Luxemburg-Gorky Effect Retooled for Elastic Waves: A Mechanism and Experimental Evidence

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A new mechanism is proposed for the linear and amplitude-dependent dissipation due to elastic-wave-crack interaction. We have observed one of its strong manifestations in a direct elastic-wave analog of the Luxemburg-Gorky effect consisting of the cross modulation of radio waves at the dissipative nonlinearity of the ionosphere plasma. The counterpart acoustic mechanism implies, first, a drastic enhancement of the thermoelastic coupling at high-compliance microdefects, and, second, the high stress-sensitivity of the defects leads to a strong stress dependence of the resultant dissipation.

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For many nonlinear effects typical in optics and plasma physics direct acoustic analogies are known. In particular, such effects have been observed in bubble-containing liquids, whose strong acoustic nonlinearity is due to the coupling to soft oscillators, the bubbles. Examples are wave phase-conjugation, stimulated scattering, and maser effects [1]. However, no acoustic analogies are known for the Luxemburg-Gorky (LG) effect, which represents one of the pioneering observations in nonlinear wave interactions [2]. It consists of the transfer of the modulation from the radiation of a powerful radio station (originally, Luxemburg and Gorky-city stations) to another carrier wave. This cross modulation is caused by variations in the absorption of the ionosphere plasma, which are induced by the amplitude-modulated stronger wave at frequencies on the same scale as the modulation frequency. The stronger wave perturbs the electronion and electron-molecule collisions in the plasma, thus influencing the dissipation and, in general, the wave velocity. The induced variations in the dissipation produce pronounced amplitude modulation of the weaker wave, whereas the role the complementary perturbations in the weaker wave velocity is of secondary importance for the considered phenomenon [2,3]. A majority of the later nonlinear research for waves of different nature focused, however, on the effects of reactive, rather than dissipative nonlinearities. In this Letter, we propose a physical mechanism for a dissipative nonlinearity due to the elastic-wave-crack interaction. Owing to the intrinsic high compliance of narrow cracks and interface contacts they both are very effective centers of elastic energy dissipation, and even relatively moderate-amplitude waves may significantly affect their state, thus strongly influencing the absorption. We will show that efficient interactions of ultrasonic or seismic waves very similar to the LG effect are possible in crack-containing solids. We believe that the mechanism should be rather general and applicable to many rocks and other microstructured materials.

It is well known [4] that for homogeneous solids the intrinsic elastic nonlinearity due to the anharmonicity of the interatomic potential is normally very weak, at least for strains much less than the damage or plastic yield threshold (typically $\varepsilon_{dmg} \sim 10^{-4} - 10^{-2}$ in strain). However, during the last few years, relatively low-amplitude nonlinear-elastic effects have been intensively studied, such effects being readily observed at strains $\varepsilon \sim 10^{-6} - 10^{-5}$ in rocks, fatigue-damaged metals, and in other microstructured materials [4–8]. For these solids, the presence of defects with highly increased compliance, such as microcracks and microcontacts, is typical. This often results in a drastic increase in nonlinear elasticity, whereas linear elastic parameters remain only slightly perturbed, which may be understood by means of instructive models describing the interplay between the strong strain-concentration at the high-compliant defects and their small density [9]. Besides the elastic nonlinearity, due to the relative ease of the breaking of interatomic bonds, the same defects induce hysteresis in the stress-strain relation [4–8]. Hysteresis, in addition to nonlinear elasticity, readily accounts for amplitude-dependent dissipation for sufficiently intensive waves. However, some observations were reported [10–12] on pronounced variations in dissipation of a weak elastic-wave (at strains down to $\varepsilon \sim 10^{-8} - 10^{-10}$) induced by another moderate-amplitude elastic-wave ($\varepsilon \sim 10^{-5} - 10^{-6} \ll \varepsilon_{dmg}$). This effect on the weaker wave can be explained neither by reactive nor by hysteretic nonlinearities [11,12] and requires assuming the existence of an additional, nonhysteretic and nonfrictional, nonlinear-dissipative mechanism.

Below we describe a physical mechanism for such a dissipative nonlinearity that should be intrinsic to a wide class of crack-containing solids. The proposed idea is based on a few reliably established microstructural features of these solids together with pertinent macroscopic experimental data on nonlinear-elastic-wave interactions.

First, it is well established that such materials contain numerous cracks or microcracks, whose strong influence on material elasticity is appreciated in seismics and fracture mechanics. This strong effect of the cracks is due to their high compliance, that is quantitatively characterized

[13] by their very small aspect ratio $d/L \ll 1$, where d is the crack opening and L is its characteristic diameter. In order to completely close a crack, it is enough to produce in the material an average strain roughly equal to the crack's aspect ratio [13]. This strain is usually rather small, $d/L \sim 10^{-3} - 10^{-4}$, but nevertheless significantly larger than the typical strain, $\varepsilon \sim 10^{-5} - 10^{-6}$, for which the above-mentioned pronounced nonlinear-dissipative effects [10–12] were observed. We stress that these statements do not depend on details of the crack model (pennylike, tapered, etc. [13]), and even for asymmetrical and corrugated real cracks the meaning of the characteristic scales L and d is quite clear (see Fig. 1).

Further condition and evidence for the generality of this mechanism comes from numerous direct images of crack-like defects in rocks and damaged solids, obtained by the methods of electron-, acoustic-, and atomic force microscopy, which indicate rather complex, wavy, or zigzag interface shapes even for micrometer-scale cracks. The cracks often have inner striplike contacts [14,15], as schematically shown in Fig. 1, or these can be small loosely separated regions, where this type of contact can be easily created or destroyed. Indeed, at these regions, local separation (or interpenetration) $\tilde{d}$ of crack interfaces is much smaller than average separation d. These contacts are extremely stress sensitive, since due to the described geometry they are strongly perturbed by the average strain, which can be orders of magnitude smaller (roughly $d/\tilde{d} \gg 1$ times) than the typical magnitude $\varepsilon \sim d/L \sim 10^{-3} - 10^{-4}$ required to close the whole crack.

Another crucial point consists in the fact that defects of a material structure are regions of very effective energy dissipation even for elastic waves whose length is much greater than the crack size L. Conventionally, this dissipation is mostly attributed to friction or adhesion hysteresis at crack interfaces [16,17]. However, it is physically clear and recently corroborated by direct nanoscale experiments [18] that for manifestation of adhesion and friction, mutual

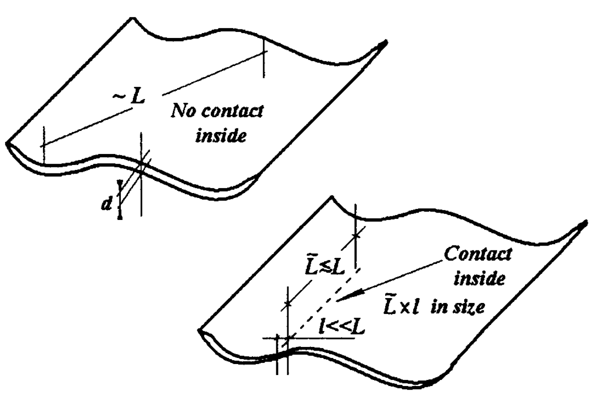

FIG. 1. A crack without and with an inner contact is shown here schematically. At $\tilde{L} \to l$ a striplike contact reduces to a pointlike contact.

displacement at interfaces should exceed the atomic size a. In this context, for a crack with diameter L, the average compressional or shear strain ε can produce maximal lateral or normal interfacial displacement [11,17] $D \sim \varepsilon L$. This estimate does not depend on the details of the crack model and agrees with the above statement that compressional strain $\varepsilon \sim d/L$ produces $D = d$, thus closing the crack completely. On the other hand, the requirement $D > a$ determines the threshold strain $\varepsilon_{th} > a/L$, below which the interfacial displacement is of a subatomic scale. For a typical atomic size $a \sim 3 \times 10^{-10}$ m and a macroscopic crack with $L \sim 10^{-3}$ m, this yields $\varepsilon_{th} \sim 0.3 \times 10^{-6}$, which should be exceeded in order to activate frictional and adhesional hysteretic losses. However, even at much smaller strains, the defects can efficiently dissipate elastic energy due to locally enhanced thermoelastic coupling. Indeed, near inhomogeneities, unlike the case of a homogeneous material, wave-induced temperature gradients are determined [19] not by the elastic-wave length, but by the much smaller defect size L and the temperature wavelength δ. When scales L and δ coincide, the "global" (over the whole crack) losses per cycle reach their maximum. This type of elastic energy dissipation was appreciated in seismics quite long ago, and rigorous analytical solutions are known for special crack models [20]. Alternatively, without specifying the crack model in detail, in order to estimate temperature gradients and the respective losses in the crack vicinity, one may use the approximate approach known for the case of polycrystals [19]. In doing so we have derived simplified asymptotic expressions for the losses per cycle in the low-frequency limit (when $L \ll \delta$), the high-frequency limit ($L \gg \delta$) and at the relaxation maximum (when $L \sim \delta$). With an accuracy of a factor of 2–3 these expressions are

$$W_{LF}^{dis} = 2\pi\omega T(\alpha^2 K^2/\kappa)L^5\varepsilon^2, \quad (1)$$

$$W_{HF}^{dis} = 2\pi T(\alpha K/\rho C)^2[1/(\kappa\rho C\omega)]^{1/2}L^2\varepsilon^2, \quad (2)$$

$$W_{\text{crack}}^{\max} = 2\pi T(\alpha^2 K^2/\rho C)L^3\varepsilon^2, \quad \omega = \omega_L \approx \kappa/(\rho C L^2), \quad (3)$$

where ω is the wave cyclic frequency, T is the temperature, α is the temperature expansion coefficient of the solid, K is the bulk elastic modulus, ρ is the density, C is the specific heat, ε is the average strain, κ is the thermal conductivity, and ω_L is the relaxation frequency for defect scale L. For example, for $L \sim 10^{-3}$ m the relaxation frequency ω_L falls between $10^{-1} - 1$ cycles/s for most rocks and metals. In the calculation of the low-frequency losses by analogy with [19] we took into account that crack size L is the characteristic scale of the transition from zero stress at the free interfaces to the applied average stress σ. The derivation of the high-frequency expression took into account that different particular crack models consistently predict that at average applied stress σ, the near-tip stress concentration has a universal form $\sigma_{\rm tip} \sim \sigma/\sqrt{r/L}$ [21] (distance r is counted from the tip), and it is just this region which gives the main contribution to the high-frequency dissipation. The validity of the approximate expressions obtained is supported by the good agreement with rigorous analysis [20].

Analogous estimates for the dissipation at the inner contact of the crack should take into account that the external applied stress is distributed between the arc crack-stiffness and the contact stiffness. The scale of the localization in the depth direction of the near-contact stress is roughly equal to the contact width $l \ll L$ [22]. At contacts that are soft (compared to the arc crack-stiffness) the corresponding magnitude of the near-contact stress, σ_c, is readily shown to be $\sigma_c \sim \sigma(L/l)$. These stress-distribution features, which do not depend on details of the crack and contact models, suffice for the estimations of the respective thermoelastic losses, which are similar to Eqs. (1)–(3), although the high-frequency asymptotic dependence of the dissipation for contacts is to ω^{-1} instead of $\omega^{-1/2}$ for cracks:

$$W_{LF}^{dis} = 2\pi\omega T(\alpha^2 K^2/\kappa) l^2 \tilde{L} L^2 \varepsilon^2, \tag{4}$$

$$W_{HF}^{dis} = (2\pi/\omega)\kappa T(\alpha K/C\rho)^2 \tilde{L}(L/l)^2 \varepsilon^2, \tag{5}$$

$$W_{\rm cont}^{\rm max} = 2\pi T(\alpha^2 K^2/\rho C)\tilde{L} L^2 \varepsilon^2, \quad \omega = \omega_l \approx \kappa/(\rho C l^2). \tag{6}$$

Comparison of Eqs. (3) and (6) indicates the striking result that, for striplike contacts with $\tilde{L} \sim L$, the maximum losses at the whole crack and at the small inner contact have the same magnitude, whereas the relaxation frequency for narrow, $l \ll L$, contacts can be 4–6 orders of magnitude higher and reaches the kHz or even the MHz band.

These results indicate, in fact, that the widely accepted opinion as to the low importance of thermoelastic coupling for seismic wave attenuation requires essential revision both for the background linear dissipation of low-strain ($\varepsilon \sim 10^{-7} - 10^{-9}$) waves and for the amplitude-dependent dissipation. In particular, Eqs. (1)–(6) demonstrate that even a single crack with a few soft inner contacts can contribute to a weakly frequency-dependent quality factor in the frequency range from fractions of a Hz to kHz frequencies. Comparison of Eqs. (3) and (6) indicates that even a single inner contact of width l in a larger crack of size L produces the same dissipation at higher frequencies as a huge number (thousands and millions) of tiny cracks of size l. For example, for a reasonable ratio $L/l \sim 10^2$, even a pointlike single contact with $\tilde{L} \sim l$ is equivalent to $(L/l)^2 \sim 10^4$ cracks, and a striplike contact with $\tilde{L} \sim L$ can dissipate the same energy as $(L/l)^3 \sim 10^6$ small cracks. Further, taking into account reasonable crack densities in the conventional way [20] allows one to readily obtain realistic estimates of the magnitude of the quality-factor in a wide frequency band.

Another essential inference comes from the fact that quite moderate average strain, say, $\varepsilon \sim 10^{-5} - 10^{-6}$, which is too small to perturb the crack as a whole, can strongly perturb sizes l and $\tilde{L}$ of soft inner contacts. According to Eqs. (4)–(6) this has a pronounced effect on the dissipation of a weaker probe wave, though neither adhesion-hysteresic, nor frictional losses are important for such a weak wave. In contrast, the complementary variation in material elastic moduli may remain very small, since the stiffness of such contacts is very low. Thus in crack-containing solids, favorable conditions should occur for the direct elastic-wave analog of the LG effect, since perturbation of the inner crack contacts by a moderate-amplitude wave via the considered mechanism can noticeably affect dissipation for another weaker wave, just as in the case of the radiowaves in the ionosphere [2,3].

We have built a setup allowing for an instructive experimental demonstration of the acoustic LG effect in the form of the cross modulation of two longitudinal modes in a glass rod containing three corrugated thermally produced cracks 2–3 mm in size (Fig. 2). In a reference rod without cracks, the modulation sidelobes (existing due to residual parasitic nonlinearities) were 25–40 dB lower than shown in Fig. 3(a). Resonance curves for the probe wave [Fig. 3(b)] demonstrate that primarily the dissipation, not the elasticity, is affected by the stronger wave. Magni tudes and frequencies, at which the observed amplitude-dependent variations in dissipation were observed, agree well with estimates based on Eqs. (4)–(6). As argued above, for small enough strains $\varepsilon \sim 10^{-8}$, estimated displacements εL of adjacent crack interfaces are subatomic in scale, so that neither hysteretic nor frictional effects can

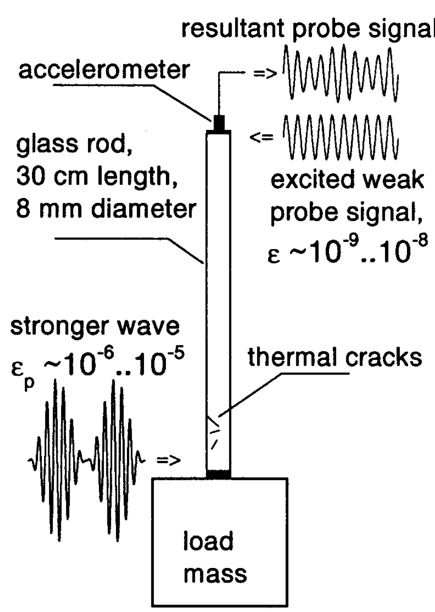

FIG. 2. Schematically shown experimental configuration.

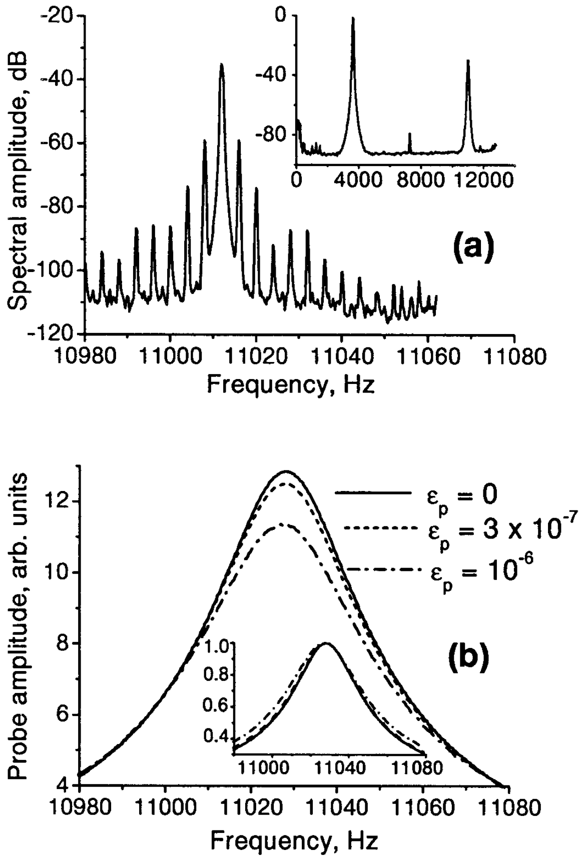

FIG. 3. Experimental observation of the elastic wave LG effect. (a) Modulation spectrum of weak second mode near 11 kHz with $\varepsilon \sim 10^{-8}$ by a stronger ($\varepsilon_p \sim 10^{-6}$) first mode wave with carrier frequency near 3.6 kHz and slow amplitude modulation at 3 Hz. The inset shows the relative levels of the stronger and the weaker waves. (b) Resonance curves for the probe wave at different stronger-wave levels, clearly illustrating a greater than 10% variation in the probe mode quality-factor. In contrast, the resonance frequency shift is hardly noticeable. The inset shows the same curves in normalized form.

be important for the probe wave dissipation. Indeed, careful experimental study of the amplitude dependencies for the observed modulation confirmed linear character of the weak wave dissipation. Quantitatively, estimates based on Eq. (6) and typical parameters for glass show that even a single contact-containing crack of a few millimeters in size suffices to explain the observed 10–12% variation in the initial magnitude of the quality factor of about 300–350 for the probe wave. In this simple experiment we have used a transparent material in which the cracks are easily visible. Their parameters may be directly and nondestructively estimated. These cracks are the only defects present, and there is no doubt that only their presence is responsible for the observed effects. Independent evidence for the formulated mechanism comes from data on the variation in dissipation of the weak wave produced by the stronger one, which were recently reported in [11,12] for copper aged by strong annealing and sandstone.

Since the described defects occur in a vast class of solids, we believe that the proposed mechanism of strong enhancement of coupling of thermal phonons and elastic waves will be found to operate widely, in particular, both for the dilatation strain responsible for conventional thermoelastic dissipation and for shear modes. The essential feature of the mechanism considered is that unlike many other nonlinear-elastic effects, even quite moderate amplitude ($\varepsilon \sim 10^{-6} - 10^{-5}$) waves can induce in another probe wave strong amplitude variations, up to tens of percents in magnitude. We hope that the presented findings will motivate new experiments. The corresponding effects, including the LG modulation, should find diagnostic applications in basic solid-state studies, in seismics, and in nondestructive testing.

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