Observation of Chaotic Dynamics in a Powerful Backward-Wave Oscillator

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Self-modulation regimes of generation in a powerful 10-μs X-band backward-wave oscillator were studied theoretically and experimentally. The sequence of the self-modulation patterns and corresponding bifurcation values observed as the current was increased were in good agreement with the results of simulations. It was found that at a current of 120 A chaotic self-modulation set in at a power of 2 MW and a relative spectral width of 4%.

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The theoretical study of the multifrequency dynamics of backward-wave oscillators (BWOs) carried out in [1] showed that, as the injection current increases above the threshold value, the regime of stationary single-frequency generation is replaced with the regime of periodic self-modulation, first in the sinusoidal form and then acquiring the shape of a sequence of spikes. As the current grows further, the signal modulation becomes chaotic. Because BWOs are among the most studied devices in high power microwave electronics [2–13], the investigation of multifrequency processes in such oscillators can be of practical interest in the development of powerful sources of modulated or noiselike signals. Such sources can be used for plasma heating, for energy transmission at large distance, as well as for technological applications. Nevertheless, until recently self-modulation regimes were observed experimentally only in nonrelativistic BWOs of the milliwatt power levels [12]. For a powerful relativistic BWO, several spikes were observed in [8] and interpreted as a manifestation of self-modulation. However, the electromagnetic pulse duration (10–20 ns) was too short to be able to identify what type of modulation (periodic or chaotic) was taking place. Note also that the BWO described in [8] operated with large (70%) end reflections from the collector side of the device. According to simulations [9], such reflections should dramatically change the dynamics of the BWO.

In the experiments discussed in this Letter, the first observation of chaotic dynamics in a long pulse BWO at megawatt power levels was carried out. The configuration of interaction space was in the form of a smoothly corrugated waveguide with a cutoff narrowing at the cathode end and diffraction output of radiation from the collector end, which ensured the absence of reflections at the device exit. Such a configuration is typical for a powerful relativistic BWO [2]. The experiments were performed using a "Saturn" accelerator which powered a thermionic cathode that allowed formation of a tubular electron beam with duration up to 10 μs, electron energies $U = 100$–300 keV, and current up to 300 A. This accelerator was used in previous experiments to drive a 10-μs relativistic X-band BWO which achieved a steady-state generation with a power level of 5 MW [6].

To observe self-modulation regimes, the operating current had to significantly exceed the threshold value. This was achieved by the choice of the TM_{01} operating mode which had a higher coupling impedance with the electron beam (compared to the TE_{11} mode), as well as by substantially increasing the length of the interaction space. At the operating point, the group velocity of the TM_{01} mode was high enough ($0.4c$) to ensure the extraction of radiation energy from the interaction space with a small reflection. It is also important to note that several microwave systems of different length have been investigated. For the shortest BWO of length $l \sim 10\lambda$ (λ is wavelength), only the steady-state regime was evident. When the interaction length was increased up to $l \sim 15\lambda$, both the steady-state and periodic self-modulation regimes were observed. Further increase of the BWO length up to $l \sim 20\lambda$ in accordance with simulations presented below resulted in a chaotic self-modulation regime.

Using a time domain approach [1,9] under the assumption that the electron beam excites a single mode of the corrugated waveguide, the dynamics of the BWO can be described by the self-consistent system of equations, which includes the equation for the field amplitude

$$\frac{\partial A}{\partial \tau} - \frac{\partial A}{\partial \zeta} = -\frac{J}{\pi}\int_0^{2\pi} \exp(-i\vartheta)\, d\vartheta_0, \tag{1}$$

and relativistic electron motion equations

$$\frac{\partial \vartheta}{\partial \zeta} = \frac{1}{\sqrt{1-\gamma^{-2}}} - \frac{1}{\sqrt{1-\gamma_0^{-2}}}, \tag{2}$$

$$\frac{\partial \gamma}{\partial \zeta} = \mathrm{Re}\left(A(\zeta,\tau)\exp(i\vartheta) + i\sigma \sum_n f_n \rho_n \exp(in\vartheta) \right). \tag{3}$$

Here $A = eE_z/(mc\omega)$ is the dimensionless resonance field amplitude at the beam radius, $\tau = \omega(t - z/v_0)/(1/\beta_0 + 1/\beta_{\mathrm{gr}})$ and $\zeta = \omega z/c$ are the dimensionless temporal variable and longitudinal coordinate, respectively, $\vartheta = \omega t - hz$ is the electron phase relative to the synchronous wave,

$J = eI|Z|/2\beta_0^2 mc^2$, Z is the coupling impedance [2], I is the beam current, γ is the relativistic mass-factor, ω is the frequency of precise synchronism, $\nu_0 = \beta_0 c$ is the initial velocity of electrons, and $\nu_{\mathrm{gr}} = \beta_{\mathrm{gr}} c$ is the group velocity of the wave. The second term in the right-hand side of Eq. (3) describes the Coulomb interaction, where $\rho_n = \frac{1}{\pi}\int_0^{2\pi} \exp(-in\vartheta)d\vartheta_0$ represent the harmonics of the space charge density, $\sigma = Ieg/(mc\omega^2 b)$ is the space charge parameter, b is the beam radius, and f_n is the reduction coefficient of the nth space charge harmonic. For the simplest model of a thin tubular beam in a smooth waveguide, this parameter can be approximated by $f_n = 1 - \exp(-2ngd)$, where d is the distance between the beam and metallic wall, and $g = \omega/c\beta\gamma$ is the transverse wave number of the synchronous harmonic. The initial and boundary conditions for Eqs. (1)–(3) have the following form:

$$\begin{aligned} A|_{\tau=0} &= A_0(\zeta), & A|_{\zeta=L} &= 0, \\ \vartheta|_{\zeta=0} &= \vartheta_0 \in (0, 2\pi), & \gamma|_{\zeta=0} &= \gamma_0, \end{aligned} \tag{4}$$

where L is the dimensionless length of the interaction space.

The simulations were carried out for the different values of electron current with BWO parameters which corresponded to the experiment. The length of the slow-wave structure was 62.3 cm, the mean waveguide radius was 1.4 cm, the period and corrugation amplitude were 1.7 and 0.25 cm, respectively, the electron beam radius was 0.6 cm, and the energy of electrons was 150 keV. An impedance of $Z = 0.5$ ohm at an operating frequency of 8.7 GHz and a starting current of 6 A were found based on the results of Ref. [2].

In Fig. 1 the output power temporal dependence as well as the signal spectra and so-called phase-plane portraits at different values of current are presented. Following [14] for plotting of phase trajectories, the points corresponding to the system states at consecutive moments in time were marked on the plane $(|A(0,t)|, |A(0, t - t^*)|)$, where t^* is the delay time, chosen to be equal to 1/4 of the self-modulation period. Figure 1(a) corresponds to a steady-state operation regime which was established for a current of 7 A. As the current was increased, the operation regimes became more complicated. Figure 1(b) shows a periodic self-modulation regime for a current of 30 A. As the current increased to 55 A, the self-modulation lost its periodicity and became chaotic [Fig. 1(c)], as corroborated by the spectra and phase portraits. Because of the space charge effect, some simplification of the output signal took place at the range of electron current 70–100 A [Fig. 1(d)]. At these currents, the self-modulation became quasiperiodic again. [When the space charge term was neglected in Eq. (3), permanent complication of the oscillation regime with an increase in current was observed.] However, above a current of 120 A as seen from the spectrum and phase portraits [Fig. 1(e)], the self-modulation again acquired chaotic behavior. It should be noted that the period of self-modulation for a current less than 30 A was equal to 7 ns [Fig. 1(b)]. When the current exceeded 70 A, i.e., after passing through the chaotic regime, the self-modulation period grew up to 13 ns [Fig. 1(d)]. The correlation function was determined as follows:

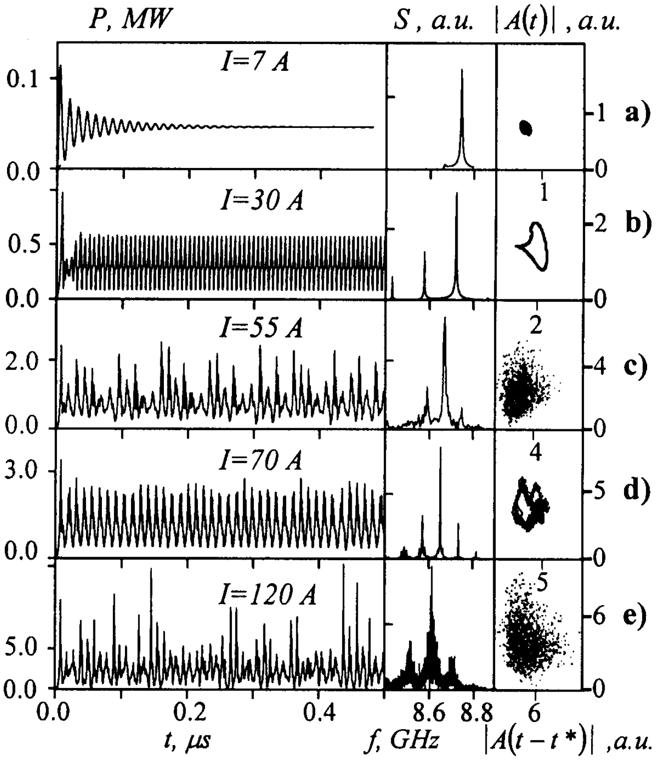

FIG. 1. The simulation of dynamics of the powerful BWO studied experimentally: $l = 62.3$ cm, $Z = 0.5$ ohm, $f_{1,2} = 0.55$, and $U = 150$ kV. The output power temporal dependence (left column), the signal spectra (middle column), and phase-plane portraits (right column) for different values of beam current.

$$\begin{aligned} K(t_{\mathrm{del}}) = \frac{1}{T}\int_0^T &(|A(0,t')| - \overline{|A|}) \\ &\cdot (|A(0, t' + t_{\mathrm{del}})| - \overline{|A|})dt', \end{aligned} \tag{5}$$

and was calculated for a current of 120 A [Fig. 2(a)], where T was the sampling time, t_{del} was the delay time, and $\overline{|A|}$ was the mean value of the amplitude. The correlation time can be estimated to be 50 ns and corresponds to a main spectrum line broadening of 0.02 GHz as shown in Fig. 1(e).

It is necessary to point out the problem of transverse mode selection. The realization of chaotic self-modulation needed substantial currents which greatly (by 20 times) exceeded the starting value. In the experiments discussed below, the operating TM_{01} mode possessed the lowest starting current, 6 A. Nevertheless, the self-excitation conditions were fulfilled for other modes (TE_{11}, TE_{21}) with transverse structures different from the operating mode.

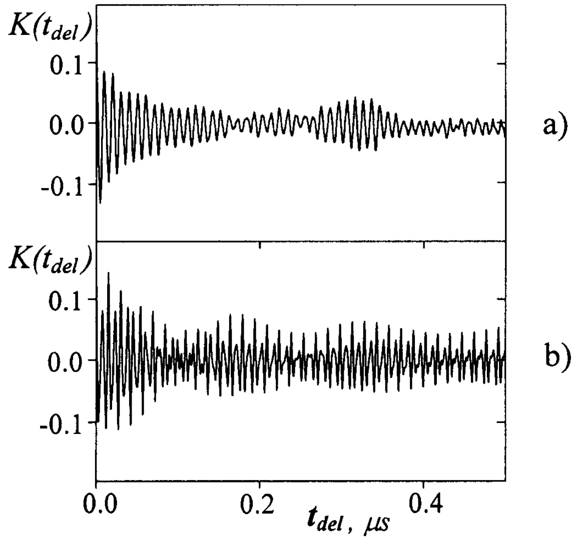

FIG. 2. The correlation function calculated from the simulation results (a) and corresponding to the experimental oscilloscope trace (b) for a current of 120 A.

The simulations [13] of competition between operating mode TM_{01} and spurious modes in a BWO with parameters closed to the ones used in the experiments showed that the spurious modes should be suppressed by the operating mode even in the chaotic self-modulation regime. This conclusion was confirmed experimentally by the absence of components corresponding to other modes in the radiation spectrum and in the radiation pattern.

In experiments at an operating voltage of 150 kV, the electron current was changed from 10 to 140 A. The thermionic cathode operated in the space charge limited regime. The current control was made by varying the potential between the three-electrode elements of the electron gun. For increasing of electrical strength, the temperature of the electrodynamic system was maintained at a level of 500 °C. The receiving horn was positioned at the maximum of the radiation pattern of the operating TM_{01} mode. The output signal was registered by a crystal detector which had a time lag of 2 ns and was recorded using a Hewlett Packard digital oscilloscope with bandpass of 500 MHz and quantization period of 1 ns. This allowed the entire microwave pulse to be stored in memory for further analysis of its separate plots. To specify the signal spectrum, a heterodyning technique was also used. The difference signal was recorded by the oscilloscope with subsequent Fourier analysis.

A typical signal oscilloscope trace in the chaotic self-modulation regime is presented in Fig. 3 for an injection current of 120 A. The self-modulation occupied almost all the rf pulse duration ~ 10 μs. Nevertheless, more detailed analysis showed that the form of the envelope was somewhat changed during the pulse due to slight variations of the voltage resulting in a variation of the coupling impedance. Note that small spurious reflections from the collector end also could lead to the output power and oscillation pattern being dependent on a frequency shift caused by voltage variations.

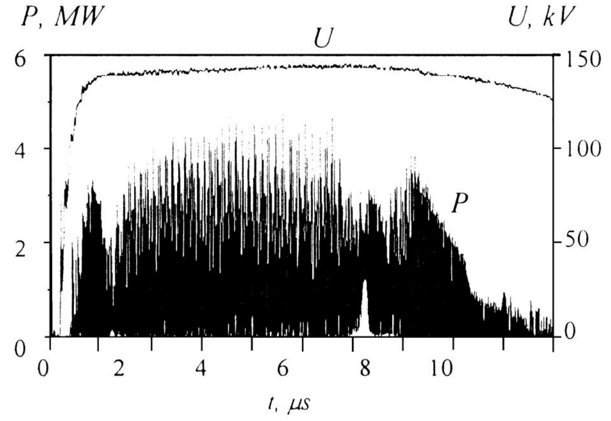

FIG. 3. The full signal oscilloscope trace for an injection current of 120 A.

The central parts of the oscillograms with duration ~ 500 ns at the flat top of voltage pulse with fixed value of 150 kV were chosen to study bifurcation currents. These parts of the oscillogram as well as spectra and phase portraits are presented in Fig. 4. For plotting the phase portraits, we calculated the electric field amplitude by the

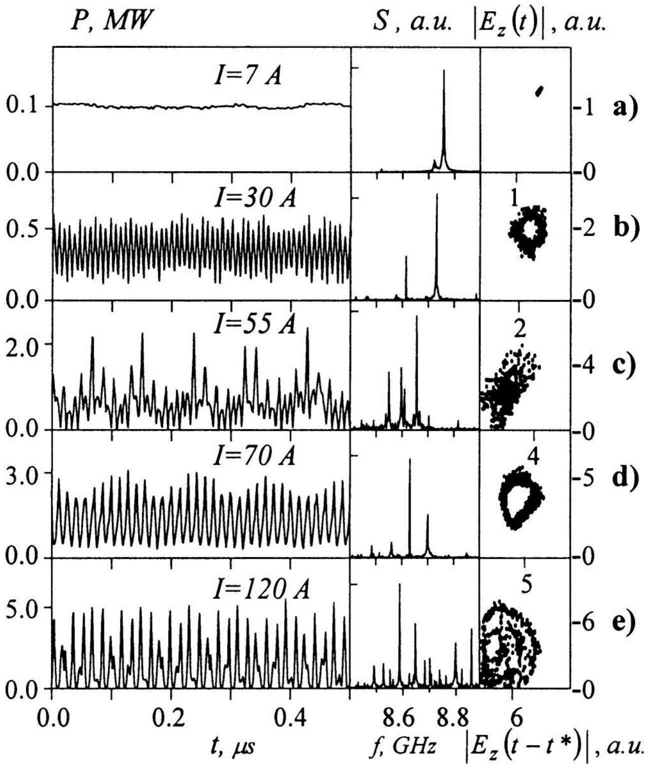

FIG. 4. Experimental oscilloscope traces of output power (left column), radiation spectra (middle column), and phase-plane portraits (right column) for different values of beam current.

measured radiation power, assuming that only the operating TM_{01} mode is excited. At a current of 7 A, a steady-state regime was established. At a current of 30 A, the regime changed to a sinusoidal self-modulation with a period of about 8 ns, and, as the current increased, the time dependence of the output power became more complicated. At a current of about 50 A [Fig. 4(c)], the self-modulation acquired a chaotic character. For higher currents some simplification of the regime was observed. It is seen from Fig. 4(d) that under a current of 70 A the self-modulation was quasisinusoidal with a period close to 14 ns. Finally, for a current greater than 100 A, the self-modulation became chaotic again with a relative spectral width of ~4% [Fig. 4(e)]. We should emphasize that bifurcation values of currents and the main self-modulation periods were in good agreement with simulations. Based on simulations, the simplification of the self-modulation pattern in the current range of 50–100 A could be explained by the influence of space charge. The correlation function [Fig. 2(b)] calculated for the signal presented in Fig. 4(e) was also in good agreement with simulations [Fig. 2(a)] for the same current of 120 A.

A high reproducibility of operating regime from pulse to pulse was achieved in our experiments. The duration of the radiation pulse became shorter with increasing acceleration potential. Because of this fact, it is very probable that the limitation in radiation power at currents more than 120 A was caused by rf breakdown. The average radiation power in the self-modulation regime measured using calorimeter techniques was 1 MW for a current of 50 A and 2 MW for a current of 120 A. At an electron accelerating potential of 150 kV, this corresponded to an electron efficiency of about 10%. The efficiency in the self-modulation regimes changed from 7% for the current of 30 A up to 14% for the current of 70 A, which corresponded to the simulation values.

In conclusion, experiments which observed the transitions from steady-state to chaotic self-modulation regime in a powerful BWO were successfully carried out. The observed sequence of operation regimes and corresponding bifurcation currents were in good agreement with the results of simulations.

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