

## Does a Randall-Sundrum Scenario Create the Illusion of a Torsion-Free Universe?

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We consider spacetime with torsion in a Randall-Sundrum scenario where torsion, identified with the rank-2 Kalb-Ramond field, exists in the bulk together with gravity. While the interactions of both graviton and torsion in the bulk are controlled by the Planck mass, an additional exponential suppression comes for the torsion zero-mode on the visible brane. This may serve as a natural explanation of why the effect of torsion is so much weaker than that of curvature on the brane. The massive torsion modes, on the other hand, are correlated with the corresponding gravitonic modes and may be detectable in TeV-scale experiments.

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Theories with large compact extra dimensions have gained considerable attention in recent times, primarily because of their role in solving the naturalness problem. A high point of such theories is the prediction of TeV-scale observable effects of the Kaluza-Klein (KK) modes of the gravitational field which is assumed to exist in the “bulk.” Such speculation can be broadly classified into two schools of thought, based essentially on the approaches of Arkani-Hamed, Dimopoulos and Dvali (ADD) [1] on one hand, and Randall and Sundrum (RS) [2] on the other. In both types of models, all visible matter is confined to a “3-brane” on which the projections of the bulk gravity give rise to KK modes. The spacings of these modes and their interactions with the SM fields are determined by specifics of the model.

In various extensions of the above models, implications of other types of bulk fields, such as scalars, gauge fields, and fermions, have been explored [3–7]. In this note, we examine what happens if bulk spacetime in an RS picture is endowed with both curvature and torsion, a possibility that is motivated from string theory [8].

The most straightforward way of including torsion is to add an antisymmetric component to the connection  $\Gamma_{\mu\nu}^{\alpha}$ . This is the essence of the so-called Einstein-Cartan theories [9]. Introduced in this manner, torsion can couple with all matter fields with spin. It can be easily seen that such interaction terms in general are of dimension five, and are suppressed by the Planck mass ( $M_P$ ), in a way similar to gravitonic couplings. Efforts have been on to constrain the torsion field and its coupling strength from a variety of considerations such as atomic energy level splitting [10] and the phenomena of optical activity in radiation from distant galactic sources [11,12]. However, it is not clearly understood from any *fundamental theoretical consideration* whether the coupling of torsion with visible matter should be different from that of curvature and, if so, why. This is precisely the question we address here, within the framework of an RS theory.

In its minimal version the RS scenario, defined in five dimensions [2], is characterized by the background metric

$$ds^2 = e^{-2\sigma} \eta_{\mu\nu} dx^\mu dx^\nu + r_c^2 d\phi^2, \quad (1)$$

with  $\eta_{\mu\nu} = (-, +, +, +)$  and  $\sigma = kr_c|\phi|$ .  $r_c$  is the compactification radius for the fifth dimension and  $k$  is on the order of the higher dimensional Planck mass  $M$ . The extra dimension, characterized by a variable  $\phi$  ranging from  $-\pi$  to  $+\pi$ , forms an  $S_1/Z_2$  orbifold. The standard model fields reside at  $\phi = \pi$ , while gravity peaks at  $\phi = 0$ . The dimensional parameters defined above are related to the four-dimensional Planck scale  $M_P$  through the relation

$$M_P^2 = \frac{M^3}{k} [1 - e^{-2kr_c\pi}]. \quad (2)$$

Clearly,  $M_P$ ,  $M$ , and  $k$  are all of the same order of magnitude. For  $kr_c \simeq 12$  the exponential factor (frequently referred to as the “warp factor”) produces TeV-scale mass parameters (of the form  $m = Me^{-kr_c\pi}$ ) on the “standard model” brane. Thus the hierarchy between the Planck and TeV scales is achieved without fine-tuning.

It is well known that the likely source of torsion is some matter field(s) with spin, just as curvature is associated with mass/energy. Attempts have been made in some earlier works [13,14] to relate torsion with fermion fields residing either on the brane or in the bulk. Here we take the standpoint that, being as much a characteristic of spacetime as curvature, it is natural for torsion to coexist with gravity in the bulk. We have earlier performed some analyses in this line in the context of an ADD model [15].

In the scenario adopted by us, the source of torsion is taken to be the rank-2 antisymmetric Kalb-Ramond (KR) field  $B_{MN}$  which arises as a massless mode in heterotic string theories [8]. To understand the above statement, let us recall that the low energy effective action for the gravity and electromagnetic sectors in  $D$  dimensions is given by

$$S = \int d^D x \sqrt{-G} \left[ R(G) - \frac{1}{4} F_{MN} F^{MN} + \frac{3}{2} H_{MNL} H^{MNL} \right]. \quad (3)$$

It has been shown earlier [16] that an action of the form

$$S = \int d^D x \sqrt{-G} \left[ R(G, T) - \frac{1}{4} F_{MN} F^{MN} - \frac{1}{2} H_{MNL} H^{MNL} + T_{MNL} H^{MNL} \right] \quad (4)$$

reproduces the low energy string effective action if one eliminates the torsion field  $T_{MNL}$  (which is an auxiliary field) by using the equation of motion  $T_{MNL} = H_{MNL}$ .

Thus torsion can be identified with the rank-3 antisymmetric field strength tensor  $H_{MNL}$  which in turn is related to the KR field  $B_{MN}$  [17] as

$$H_{MNL} = \partial_{[M} B_{NL]}, \quad (5)$$

with each Latin index running from 0 to 4. (Greek indices, on the other hand, run from 0 to 3.) Furthermore, we use the KR gauge fixing conditions to set  $B_{4\mu} = 0$ . Therefore, the only nonvanishing KR field components correspond to the brane indices. These components, of course, are functions of both compact and noncompact coordinates.

The five-dimensional action for the curvature-torsion sector in this case is

$$S_G = \int d^4 x \int d\phi \sqrt{-G} 2M^3 R(G, H), \quad (6)$$

where  $G_{MN}$  is given by Eq. (1) and  $R(G, H)$  is the scalar curvature constructed from the modified affine connection,

$$\bar{\Gamma}_{NL}^K = \Gamma_{NL}^K - \frac{1}{M^2} H_{NL}^K. \quad (7)$$

Clearly, the action can be decomposed into two independent parts—one consisting of pure curvature, and the other consisting of torsion:

$$S_G = \int d^4 x \int d\phi \sqrt{-G} 2[M^3 R(G) - H_{MNL} H^{MNL}]. \quad (8)$$

Thus the five-dimensional action corresponding to the Kalb-Ramond field, up to a dimensionless multiplicative constant, is given by

$$S_H = \int d^4 x \int d\phi \sqrt{-G} H_{MNL} H^{MNL}. \quad (9)$$

Using the explicit form of the RS metric, and remembering that  $B_{4\mu} = 0$ , we have

$$S_H = \int d^4 x \int d\phi r_c e^{2\sigma(\phi)} \left[ \eta^{\mu\alpha} \eta^{\nu\beta} \eta^{\lambda\gamma} H_{\mu\nu\lambda} H_{\alpha\beta\gamma} - \frac{3}{r_c^2} e^{-2\sigma(\phi)} \eta^{\mu\alpha} \eta^{\nu\beta} B_{\mu\nu} \partial_\phi^2 B_{\alpha\beta} \right]. \quad (10)$$

Next, we consider Kaluza-Klein decomposition for the Kalb-Ramond field:

$$B_{\mu\nu}(x, \phi) = \sum_{n=0}^{\infty} B_{\mu\nu}^n(x) \frac{\chi^n(\phi)}{\sqrt{r_c}}. \quad (11)$$

In terms of the four-dimensional projections  $B_{\mu\nu}^n$ , an effective action of the form

$$S_H = \int d^4 x \sum_{n=0}^{\infty} \left[ \eta^{\mu\alpha} \eta^{\nu\beta} \eta^{\lambda\gamma} H_{\mu\nu\lambda}^n H_{\alpha\beta\gamma}^n + 3m_n^2 \eta^{\mu\alpha} \eta^{\nu\beta} B_{\mu\nu}^n B_{\alpha\beta}^n \right] \quad (12)$$

can be obtained provided

$$-\frac{1}{r_c^2} \frac{d^2 \chi^n}{d\phi^2} = m_n^2 \chi^n e^{2\sigma} \quad (13)$$

and subject to the orthonormality condition

$$\int e^{2\sigma(\phi)} \chi^m(\phi) \chi^n(\phi) d\phi = \delta_{mn}, \quad (14)$$

where  $H_{\mu\nu\lambda}^n = \partial_{[\mu} B_{\nu\lambda]}^n$  and  $\sqrt{3}m_n$  gives the mass of the  $n$ th mode. In terms of  $z_n = \frac{m_n}{k} e^{\sigma(\phi)}$ , Eq. (11) can be recast in the form

$$\left[ z_n^2 \frac{d^2}{dz_n^2} + z_n \frac{d}{dz_n} + z_n^2 \right] \chi^n = 0. \quad (15)$$

The above equation admits the following solution:

$$\chi^n = \frac{1}{N_n} [J_0(z_n) + \alpha_n Y_0(z_n)], \quad (16)$$

where  $J_0(z_n)$  and  $Y_0(z_n)$  are, respectively, Bessel and Neumann functions of the order of 0.  $\alpha_n$  as well as  $m_n$  can be determined from the continuity conditions for the derivative of  $\chi_n$  at  $\phi = 0$  and  $\pi$ , which are dictated by self-adjointness of the left-hand side of Eq. (11). On using the fact that  $e^{kr_c\pi} \gg 1$  and the mass values  $m_n$  on the brane should be on the order of the TeV-scale ( $\ll k$ ), we obtain from the continuity condition at  $\phi = 0$

$$\alpha_n \simeq x_n e^{-2kr_c\pi}, \quad (17)$$

with  $x_n = z_n(\pi)$ . The boundary condition at  $\phi = \pi$  gives

$$J_1(x_n) \simeq \frac{\pi}{2} x_n e^{-2kr_c\pi}. \quad (18)$$

Since the right-hand side of the above equation is very small, the roots can be closely approximated to the zeros of

$J_1(x_n)$ . These roots of  $J_1$  give  $m_n$  on the TeV range, as expected initially.

Since  $x_n \approx 1$ , from Eq. (15)  $\alpha_n$  becomes  $\ll 1$ . The normalization condition yields

$$N_n = \frac{\pi}{2\sqrt{kr_c}} x_n e^{-kr_c \pi}. \quad (19)$$

Thus the final solution for the massive modes turns out to be

$$\chi^n(z_n) = \frac{2\sqrt{kr_c}}{\pi x_n} e^{kr_c \pi} J_0(z_n). \quad (20)$$

At this point it is useful to compare the solutions with those for bulk gravitons [18] and gauge fields [4]. First, here the massive solutions are governed by zeroth-order Bessel functions, as against those of second and first orders in the two other cases. Furthermore, a comparison with the above references shows that, whereas a massive gravitonic mode contains the same exponential enhancement factor as that in Eq. (18), it is absent in the case of bulk gauge fields. This difference can be attributed essentially to the tensorial structures of the different types of bulk fields as well as to the characteristic forms of the four-dimensional effective actions into which the theory must reduce in the different cases.

As Table I shows, the mass spectrum here is correlated with the masses of the gravitonic modes. This is because the masses in both cases are effectively given, up to an overall factor of  $\sqrt{3}$ , by the zeros of  $J_1(x_n)$  [as we have already noticed, the right-hand side of Eq. (16) is negligibly small]. Thus the scenario proposed here has an added

element of predictability as far as the graviton and torsion KK modes are concerned. The  $B_{\mu\nu}^n$  spacings ( $n = 1$  onwards) are just scaled with respect to the gravitonic modes by a factor of  $\sqrt{3}$  to a close approximation, and therefore the low-lying states in the spectrum should be within the reach of TeV-scale collider experiments.

However, a more drastic difference is noticed when we consider the massless mode. In this case the solution to Eq. (11) turns out to be

$$\chi^0(\phi) = c_1 |\phi| + c_2. \quad (21)$$

The condition of self-adjointness leaves the scope of only a constant solution. Using the normalization condition, one obtains

$$\chi^0 = \sqrt{kr_c} e^{-kr_c \pi}. \quad (22)$$

Thus, in contrast to the other types of bulk fields mentioned above, the zero-mode  $\chi^0$  exhibits a *suppression by a large exponential factor*. This causes the massless KR mode to be severely suppressed on the visible brane, granting a practically imperceptible presence to torsion.

This can be seen more clearly if we consider the coupling of torsion to matter fields on the visible brane. Let us, for example, consider the interaction with spin-1/2 fields [19]. Starting with a five-dimensional action and remembering that the fermion and all its interactions are confined to the brane at  $\phi = \pi$ , the fermionic action in terms of the modified affine connection is given by:

$$S_\psi = i \int d^4x \int d\phi [\det V] \bar{\psi} \left[ \gamma^a v_a^\mu \left( \partial_\mu - \frac{i}{2} G_{LN} \sigma^{ab} v_a^\nu \partial_\nu v_b^\lambda \delta_\nu^N \delta_\lambda^L - G_{AD} \sigma^{ab} v_a^\beta v_b^\delta \Gamma_{MB}^A \delta_\mu^M \delta_\beta^B \delta_\delta^D \right) \right] \psi \delta(\phi - \pi), \quad (23)$$

where  $G_{MN}$  is given by

$$G_{MN} = v_M^a v_N^b \eta_{ab}, \quad (24)$$

and the vierbein  $v_\mu^a$  is given in this case by

$$v_4^4 = 1, \quad v_\mu^a = e^{-\sigma} \delta_\mu^a, \quad \det V = e^{-4\sigma}, \quad (25)$$

$a, b$ , etc., being tangent space indices.

Integrating out the compact dimension and using the fact that the fermion field on the brane is consistently renormalized as  $\psi \rightarrow e^{3kr_c \pi/2} \psi$ , one obtains the effective four-dimensional fermion KR interaction as

$$\mathcal{L}_{\psi\bar{\psi}H} = -\bar{\psi} \left[ i \gamma^\mu \sigma^{\nu\lambda} \left\{ \frac{1}{M_P e^{kr_c \pi}} H_{\mu\nu\lambda}^0 + \frac{1}{\Lambda_\pi} \frac{2J_0(x_n)}{\pi x_n} \sum_{n=1}^{\infty} H_{\mu\nu\lambda}^n \right\} \right] \psi, \quad (26)$$

where  $H_{\mu\nu\lambda}^n = \partial_{[\mu} B_{\nu\lambda]}^n$  and  $\Lambda_\pi = M_P e^{-kr_c \pi}$ .

A rather remarkable fact becomes evident from above. Although gravity and torsion are treated at par on the bulk, with the Planck mass characterizing any dimensional parameter controlling their interactions, the coupling of the zero-mode torsion field with fermionic matter suffers an enormous additional suppression via the warp factor when one compactifies the extra dimension in the RS scheme. However, the massless graviton continues to have interactions driven by  $1/M_P$  on the brane. Therefore, the experimental signatures will be elusive so long as we are sitting on the brane, with the apparent feeling that we are living in a torsionless universe.

The massive modes  $H_{\mu\nu\lambda}^n$ , however, have enhanced coupling with matter, caused by the usual warp factor. The lowest-lying modes are in the TeV scale and their interaction strength is suppressed by a mass of similar magnitude. One can see observable effects of these modes through new resonances in TeV-scale accelerators.

TABLE I. The masses of a few low-lying gravitonic modes vis-a-vis the massive KR modes for  $kr_c = 12$  and  $k = 10^{19}$  GeV.

$n$	1	2	3	4
$m_n^{\text{grav}}$ (TeV)	1.66	3.04	4.40	5.77
$m_n^{\text{tor}}$ (TeV)	2.87	5.26	7.62	9.99

As far as the interaction of bulk torsion with standard model gauge fields is concerned, very similar effects can be seen. We show this in the context of the  $U(1)_{EM}$  gauge field. First it has to be remembered that gauge invariant interactions of this type cannot be obtained without introducing some amount of nonminimality into the theory. A rather convenient way of doing so is to augment the brane components of the torsion tensor  $H_{MNL}$  with a so-called Chern-Simons term [8],

$$H_{\mu\nu\lambda}(x, \phi = \pi) \rightarrow H_{\mu\nu\lambda}(x, \phi = \pi) + \frac{1}{M^{1/2}} A_{[\mu}(x) F_{\nu\lambda]}(x). \quad (27)$$

Such a term is natural in the context of gauge anomaly cancellation in a heterotic string theory [8]. On considering the KK modes  $B_{\mu\nu}^n$ , and appropriately redefining the electromagnetic field on the brane, one ends up with interaction terms of the following nature:

$$\mathcal{L}_{EM-H} = -\frac{1}{M_P e^{kr_c \pi}} A^{[\mu} F^{\nu\lambda]}(x) H_{\mu\nu\lambda}^0(x) - \frac{1}{\Lambda_\pi} \frac{2J_0(x_n)}{\pi x_n} A^{[\mu} F^{\nu\lambda]}(x) \sum_{n=1}^{\infty} H_{\mu\nu\lambda}^n(x), \quad (28)$$

where again we notice an additional suppression for the massless modes and an enhancement for the massive modes, just as in the case of spin-1/2 particles coupling to torsion. This extreme suppression of the zero-mode coupling could weaken torsion-induced optical activities, as suggested in some recent works [11,12].

The following picture emerges from the above analysis. Torsion, caused by a KR field, can be postulated to exist in the bulk in a five-dimensional RS scenario, together with gravity. At that level, only one mass parameter (of the order of the Planck scale) controls the actions for both gravity and torsion. The compactification of the fifth dimension gives rise to a spectrum of Kaluza-Klein modes for the KR field, just as in the case of gravity. However, the torsion zero-mode on the visible brane has an added suppression through the warp factor in its interaction with matter fields. This indicates that it is going to be nearly impossible to see its trace in any observation performed on the visible brane. Therefore, we shall continue to have the impression of residing in a torsionless universe if this scenario is correct. On the other hand, the massive KR spectrum gets corre-

lated with the spectrum of graviton, and their signals in TeV-scale accelerator experiments can make the hypothesis of bulk torsion verifiable.

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