Observability of the Neutrino Charge Radius

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It is shown that the probe-independent charge radius of the neutrino is a physical observable; as such, it may be extracted from experiment, at least in principle. This is accomplished by expressing a set of experimental ν_μ-e cross sections in terms of the finite charge radius and two additional gauge- and renormalization-group-invariant quantities, corresponding to the electroweak effective charge and mixing angle.

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Within the standard model the photon (A) does not interact with the neutrino (ν) at tree-level; however, an effective photon-neutrino vertex $\Gamma^{\mu}_{A\nu\bar{\nu}}$ is generated through one-loop radiative corrections, giving rise to a nonzero neutrino charge radius (NCR) [1], which contributes nontrivially to the full electron-neutrino scattering amplitude. Even though the one-loop computation of the *entire* S-matrix element describing the aforementioned amplitude is conceptually straighforward, the identification of a *subamplitude,* which would serve as the effective $\Gamma^{\mu}_{A\nu\bar{\nu}}$ has been faced with serious complications, associated with the simultaneous reconciliation of crucial requirements such as gauge invariance, finiteness, and target independence [2]. The crux of the problem is that, since in non-Abelian gauge theories individual off shell Green's functions are in general unphysical, the definition of quantities familiar from scalar theories or QED, such as effective charges and form factors, is in general problematic. Thus, whereas a pion form factor may be defined perfectly well in the one-photon approximation, the same definition leads to unphysical results in the case of the NCR. The above difficulties have been conclusively settled in [3] by resorting to the well-defined electroweak gauge-invariant separation of physical amplitudes into effective self-energy, vertex and box subamplitudes, implemented by the pinch technique formalism [4]. These effective Green's functions are completely independent of the gauge-fixing parameter regardless of the gauge-fixing scheme chosen, and satisfy simple, QED-like Ward identities, instead of the complicated Slavnov-Taylor identities. The NCR obtained in [3] is (i) independent of the gauge-fixing parameter, (ii) ultraviolet finite, (iii) couples electromagnetically to the target, and (iv) process (target) independent and can therefore be considered as an intrinsic property of the neutrino. In particular, from the gauge-invariant one-loop *proper* vertex $\hat{\Gamma}^{\mu}_{A\nu_i\bar{\nu}_i}$ constructed using this method one extracts the dimension-full electromagnetic form factor $\hat{F}_{\nu_i}(q^2)$ as $\hat{\Gamma}^{\mu}_{A\nu_i\bar{\nu}_i} = ieq^2\hat{F}_{\nu_i}(q^2)\gamma_\mu(1-\gamma_5)$. The NCR, to be denoted by $\langle r^2_{\nu_i}\rangle$, is then defined as $\langle r^2_{\nu_i}\rangle = 6\hat{F}_{\nu_i}(0)$, and thus one obtains

$$\langle r^2_{\nu_i}\rangle = \frac{G_F}{4\sqrt{2}\pi^2}\left[\frac{25}{6} - \log\left(\frac{m_i^2}{M_W^2}\right)\right], \qquad i = e, \mu, \tau, \quad (1)$$

where m_i denotes the mass of the charged isodoublet partner of the neutrino under consideration, and G_F is the Fermi constant. The numerical values of the NCR given in Eq. (1) are $\langle r^2_{\nu_e}\rangle = 2.3\times10^{-33}$ cm^2, $\langle r^2_{\nu_\mu}\rangle = 1.4\times10^{-33}$ cm^2, and $\langle r^2_{\nu_\tau}\rangle = 0.9\times10^{-33}$ cm^2. The classical definition of the NCR (in the static limit) as the second moment of the spatial neutrino charge density $\rho_\nu(\mathbf{r})$, i.e., $\langle r^2_\nu\rangle = e^{-1}\int d\mathbf{r}r^2\rho_\nu(\mathbf{r})$, suggests the heuristic interpretation of the above numbers as a measure of the "size" of the neutrino ν_i when probed electromagnetically.

The unambiguous resolution of the theoretical issues which was accomplished in [3], together with the definite numerical predictions quoted above, inevitably leads to the next important questions: Can the NCR be measured, even in principle? Does it qualify as a "physical observable"? In this Letter we will show that the answer to the above questions is affirmative.

It is important to clarify from the outset what we mean by "measuring" the NCR, especially in light of the fact that bounds on the NCR already appear in the literature [5]. From our point of view, measuring the entire process $f^\pm\nu \to f^\pm\nu$ does *not* constitute a measurement of the NCR, because by changing the target fermions $f^\pm$ one will generally change the answer, thus introducing a target dependence into a quantity which (supossedly) constitutes an intrinsic property of the neutrino. Instead, what we want to measure is the target-independent standard model NCR only, stripped of any target-dependent contributions. Specifically, as mentioned above, the pinch technique rearrangement of the S-matrix makes possible the definition of distinct, physically meaningful subamplitudes, one of which, $\hat{\Gamma}^{\mu}_{A\nu_i\bar{\nu}_i}$, is finite and directly related to the NCR. However, the remaining subamplitudes, such as self-energy, vertex and box corrections, even though they do no enter into the definition of the NCR, still contribute numerically to the entire S matrix; in fact, some of them

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combine to form additional physical observables of interest, most notably the effective (running) electroweak charge and mixing angle. Thus, in order to isolate the NCR, one must conceive of a combination of experiments and kinematical conditions, such that all contributions not related to the NCR will be eliminated.

In this paper we propose a set of such (thought) experiments involving neutrinos and antineutrinos. Consider the elastic processes $f\nu \to f\nu$ and $f\bar{\nu} \to f\bar{\nu}$, where f denotes an electrically charged fermion belonging to a different isodoublet than the neutrino ν, in order to eliminate the diagrams mediated by a charged W boson. The Mandelstam variables are defined as $s = (k_1 + p_1)^2 = (k_2 + p_2)^2$, $t = q^2 = (p_1 - p_2)^2 = (k_1 - k_2)^2$, $u = (k_1 - p_2)^2 = (k_2 - p_1)^2$, and $s + t + u = 0$ (see Fig. 1). In what follows we will restrict ourselves to the limit $t = q^2 \to 0$ of the above amplitudes, assuming that all external (on shell) fermions are massless. As a result of this special kinematic situation we have the following relations: $p_1^2 = p_2^2 = k_1^2 = k_2^2 = p_1 \cdot p_2 = k_1 \cdot k_2 = 0$ and $p_1 \cdot k_1 = p_1 \cdot k_2 = p_2 \cdot k_1 = p_2 \cdot k_2 = s/2$. In the center-of-mass system we have that $t = -2E_\nu E'_\nu(1 - x) \leq 0$, where E_ν and E'_ν are the energies of the neutrino before and after the scattering, respectively, and $x \equiv \cos\theta_{cm}$, where θ_{cm} is the scattering angle. Clearly, the condition $t = 0$ corresponds to the exactly forward amplitude, with $\theta_{cm} = 0$, $x = 1$. Equivalently, in the laboratory frame, where the (massive) target fermions are at rest, the condition of $t = 0$ corresponds to the kinematically extreme case where the target fermion remains at rest after the scattering.

At tree-level the amplitude $f\nu \to f\nu$ is mediated by an off shell Z boson, coupled to the fermions by means of the bare vertex $\Gamma^\mu_{Zf\bar{f}} = -i(g_w/c_w)\,\gamma^\mu\,[v_f + a_f\gamma_5]$ with $v_f = s_w^2 Q_f - \frac{1}{2}T_z^f$ and $a_f = \frac{1}{2}T_z^f$; Q_f is the electric charge of the fermion f, T_z^f its z component of the weak isospin, $c_w = \sqrt{1 - s_w^2} = M_W/M_Z$, and the electric charge e is related to the $\mathrm{SU}(2)_L$ gauge coupling g_w by $e = g_w s_w$. At one loop, the relevant contributions may be unambiguously determined through the standard pinch technique rearrangement of the amplitude, giving rise to gauge-independent subamplitudes. In particular, the one-loop AZ self-energy $\hat{\Sigma}^{\mu\nu}_{AZ}(q^2)$ obtained is transverse, for *both* the fermionic and the bosonic contributions, i.e., it may be written in terms of the dimensionless scalar function $\hat{\Pi}_{AZ}(q^2)$ as $\hat{\Sigma}^{\mu\nu}_{AZ}(q^2) = (q^2 g^{\mu\nu} - q^\mu q^\nu)\hat{\Pi}_{AZ}(q^2)$. Of course, the ZZ self-energy $\hat{\Sigma}^{\mu\nu}_{ZZ}(q^2)$ is not transverse; in what follows we will discard all longitudinal pieces, since they vanish between the conserved currents of the massless external fermions, and will keep only the part proportional to $g^{\mu\nu}$, whose dimension-full cofactor will be denoted by $\hat{\Sigma}_{ZZ}(q^2)$. If the fermion mass m were nonvanishing, the longitudinal pieces would induce additional terms proportional to positive powers of (m/M_W) and/or $(m/\sqrt{s})$; the former are naturally suppressed, whereas the latter may be made arbitrarily small, by adjusting appropriately the value of s. Furthermore, as is well known, the one-loop vertex $\hat{\Gamma}^\mu_{ZF\bar{F}}(q, p_1, p_2)$, with $F = f$ or $F = \nu$, satisfies a QED-like Ward identity, relating it to the one-loop inverse fermion propagators $\hat{\Sigma}_F$, i.e., $q_\mu\hat{\Gamma}^\mu_{ZF\bar{F}}(q, p_1, p_2) = \hat{\Sigma}_F(p_1) - \hat{\Sigma}_F(p_2)$. It is then easy to show that, in the limit of $q^2 \to 0$, $\hat{\Gamma}^\mu_{ZF\bar{F}} \sim q^2\gamma^\mu(c_1 + c_2\gamma_5)$; since it is multiplied by a massive Z boson propagator $(q^2 - M_Z)^{-1}$, its contribution to the amplitude vanishes when $q^2 \to 0$. This is to be contrasted with the $\hat{\Gamma}^\mu_{A\nu_i\bar{\nu}_i}$, which is accompanied by a $(1/q^2)$ photon propagator, thus giving rise to a contact interaction between the target fermion and the neutrino, described by the NCR.

We next proceed to eliminate the target-dependent box contributions; to accomplish this we resort to the "neutrino–antineutrino" method. The basic observation is that the tree-level amplitudes $\mathcal{M}^{(0)}_{\nu f}$, as well as the part of the one-loop amplitude $\mathcal{M}^{(B)}_{\nu f}$ consisting of the propagator and vertex corrections (the "Born-improved" amplitude), are proportional to $[\bar{u}_f(k_2)\gamma_\mu(v_f + a_f\gamma_5)u_f(k_1)][\bar{v}(p_1)\gamma_\mu P_L\, v(p_2)]$, and therefore transform differently than the boxes under the replacement $\nu \to \bar{\nu}$. In particular, the coupling of the Z boson to a pair of on shell antineutrinos may be written in terms of on shell neutrinos provided that one changes the chirality projector from $P_L = \frac{1}{2}(1 - \gamma_5)$ to $P_R = \frac{1}{2}(1 + \gamma_5)$, and supplies a relative minus sign [6], i.e.,

$$\begin{aligned}\bar{v}(p_1)\,\Gamma_{Z\bar{\nu}\bar{\nu}}\,v(p_2) &= i\left(\frac{g_w}{2c_w}\right)\bar{v}(p_1)\gamma_\mu P_L v(p_2)\\ &= -i\left(\frac{g_w}{2c_w}\right)\bar{u}(p_2)\gamma_\mu P_R u(p_1). \quad (2)\end{aligned}$$

To obtain the above results, we simply use the fact that since the quantities considered are scalars in the spinor space their values coincide with those of their transposed, and employ subsequently $\gamma_\mu^T = -C\gamma_\mu C^{-1}$, $\gamma_5^T = C\gamma_5 C^{-1}$, $v^T(p)C = \bar{u}(p)$, $C^{-1}\bar{v}^T(p) = u(p)$, where C is the charge conjugation operator. Thus, under the above transformation, $\mathcal{M}^{(0)}_{\nu f} + \mathcal{M}^{(B)}_{\nu f}$ reverse sign once, whereas the box contributions reverse sign twice. These distinct transformation properties allow for the isolation of the box contributions when judicious combinations of the forward differential cross sections $(d\sigma_{\nu f}/dx)_{x=1}$

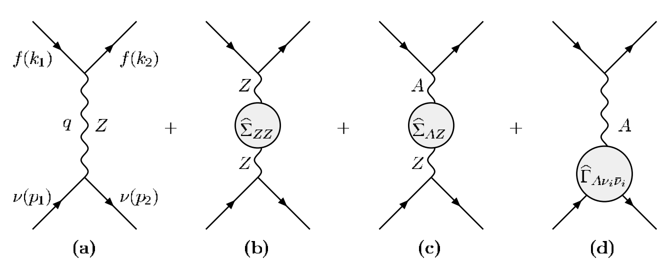

FIG. 1. The universal (a)–(c) and flavor-dependent (d) contributions to $\sigma^{(+)}_{\nu f}$.

and $(d\sigma_{\bar{\nu}f}/dx)_{x=1}$ are formed. In particular, $\sigma_{\nu f}^{(+)} \equiv (d\sigma_{\nu f}/dx)_{x=1} + (d\sigma_{\bar{\nu}f}/dx)_{x=1}$ does not contain boxes, i.e.,

$$\sigma_{\nu f}^{(+)} = \frac{1}{16\pi s}[\mathcal{M}_{\nu f}^{(0)} * \mathcal{M}_{\nu f}^{(0)\dagger} + 2\,\mathrm{Re}(\mathcal{M}_{\nu f}^{(0)} * \mathcal{M}_{\nu f}^{(B)\dagger})], \tag{3}$$

whereas the conjugate combination $\sigma_{\nu f}^{(-)} \equiv (d\sigma_{\nu f}/dx)_{x=1} - (d\sigma_{\bar{\nu}f}/dx)_{x=1}$ isolates the contribution of the boxes. The $*$ in the above formulas denotes that the trace over initial and final fermions must be taken.

Finally, a detailed analysis [7] shows that, in the kinematic limit we consider, the Bremsstrahlung contribution vanishes, due to a a completely destructive interference between the two relevant diagrams corresponding to the processes $fA\nu(\bar{\nu}) \to f\nu(\bar{\nu})$ and $f\nu(\bar{\nu}) \to fA\nu(\bar{\nu})$. The absence of such corrections is consistent with the fact that there are no infrared divergent contributions from the (vanishing) vertex $\hat{\Gamma}^{\mu}_{ZF\bar{F}}$, to be canceled against.

From Eq. (3) and Fig. 1 we see that $\sigma_{\nu f}^{(+)}$ receives contributions from the tree-level exchange of a Z boson [Fig. 1(a)], the one-loop contributions from the ultraviolet divergent quantities $\hat{\Sigma}_{ZZ}(0)$ and $\hat{\Pi}^{AZ}(0)$ [Figs. 1(b) and 1(c), respectively], and the (finite) NCR, coming from the proper vertex $\hat{\Gamma}^{\mu}_{A\nu_i\bar{\nu}_i}$ [Fig. 1(d)]. The first three contributions are universal, i.e., common to all neutrino species, whereas that of the proper vertex $\hat{\Gamma}^{\mu}_{A\nu_i\bar{\nu}_i}$ is flavor dependent. As a consequence, the flavor-dependent part of the NCR can be immediately separated out by taking in $\sigma_{\nu f}^{(+)}$ the difference for two neutrino species. In particular, for the case of ν_μ and ν_τ, we obtain from Eq. (3)

$$\sigma_{\nu_\mu e}^{(+)} - \sigma_{\nu_\tau e}^{(+)} = \lambda\,(1 - 4s_w^2)\,(\langle r_{\nu_\mu}^2\rangle - \langle r_{\nu_\tau}^2\rangle), \tag{4}$$

where $\lambda \equiv (2\sqrt{2}/3)s\alpha\, G_F$, $\alpha = e^2/4\pi$ is the fine-structure constant. *A priori,* the difference in the forward amplitudes $\mathcal{M}_{\nu_\mu e} - \mathcal{M}_{\nu_\tau e}$ would contribute to a difference for the neutrino index of refraction [8] in electron matter; this difference vanishes, however, for ordinary matter due to its neutrality.

Next we will demonstrate that one can actually do better than that, obtaining from experiment not only the difference but even the absolute value of the NCR for a given neutrino flavor. To discuss this methodology, the renormalization of $\hat{\Sigma}_{ZZ}(0)$ and $\hat{\Pi}_{AZ}(0)$ must be carried out. It turns out that, by virtue of the Abelian-like Ward identities enforced after the pinch technique rearrangement [4], the resulting expressions combine in such a way as to form manifestly renormalization-group invariant combinations [9,10]. In particular, after carrying out the standard rediagonalization [11], two such quantities may be constructed (see third paper in [10]):

$$\begin{aligned} \bar{R}_Z(q^2) &= \frac{1}{4\pi}\left(\frac{g_w}{c_w}\right)^2[q^2 - M_Z^2 + \mathrm{Re}\{\hat{\Sigma}_{ZZ}(q^2)\}]^{-1}, \\ \bar{s}_w^2(q^2) &= s_w^2\left(1 - \frac{c_w}{s_w}\,\mathrm{Re}\{\hat{\Pi}_{AZ}(q^2)\}\right). \end{aligned} \tag{5}$$

where $\mathrm{Re}\{\cdots\}$ denotes the real part. These quantities retain the same form when written in terms of unrenormalized or renormalized quantities, due to the special conditions enforced on the renormalization constants, analogous to the textbook QED relation $Z_1 = Z_2$ between the renormalization constants of the vertex and the fermion self-energy. In addition to being renormalization-group invariant, both quantities defined in Eq. (5) are universal (process independent); $\overline{R}_z(q^2)$ corresponds to the Z-boson effective charge, while $\overline{s}_w^2(q^2)$ corresponds to an effective mixing angle. We emphasize that the renormalized $\hat{\Pi}_{AZ}(0)$ *cannot* form part of the NCR, because it fails to form a renormalization-group invariant quantity on its own. Thus, if $\hat{\Pi}_{AZ}(0)$ were to be considered as the "universal" part of the NCR, to be added to the finite and flavor-dependent contribution coming from the proper vertex, then the resulting NCR would depend on the subtraction point and scheme chosen to renormalize it, and would therefore be unphysical. Instead, $\hat{\Pi}_{AZ}(0)$ must be combined with the appropriate tree-level contribution (which evidently does not enter into the definition of the NCR, since it is Z mediated) in order to form the effective $\overline{s}_w^2(q^2)$ acting on the electron vertex, whereas the finite NCR will be determined from the proper neutrino vertex only.

After recasting $\sigma_{\nu f}^{(+)}$ of Eq. (3) in terms of manifestly renormalization-group invariant building blocks, one may fix $\nu = \nu_\mu$, and then consider three different choices for f: (i) right-handed electrons, e_R, (ii) left-handed electrons, e_L, and (iii) neutrinos, ν_i other than ν_μ, i.e., e, τ. It is then straightforward to verify from Eqs. (3) and (5) that $R^2(0)$ is directly written in terms of the physical cross section $\sigma_{\nu_\mu\nu_i}^{(+)}$ as

$$\sigma_{\nu_\mu\nu_i}^{(+)} = s\pi\bar{R}^2(0). \tag{6}$$

This cross section constitutes a fundamental ingredient for neutrino propagation in a neutrino medium [12] and is relevant for astrophysical and cosmological scenarios. Similarly, for the electron target we obtain the system

$$\begin{aligned} \sigma_{\nu_\mu e_R}^{(+)} &= s\pi\bar{R}^2(0)\bar{s}_w^4(0) - 2\lambda s_w^2\langle r_{\nu_\mu}^2\rangle, \\ \sigma_{\nu_\mu e_L}^{(+)} &= s\pi\bar{R}^2(0)\left(\frac{1}{2} - \bar{s}_w^2(0)\right)^2 + \lambda(1 - 2s_w^2)\langle r_{\nu_\mu}^2\rangle \end{aligned} \tag{7}$$

At this point one possibility would be to extract *indirectly* the value of the NCR, using the precision electroweak predictions for $\bar{R}^2(0)$ and $\bar{s}_w^2(0)$ [9] as input in Eq. (7). Much better, there is a second possibility, whereby $\bar{R}^2(0)$, $\bar{s}_w^2(0)$, and $\langle r_{\nu_\mu}^2\rangle$ are treated as three unknown quantities, to be determined from the above equations. This procedure, although more involved, allows (at least conceptually) for a *direct* measurement of NCR. Substituting $s\pi\bar{R}^2(0) \to \sigma_{\nu_\mu\nu_i}^{(+)}$ into Eq. (7) we arrive at a system which is linear in the unknown quantity $\langle r_{\nu_\mu}^2\rangle$ and quadratic in $\bar{s}_w^2(0)$. The corresponding solutions are given by

$$\bar{s}_w^2(0) = s_w^2 \pm \Omega^{1/2},$$
$$\langle r_{\nu_\mu}^2 \rangle = \lambda^{-1}\left[\left(s_w^2 - \frac{1}{4} \pm \Omega^{1/2}\right)\sigma^{(+)}_{\nu_\mu \nu_i} + \sigma^{(+)}_{\nu_\mu e_L} - \sigma^{(+)}_{\nu_\mu e_R}\right], \tag{8}$$

where the discriminant Ω is given by

$$\Omega = (1 - 2s_w^2)\left(\frac{\sigma^{(+)}_{\nu_\mu e_R}}{\sigma^{(+)}_{\nu_\mu \nu_i}} - \frac{1}{2}s_w^2\right) + 2s_w^2 \frac{\sigma^{(+)}_{\nu_\mu e_L}}{\sigma^{(+)}_{\nu_\mu \nu_i}} \tag{9}$$

and must satisfy $\Omega > 0$. The actual sign in front of Ω may be chosen by requiring that it correctly accounts for the sign of the shift of $\bar{s}_w^2(0)$ with respect to s_w^2 predicted by the theory [9].

To extract the experimental values of the quantities $\bar{R}^2(0)$, $\bar{s}_w^2(0)$, and $\langle r_{\nu_\mu}^2 \rangle$, one must substitute in Eqs. (8) and (9) the experimentally measured values for the differential cross sections $\sigma^{(+)}_{\nu_\mu e_R}$, $\sigma^{(+)}_{\nu_\mu e_L}$, and $\sigma^{(+)}_{\nu_\mu \nu_i}$. This means that to solve the system one would have to carry out three different pairs of experiments.

The theoretical values of the $\bar{R}^2(0)$ and $\bar{s}_w^2(0)$ are obtained from Eq. (5). Since (by construction) these two quantities are renormalization-group invariant, one may choose any renormalization scheme for computing their value. In the "on shell" (OS) scheme [13] the experimental values for the input parameters s_w and α are $s_w^{(OS)} = 0.231$ and $\alpha^{(OS)} = 1/128.7$; the renormalized self-energies $\hat{\Sigma}^R_{ZZ}(q^2)$ and $\hat{\Pi}^R_{AZ}(q^2)$ are defined as $\hat{\Sigma}^R_{ZZ}(q^2) = \hat{\Sigma}_{ZZ}(q^2) - \hat{\Sigma}_{ZZ}(M_Z^2) - (q^2 - M_Z^2)\hat{\Sigma}'_{ZZ}(q^2)|_{q^2=M_Z^2}$, where the prime denotes differentiation with respect to q^2, and $\hat{\Pi}^R_{AZ}(q^2) = \hat{\Pi}_{AZ}(q^2) - \hat{\Pi}_{AZ}(M_Z^2)$. Substituting in the resulting expressions (see, for example, [9]) standard values for the quark and lepton masses, and choosing for the Higgs boson a mass $M_H = 150$ GeV, we obtain $\bar{R}^2(0) = 1.86 \times 10^{-3}/M_Z^4$ and $\bar{s}_w^2(0) = 0.239$.

To summarize, we have found that the interaction of ν_μ's with other neutrino species and with left- and right-handed electrons provides at $q^2 = 0$ a definite framework for separating out the probe-independent NCR from other gauge- and renormalization-group-invariant quantities, i.e., the effective electroweak charges $\bar{R}^2(0)$ and $\bar{s}_w^2(0)$. The analysis has used the symmetric combination of neutrinos and antineutrinos to avoid contributions from box diagrams. Once the observable character of the NCR has been established, we plan to extend the method to the entire electromagnetic form-factor analysis by means of the coherent neutrino-nuclear scattering [14]. Finally, note that, for the Dirac neutrinos that we consider, the neutrino anapole moment [2] is simply equal to $\frac{1}{6}\langle r_\nu^2 \rangle$, due to the $(1 - \gamma_5)$ character of the vertex. Therefore, all theoretical properties of the NCR, as well as its observability, carry over automatically to this quantity as well.

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