Polymers in 2D Turbulence: Suppression of Large Scale Fluctuations

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Small quantities of a long chain molecule or polymer affect two-dimensional turbulence in unexpected ways. Their presence inhibits the transfers of energy to large scales causing their suppression in the energy density spectrum. This also leads to the change of the spectral properties of a passive scalar which turns out to be highly sensitive to the presence of energy transfers.

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The effects of soluble polymers on the flow properties of liquids can be quite dramatic and sometimes useful [1,2]. These long chain molecules can couple to the flow and change their conformation: at rest, they are coiled up into very small balls; under shear or elongation they may stretch considerably [3]. A long-standing issue is the Tom's effect [2]: the important reduction of the drag force through the addition of minute amounts of polymers to a turbulent flow [4]. The interaction between the polymer and the walls of the container is apparently at the heart of the problem according to some recent investigations [5]. Others indicate effects related to the interaction between the turbulent flow itself and the long chain molecules anticipating a suppression of small scale fluctuations [6]. The two principle ideas to explain this drag reduction invoke either the importance of an elongational viscosity which can be considerably large [4,7] or the role of the elastic modulus of the polymer solution which may truncate the energy cascade to small scales [8]. Recent theory advances the issue by showing that the polymers may stretch considerably in a random flow [9].

We undertook a study of the effects of a polymer additive on the properties of decaying quasi-two-dimensional turbulence obtained in fast flowing soap films past an array of cylinders. By changing the dimensionality of the turbulence and therefore the cascade mechanisms at play [10] we hope to bring new insight into the problem [11]. Some of the effects we find bring some surprises and probably new hints on how and why the polymer affects high Reynolds number flows. One of the main effects, which is the suppression of the large scale fluctuations, appears quite spectacular and seems to be a direct consequence of the two dimensionality of the film and the interaction of the polymer with the flow reducing the elongation rates present. As a prelude to the effects of polymers on the flow studied, we show, in Fig. 1, interferograms of the thickness field behind an array of cylinders for the two cases. The thickness field in these films acts as a passive scalar as we will see below and follows the flow field just like smoke in air or dye in water. The polymer clearly makes for smaller scales as seen in the apparent sizes of the eddies. The polymer must inhibit vortex mergers, a signature of inverse transfers of energy.

Forced two-dimensional turbulence admits two cascades, an inverse energy cascade to large scales and an enstrophy cascade to the small scales, as both energy and enstrophy (mean squared vorticity) are conserved. These two ranges are believed to be separated by an injection scale l_{inj}. The energy density spectrum $E(k)$ of two-dimensional turbulence is predicted to have two different scalings: for scales larger than l_{inj}, $E(k)$ scales as $k^{-5/3}$, while for smaller scales $E(k)$ scales as k^{-3} [10]. During the last decade soap films have provided a test bed system for the study of 2D turbulence. There have been several reports so far of the properties of turbulence generated by a horizontal array of cylinders in vertically standing and fast flowing soap films [12]. Their mean thickness is of the order of a few μm; the rms fluctuations of the thickness are of the order of 5% [13,14]. The turbulence intensities engendered can reach 15% of the mean speed which hovers around 2 m/s. While the viscosity of these films is somewhat ill known and is thought to depend on the thickness of the film [15], the channels Reynolds number can be of the order of 10^4 (for a film kinematic viscosity of five times that of water [15]). The turbulence generated is decaying, presents isotropy at the small scales (between about 2 cm and 1 mm), and a power law scaling of the energy density spectrum $E(k) \sim k^{\beta}$ close to that predicted by Batchelor for two-dimensional decaying turbulence [10]. The exponent β is close to -3 and the turbulence is believed to be governed by a direct enstrophy cascade to the small scales.

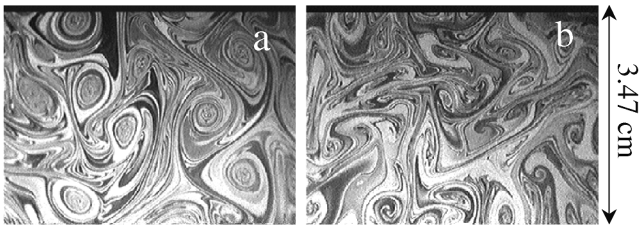

FIG. 1. Interferograms of the thickness field: (a) polymer free solution; (b) 25 ppm PEO solution for a flux of 0.3 ml/s.

Measurements of the integral scale also show that this scale increases with the distance from the grid in a manner consistent with a prediction by Batchelor. The third moment of longitudinal velocity differences $S_3(r) = \langle \delta u^3(r) \rangle = \langle (u(y + r) - u(y))^3 \rangle$ (the brackets indicate temporal averaging or ensemble averaging) was found non-negligible and positive, indicating the presence of some inverse transfer of the energy probably leading to the growth of the integral scale [12]. However, the energy density spectrum does not show a $-5/3$ scaling range.

The measurements use laser doppler velocimetry and interferometry to measure the velocity field and the thickness field. Both measurements are made in one point in the flow and the Taylor hypothesis, which has been thoroughly tested in these soap films [12], is used. The Taylor frozen turbulence hypothesis assumes that the eddies are being convected past the observation point without much deformation; this permits one to convert a time scale or frequency f to a length scale $l : k_y = 2\pi/l = 2\pi f/V$, where k_y is the longitudinal wave number in the direction of the mean velocity V. LDV gives the instantaneous velocity of small seed particles (diameter: 0.8 μm) in the flow. The interferometry technique has been recently used to study the thickness fluctuations in turbulent soap films [13]. A piezoelectric transducer is used on one of the arms of the interferometer; the other arm goes through the film and the combination of the two beams is monitored with a photodiode coupled to a feedback loop. The response of such a system has been optimized to reach 4 kHz. The polymer used is polyethyleneoxide (PEO, molecular weight of 4×10^6, or 8×10^6) at concentrations in the dilute regime ranging from a few parts per million by weight (ppm) to 100 ppm. The soap solution contains 1% of a commercial detergent and is injected at the top of the channel (width of 6 cm and length of 2 m), made of nylon wires, at controlled flux (ranging from 0.1 to 0.6 ml/s). The grids used are horizontal arrays of seven equally spaced cylinders with diameters of 3 or 1 mm.

Quantitatively, the first effect of the presence of polymers is to reduce the turbulent intensity of the flow as can be seen from the longitudinal and transverse velocity probability density functions (pdfs) shown in Fig. 2. The second effect which is manifest in the power spectra of the velocity components is the reduction in amplitude of the low frequencies or large scales greater than 1 cm; the higher the polymer concentration, the more important is this reduction. The concentrations used are all in the dilute regime for which the shear viscosity is hardly affected with respect to that for the soap solution.

As shown in the inset to Fig. 2 for the 0 and 25 ppm solutions, the longitudinal velocity power spectra are relatively similar for locations near the grid (at 1.5 cm) where the two spectra show a broad peak centered at the shedding frequency of the individual cylinders. The differences are much greater farther away from the grid (at 7 cm) where the spectra are broad band with a smaller amplitude at large scales for the polymer solution. The effects of the boundary layers around the cylinders seem to be minor in our case. The initial stages of the flow are roughly the same. Since the turbulence is produced by the interactions of the different wakes behind the individual cylinders and evolves with downstream distance we believe that the effects described here are the outcome of these interactions. A recent study of the wake of a single cylinder in these fast flowing soap films [16] found a modification of the wake by the addition of polymers: the formation length of the alley was increased further downstream. The authors specified that the boundary layer around the cylinder is different. However, beyond this effect, the spectrum of the velocity was changed only slightly at locations farther downstream.

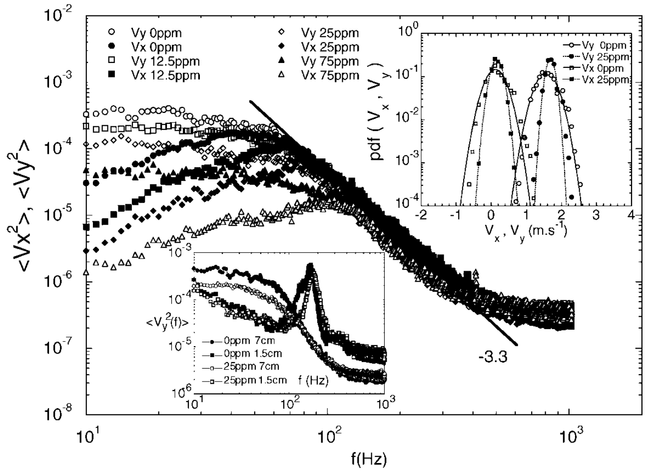

FIG. 2. Longitudinal (V_x) and transverse (V_y) velocity power spectra for different polymer concentrations as indicated in ppm (4×10^6 molecular weight) at a distance of 7 cm from the grid and a flux of 0.4 ml/s . Upper inset: pdfs of V_x and V_y for similar conditions. Lower inset: longitudinal velocity spectra for the 0 and 25 ppm solutions at 1.5 and 7 cm from the grid.

The high frequency part of the velocity power spectra in Fig. 2, which corresponds to the small scales (smaller than 1 cm) seems hardly affected. A slight decrease is observed, however, at the highest concentration used (75 ppm) but it is much smaller than the decrease observed at the low frequency end of the spectrum. The power law decrease at high frequencies seems to be the same as that for the polymer free solution. The exponent is consistent with -3 or even slightly higher. *A priori* one expects the polymers to change the small scale structure of a turbulent flow; the contrary seems to occur in a quasi-two-dimensional flow. The mechanisms leading to the merging of vortices may have changed. One way to examine the changes to the transfer of energy is a look at the third moment of velocity differences $S_3(r)$ which measures an energy flux. Without polymers, this moment is positive. It increases versus the increment r, goes through a maximum, and decreases to zero at larger scales as seen Fig. 3. The presence of the polymer depresses the amplitude of the third moment and seems to shift the small maximum to smaller scales. Still compared to the no-polymer case the third moment seems almost negligible. The third moment in turbulent flows measures the energy flux and may be related to the energy transfer rate which may be generalized as a function of the scale [17]. The polymer suppresses the third moment indicating a much smaller energy flux. The presence of the polymer affects not only the mean value of $\delta u^3(r)$ as seen through $S_3(r)$ but also affects the fluctuations of this quantity. The inset of Fig. 3 shows the pdfs of $\delta u^3(r)/r$ which has the dimensions of an energy transfer rate $\epsilon(r)$. For different increments r in the -3 scaling range for the energy spectrum, the polymer suppresses the tails of the pdfs of $\epsilon(r)$ which are indicative of very large deviations from the mean.

Let us now turn to the thickness fluctuations of the soap film. As we have recently reported, the power spectrum of the thickness fluctuations in these turbulent soap films showed a clear scaling law $[\langle h^2(f)\rangle \equiv \langle h^2(k_y)\rangle \sim f^\alpha \sim k_y^\alpha]$ with an exponent α close to $-5/3$ as would be expected for a passive scalar if the turbulence were in the inverse cascade range [13]. As mentioned earlier, the turbulence is decaying and shows a -3.3 scaling for the energy density spectrum consistent with the existence of an enstrophy cascade but no $-5/3$ range associated with the inverse cascade. Theoretical expectations based either on dimensional analysis or more elaborate analytical work indicate that the passive scalar should display a scaling with an exponent of -1 in the enstrophy cascade range. It was proposed that the positivity of the third moment may be related to this anomalous scaling of the thickness field. When the polymers are used, the $-5/3$ scaling for the scalar ceases to exist at concentrations as low as 12.5 ppm for the 4×10^6 molecular weight polymer. Figure 4 shows the power spectrum of the thickness fluctuations for the no-polymer case and a 25 ppm polymer solution. For both configurations of the grid, the polymer changes the scaling of the spectrum with respect to the polymer free case. The scaling comes out to be close to -1. This is the case for other polymer concentrations as well, as seen in Fig. 4. The second order structure function $S_2^h(r) = \langle \delta h^2(r)\rangle = \langle (h(y+r) - h(y))^2\rangle$ also shows logarithmic-like behavior as seen in the inset of Fig. 5. Such a variation is consistent with theoretical expectations for the passive scalar in an enstrophy cascade range [18,19] (also known as "Batchelor scaling" [20]). The pdfs of the scalar differences are shown in Fig. 5; a conspicuous feature is their exponential shape which is also expected theoretically [19]; their width, however, barely varies with the increment in accordance with the logarithmic variation of the second order structure function. Our results are also consistent with recent experiments on the dispersion of a passive scalar in the enstrophy cascade range produced in electromagnetically driven flows in thin layers of electrolyte solutions [21]. What prevents the polymer free case from exhibiting a Batchelor like scaling? As we mentioned earlier, the third moment of velocity differences indicates that the energy flux to the large scales is suppressed by the polymer. This is also indicated by the suppression of the large scale fluctuations in the energy spectra. We believe that by suppressing this energy flux or by making it very small, the polymer allows the thickness fluctuations to be governed by the enstrophy flux alone. From the similarity of the spectra at high frequencies one may conclude that the rate of enstrophy transfer remains roughly the same despite the large decrease of the energy transfer rate. In the no-polymer case, two fluxes exist: that of energy and that of enstrophy as has been noted before for decaying 2D turbulence [22]. For an unknown reason so far, the thickness field in the presence of the two fluxes follows the energy flux rather than the enstrophy flux. Since the vorticity and the thickness dynamics are believed to be related for these soap films we may expect that the scaling of the enstrophy may show similar trends. The scaling exponent for the enstrophy density spectrum was found larger than -1 in these turbulent soap films [12].

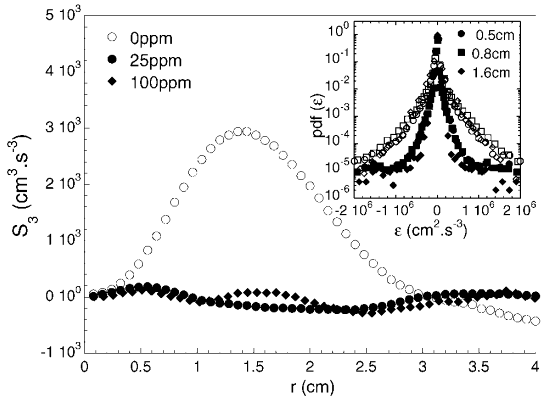

FIG. 3. $S_3(r)$ for different polymer concentrations (4×10^6 molecular weight) at a distance of 7 cm from the grid and a flux of 0.4 m/s. Inset: pdf$(\delta u^3(r)/r)$ for different separations r; open symbols are for polymer free solution and closed symbols for the 25 ppm PEO solution.

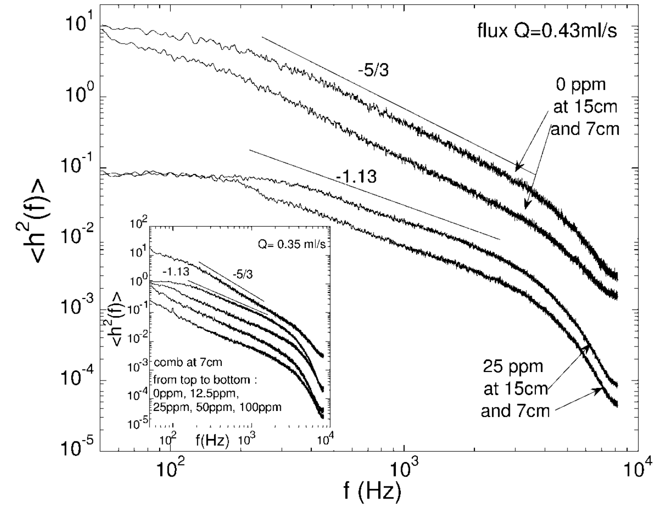

FIG. 4. Power spectra of the thickness fluctuations for the 0 and 25 ppm PEO solution at different locations, and different polymer concentrations (inset).

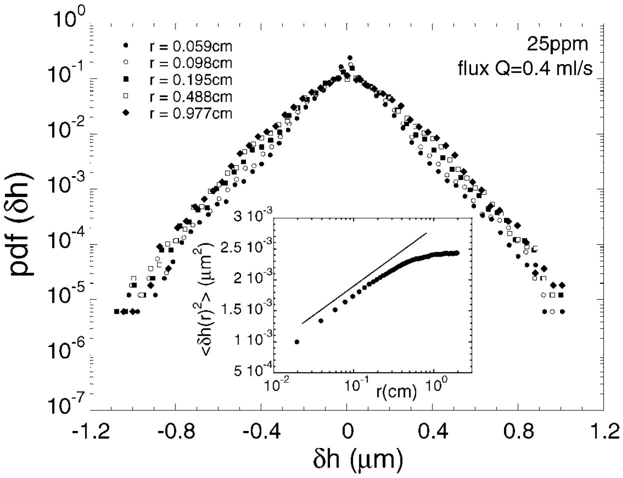

FIG. 5. Pdfs of $\delta h(r)$ for different separations r and at 7 cm from the grid for the 25 ppm PEO solution (4×10^6 molecular weight). Inset: $S_2^h(r)$ for similar conditions.

The use of polymers in turbulence can be twofold: understanding the dynamics of these long chain molecules in complex flows as well as a tool to change and modify some subtle properties of turbulence. Here in a two-dimensional flow, the polymer probably suppresses or reduces vortex mergers responsible for the transfer of energy up scale. The polymer must stretch under such conditions and such stretching could inhibit such mergers through a local increase in elongational viscosity. One can imagine that the polymers trapped in the regions between vortices, otherwise called saddle points, may experience large elongation rates and for relatively long periods of time. In recent experiments on the effects of flexible polymers on detaching droplets it was found that the elongational viscosity can be very large at values of the applied strain (the product of the elongation rate by its persistence time) of the order of 10^{-1} and higher [23]. As the elongation rates present in the flow studied here are quite large and the residence times quite sufficient to reach applied strains greater than 10^{-1}, the effects of the elongational viscosity may become important. An estimate of the elongation rate is $[(\delta u^3(r)/r)^{1/3}/r^{2/3}]$. As shown in Fig. 3 (inset), the pdfs of $\delta u^3(r)/r$ have a much narrower distribution when the polymers are present. The tails are simply cut off by the polymers indicating the suppression of large elongation rates due to the strong resistance of the polymers to elongation. While the mean elongation rates estimated from $S_3(r)/r^3$ are at most of the order of 10 Hz, the pdfs of $\delta u^3(r)/r$ show the presence of variations of the order of 100 Hz. In light of these results we may speculate that similar mechanisms may also occur in 3D drag reduction: suppression of large elongation rates and transfers of energy down to small scales reducing the amplitude of the small scale fluctuations as observed in experiments [24].

We present the effects of long flexible polymers on a quasi-two-dimensional turbulent flow obtained in fast flowing soap films. The effects are quite surprising; the polymer leaves the small scale fluctuations of the same spectral amplitude but inhibits the large scales. By suppressing the flux of energy to large scales, the polymers restore the Batchelor scaling to the thickness field which acts as a passive scalar. The two-dimensional nature of the flow studied brings in these somewhat unusual effects. The suppression of inverse energy transfers renders decaying turbulence somewhat more tame having only a single flux: that of enstrophy.

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