dilaton SUSY breaking ($F_S/F_{T_i} \sim 10$), the arising *A* term texture is

$$A^d \sim -\sqrt{3}m_{3/2}\begin{pmatrix} 0 & 0 & -2 \\ 1 & 1 & 3 \\ -1 & 2 & 1 \end{pmatrix}. \tag{11}$$

Here $m_{3/2}$ is the gravitino mass and we have assumed real F_S and F_T. The resulting mass insertions are $(\delta^d_{LR})_{11} \simeq -3\times 10^{-5} - 3\times 10^{-5}i$, $(\delta^d_{LR})_{12} \simeq -2\times 10^{-5} + 2\times 10^{-5}i$, $|(\delta^d_{LR})_{23}| \sim \mathcal{O}(10^{-4})$, $|(\delta^d_{LR})_{13}| \sim \mathcal{O}(10^{-5})$. Thus, the SUSY contribution to $K - \bar{K}$ mixing is negligible (only the contribution to ϵ' is significant). The FCNC bounds involving the third generation are much looser and easily satisfied. On the other hand, the neutron and mercury EDMs are overproduced by 1–2 orders of magnitude. We found that these results are qualitatively stable under variation of the Goldstino angle. Clearly, this situation is quite general and holds in most models with $\mathcal{O}(1)$ nonuniversality in the *A* terms.

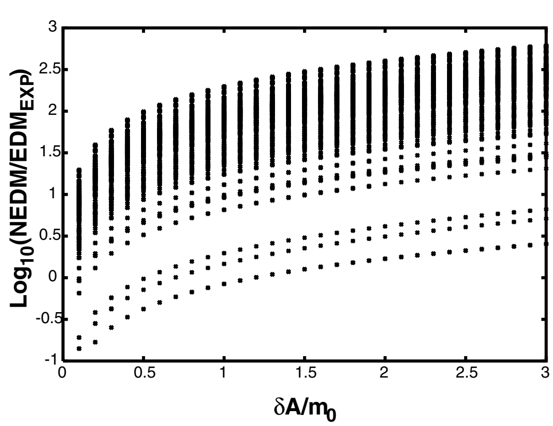

FIG. 1. Neutron EDM versus δA.

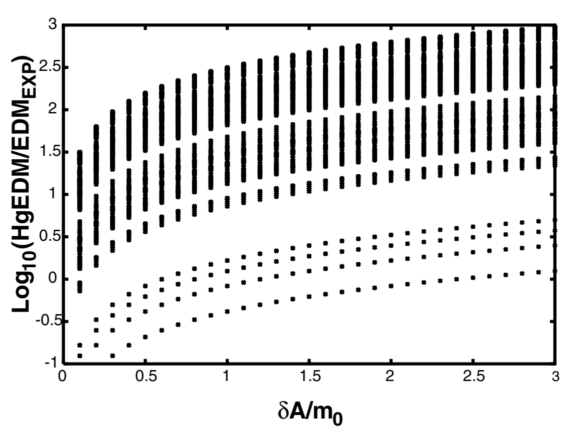

FIG. 2. Mercury EDM versus δA.

The additional EDM contributions emerge due to the fact that string theory, as a fundamental theory, must explain the fermion mass patterns as well as accommodate a large CKM phase. The underlying mechanism generally affects the soft terms thereby inducing large EDMs. The two sectors, the standard model and the supersymmetric sector, which can be assumed to be disconnected in the context of the minimal supersymmetric standard model, are tightly related in string models and it is the absence of *CP* violation in one of them and its presence in the other that leads to the conflict.

In this context, EDMs serve as a probe of supersymmetric flavor physics as well as supersymmetry breaking. Nonobservation of the EDMs implies that string theory selects special flavor structures or a special pattern of SUSY breaking. The nonuniversal contributions to the EDMs are absent if the A terms happen to be *universal*. This requires that (i) the fields that generate the Yukawa matrices do not break SUSY,

$$F_T \simeq 0, \qquad F_\phi \simeq 0, \tag{12}$$

and (ii) that there are no generation-dependent contributions from the Kähler potential in Eq. (1). This occurs if different moduli are responsible for SUSY breaking and generating the Yukawa couplings, yet it remains to be understood why the latter moduli do not break supersymmetry. Exact universality can be a result of dilaton-dominated SUSY breaking. This scenario is known to lead to charge and color breaking minima which, however, can be avoided by lowering the string scale to intermediate values. On the other hand, dilaton domination would be hard to implement in semirealistic string models which exhibit *CP* violation and dilaton stabilization [8,15].

The problem can also be avoided if the flavor structures are *Hermitian*,

$$Y^a = Y^{a\dagger}, \qquad A^a = A^{a\dagger}. \tag{13}$$

In this case, the $\hat{A}$ matrices are also Hermitian and, since $V_L^a = V_R^a$, the Hermiticity is preserved by the basis rotation [11]. This eliminates the flavor-diagonal phases in the A terms. The non-Hermitian corrections appear only due to small renormalization group effects and lead to naturally suppressed EDMs. To justify the Hermiticity requires the presence of an additional symmetry [16] and it is a nontrivial task to embed such models in string theory.

The problem becomes milder (yet it exists) if one assumes that the up Yukawa matrix is diagonal while the down one is proportional to the CKM matrix. In this case, the *CP* phase appears only in the (13) and (31) entries of the down Yukawa matrix. This suppresses the effect of the super-CKM rotation, yet the mercury EDM often exceeds the limit by about an order of magnitude. Another interesting possibility is to have *matrix*-factorizable A terms: $\hat{A} = A \cdot Y$ or $Y \cdot A$. This suppresses the magnitude of the mass insertions since now there is a contribution from only one generation, $(\delta_{LR})_{ii} \propto m_i$. However, again it is not clear whether these special cases can be obtained in realistic string models. In any case, the above considerations provide a selection rule: any realistic string model must satisfy the requirement of absence of the nonuniversal contributions to the EDMs.

To summarize, we have argued that string models which accommodate the fermion mass hierarchy and mixings generally lead to large EDMs even in the absence of *CP* phases in the SUSY breaking auxiliary fields and the μ term. Nonobservation of the EDMs implies that the supersymmetric structures have a special flavor or SUSY breaking pattern, which can be probed in current and future particle experiments.

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