Hall Magnetic Shocks in Plasma Current Layers

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We present new analytical and numerical results of the dynamics of reversed field current layers in the Hall limit (i.e., characteristic length scales smaller than the ion inertial length). A rapid, localized thinning of the current layer leads to the generation of a nonlinear, shocklike structure that propagates in the $\mathbf{B} \times \nabla n$ direction. This magnetic structure is self-supportive and can lead to a nonlocal thinning of the current layer and the release of magnetic energy.

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The objective of this paper is to describe new Hall magnetohydrodynamic (MHD) physics of plasma current sheets: the asymmetric propagation of magnetic shocklike structures that can result in the nonlocal thinning of the current layer and the release of magnetic energy. During the past 15 years, a significant interest in Hall MHD physics has been aroused because of the rapid penetration of a pulsed magnetic field into a plasma [1–5], the rapid structuring of sub-Alfvénic plasma expansions [6–9], and whistler-mediated magnetic reconnection processes [10]. Recently, Lee and Wu [11] reported the existence of gasdynamic subshocks in the Hall regime; this phenomenon is distinct from the magnetic shocklike structures described here.

The typical laboratory experiment for studying the dynamics of magnetic field penetration into plasma is the plasma filled diode. A current and a magnetic field are generated at the edge of the diode. The magnetic field penetration is measured by magnetic probes, while the compression wave of the plasma density is measured by a laser interferometer. It was found that, in low-density plasma, the magnetic signal propagates without noticeable plasma compression [1,3]. The theory of this phenomenon is straightforward. We assume a quasineutral plasma in which the electrons are isothermal (i.e., $T = \text{const}$) and drifting, while the ions are unmagnetized and motionless. This limit is also referred to as electron magnetohydrodynamics. In this case, the evolution of the magnetic field is described by the following equations:

$$\mathbf{E} + \frac{1}{c}\mathbf{V}_e \times \mathbf{B} = \mathbf{J}/\sigma, \tag{1}$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} = -\frac{4\pi}{c} ne\mathbf{V}_e, \tag{2}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla \times \mathbf{E}. \tag{3}$$

We use a Cartesian geometry in which the magnetic field $\mathbf{B} = B\,\mathbf{e}_z$ is immersed in an inhomogeneous plasma [i.e., $n = n(y)$]. The evolution of the magnetic field is written as

$$\frac{\partial B}{\partial t} + \frac{cB}{4\pi e}\frac{\partial B}{\partial x}\frac{\partial}{\partial y}\frac{1}{n} = \frac{\partial}{\partial x}\frac{c^2}{4\pi\sigma}\frac{\partial B}{\partial x}, \tag{4}$$

which is Burgers equation. Equation (4) has a nontrivial analytical solution in a two-dimensional system where $\nabla(1/n)$ exists along the electron flux nV_{ey}. The magnetic field forms a shock structure that penetrates the plasma in the $\mathbf{B} \times \nabla n$ direction with a velocity [4]

$$V_H = \frac{cB}{4\pi ne}\frac{\partial}{\partial y}\frac{1}{n} \tag{5}$$

and a shock width $\Delta = c^2/4\pi\sigma V_H$.

In order to neglect ion motion, the condition $V_H \gg V_A$ must be satisfied where $V_A = B/(4\pi n m_i)^{1/2}$ is the Alfvén velocity. This is the case if $(c/\omega_{pi})(\partial \ln n/\partial y) = c/\omega_{pi}L_n \gg 1$, where $\omega_{pi} = (4\pi n e^2/m_i)^{1/2}$. This condition is also satisfied by assuming the ions are unmagnetized, i.e., $\omega_{ci} = eB/m_i c \ll V_H/L_n$. A detailed analysis of the Hall drift mode based on fluid and kinetic theory, as well as simulation results, is given in Ref. [12]. Analysis of the development of fast, two-dimensional shocks in nearly collisionless and homogeneous plasmas was carried out in Ref. [13].

We now consider the situation of a plasma current layer where the magnetic field reverses direction (i.e., a neutral sheet). The equilibrium satisfies $B_0^2(y)/8\pi + n(y)T = \text{const}$ with a scale length of L_{y0}. The density has a maximum value n_0 in the center of the current sheet at $y = 0$ and a minimum value n_1 away from the current sheet at $|y| > L_{y0}$. We reduce the width of the current sheet to $L_{y1} < L_{y0}$ for $x < x_0$. The enhanced current structure at $x = x_0$ acts as a piston, and we find that a shocklike solution exists for $x > x_0$. We assume that the one-dimensional magnetic field has the form $\mathbf{B} = B(x - ut, y)\,\mathbf{e}_z$ with $\partial B/\partial x \gg \partial B/\partial y$. For this situation (4) has the first integral,

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$$-u[B - B_0(y)] + \frac{c[B^2 - B_0(y)^2]}{8\pi e}\frac{\partial}{\partial y}\frac{1}{n} = \frac{c^2}{4\pi\sigma}\frac{\partial B}{\partial x}. \quad (6)$$

Equation (6) may have a stationary, shocklike solution if the following relationship is fulfilled:

$$u = \frac{c}{8\pi e}[B_1 + B_0(y)]\frac{\partial}{\partial y}\frac{1}{n(y)}. \quad (7)$$

The post-shock magnetic field B_1 is the magnetic field at $|y| > L_{y0}$, i.e., $B_1^2/8\pi = (n_0 - n_1)T$. Solving (7) numerically, one finds that the plasma density scales approximately as $1/(y + L_{y0})$ for $y > 0$ and as $1/(-y + L_{y0})$ for $y < 0$. In the limit of a weak shock $B_1 \simeq B_0(y)$, the shock velocity is $u \simeq cB_1/4\pi neL_{y0}$; in the case of a strong shock $B_1 \gg B_0(y)$, the shock velocity is reduced by half to $u \simeq cB_1/8\pi neL_{y0}$.

We now present 2D simulation results of this phenomenon, both in the electron MHD limit (i.e., $\mathbf{V}_i = 0$) and the Hall MHD limit (i.e., $\mathbf{V}_i \neq 0$). The Hall MHD equations are given by

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{V} = 0, \quad (8)$$

$$\frac{\partial \rho \mathbf{V}}{\partial t} + \nabla \cdot [\rho \mathbf{V}\mathbf{V} + (P + B^2/8\pi)\underline{\underline{\mathbf{I}}}] = 0, \quad (9)$$

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla \times \mathbf{E} = \nabla \times (\mathbf{V} - \mathbf{J}/ne) \times \mathbf{B}, \quad (10)$$

where $\underline{\underline{\mathbf{I}}}$ is the unit dyad and $P = nT$, where $T = \text{const}$. We use the recently developed 3D Hall MHD code VOODOO [14]. VOODOO is a high-order, finite-volume code that uses a distribution function scheme to calculate the fluxes of mass, momentum, and energy at cell interfaces, as well as the convective electric field [15]. The Hall electric field is updated based on an upwinding scheme using high-order magnetic field values. The partial donor cell method is used to limit fluxes at sharp discontinuities [16]. The code is "collisionless" in that there is no resistive term in the Hall MHD equations that are solved [i.e., $\sigma \rightarrow \infty$ in (1)]; however, there is numerical dissipation in the code.

We consider a two-dimensional system in the x-y plane. The density profile at $t = 0$ is given by

$$n(y) = \begin{cases} n_1 & y < -y_{L0} \\ n_0 A y_{L0}/(-y + A y_{L0}) & y_{L0} < y < 0 \\ n_0 A y_{L0}/(y + A y_{L0}) & y_{L0} > y > 0 \\ n_1 & y > y_{L0}, \end{cases} \quad (11)$$

where $A = n_1/(n_0 - n_1)$ and L_{y0} is the scale size of the current sheet. The magnetic field is calculated using pressure balance $B^2/8\pi + nT = n_0T$. The temperature T is defined from $\beta_0 = 8\pi n_0 T/B_1^2 = 1.42$, where $B_1 = B(|y| > y_{L0})$. The mesh size used in the simulations is 200×200 and we use $A = 1$.

In the first simulation, we consider a system size $L_x = L_y = 0.4c/\omega_{pi}$. The scale length of the neutral sheet is $y_{L0} = 0.1c/\omega_{pi}$. We rapidly narrow the neutral sheet width to $y_{L1} = 0.1y_{L0}$ on the left boundary (i.e., $x = -0.2c/\omega_{pi}$) and maintain this value. The temporal evolution of the plasma is shown in Fig. 1 for the electron MHD case and the Hall MHD case at two times: $t/\tau_A = 0.029$ and 0.087, where $\tau_A = V_{A0}t/L_x$ is the normalized transit time for an Alfvén wave across the system. In the electron MHD case, the thinning of the current sheet rapidly propagates in the positive x direction (i.e., the $\mathbf{B} \times \nabla n$ direction). In the Hall MHD case, the thinning proceeds at the same rate; however, there is also a density compression behind the shock followed by a slight expansion of the current sheet. For example, this is evident at time $\tau_A = 0.087$. The plasma sheet density is compressed above its initial value and subsequently undergoes an expansion. This is an MHD response to the rapid thinning of the current sheet.

In Fig. 2, we plot the magnetic field at three times ($t_1 = 0.029\tau_A$, $t_2 = 0.055\tau_A$, and $t_3 = 0.087\tau_A$) as a function of $x/(c/\omega_{pi})$ at two positions in the neutral layer: $y/(c/\omega_{pi}) = 0.06$ and 0.02. This is for the electron MHD case. The speed of the shocklike structure is $u \simeq 9.44V_{A0}$ for $y/(c/\omega_{pi}) = 0.06$ and $u \simeq 7.61V_{A0}$ for $y/(c/\omega_{pi}) = 0.02$. These values are consistent with the theoretical values obtained from (7): $u \simeq 9.47V_{A0}$ for $y/(c/\omega_{pi}) = 0.06$, and $u \simeq 7.99V_{A0}$ for $y/(c/\omega_{pi}) = 0.02$. Thus, we confirm the theoretical prediction that the shock speed decreases with shock strength. The oscillations behind the shock are not evident in Fig. 1 because of the limited contour level scaling.

Associated with the propagation of the nonlinear Hall shock structure in Fig. 1 is the increase in magnetic energy

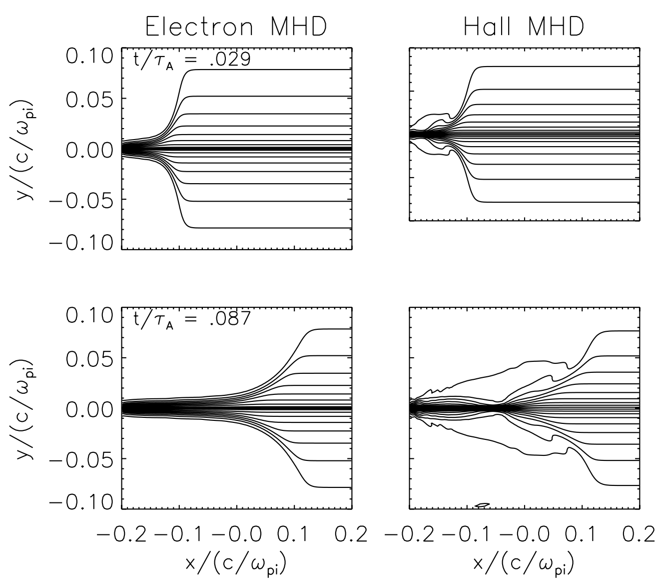

FIG. 1. Contour plot of the magnetic field as a function of time showing the shocklike propagation of the magnetic field in the electron and Hall MHD limits.

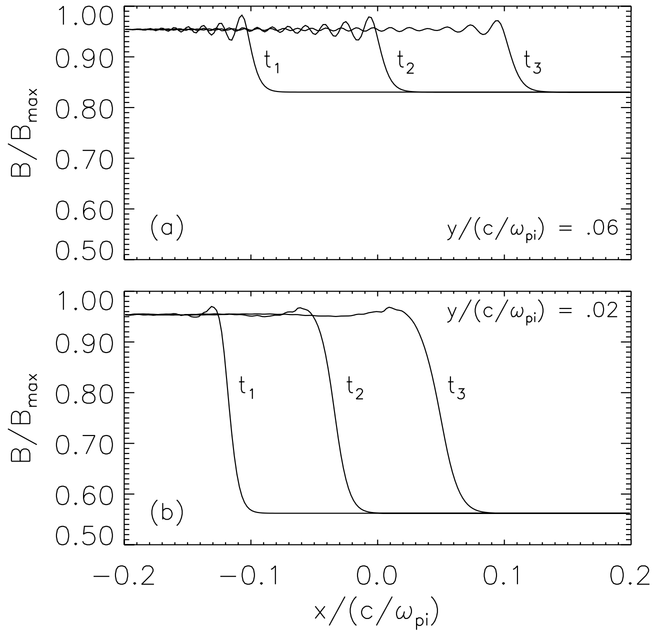

FIG. 2. The magnetic field as a function of $x/(c/\omega_{pi})$ at three times and two locations in the neutral layer.

of the system. A plot of the total magnetic energy in the system $L_x \times L_y$ (normalized to the initial magnetic energy) as a function of time is shown in Fig. 3. The solid curve is for the electron MHD case and the dashed curve is for the Hall MHD case. As time increases, a difference in magnetic energy between the two cases develops. This is because the Hall MHD case develops plasma motion and a portion of the magnetic energy is converted to kinetic energy. However, the important point is that the magnetic energy of the system increases with time, and Hall physics provides a new energy transport and dissipation mechanism.

The physics of the Hall MHD shock waves is the following. The magnetic field is frozen into the electron fluid according to (1) in the nearly collisionless limit $\sigma \rightarrow \infty$. In conventional MHD, the electron and the ion velocities in $\mathbf{J} = en(\mathbf{V}_e - \mathbf{V}_i)$ are close to each other; therefore, the

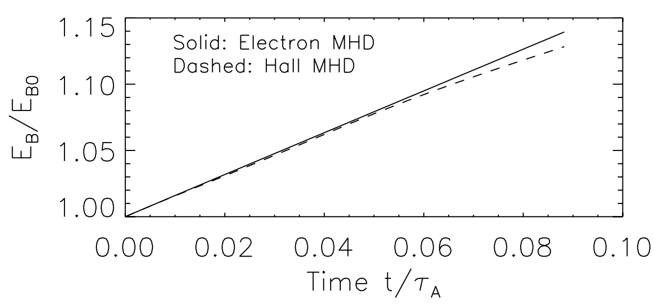

FIG. 3. The total magnetic energy (normalized to the initial magnetic energy) as a function of time in the electron and Hall MHD limits.

relative displacement of these two components is small compared to the characteristic spatial scale of the system on MHD time scales. However, this is not the case in Hall MHD: The electron displacement is finite. Electrons transfer the frozen-in magnetic field and the magnetic energy. The voltage (i.e., magnetic field flux) U and the magnetic energy flux S across the magnetic neutral sheet in Fig. 1 are

$$U = \int V_{ex} B dy = -\int \frac{c}{8\pi ne} \frac{\partial B^2}{\partial y} dy, \qquad (12)$$

$$S = \int V_{ex} \frac{B^2}{8\pi} dy = -\int \frac{c}{4\pi ne} \frac{B^2}{8\pi} \frac{\partial B}{\partial y} dy. \qquad (13)$$

The wave described by (4) exists because electrons bring the magnetic field into plasma along the wave front. The electrons preserve the frozen-in condition ($B/n = \text{const}$): They move towards the increasing density n as the magnetic field increases in time. However, there is no frozen-in law for ions in such a wave. Thus, in contrast to the conventional MHD approximation, the magnetic field and energy are flowing in and out of a fixed volume of the neutral sheet on a spatial scale c/ω_{pi} according to (12) and (13). The simulation results show that this can result in a nonstationary process of ''zippering'': the self-supportive thinning of a current layer (not the plasma density) and a concomitant energy increase.

An alternative physical interpretation is the following. The plasma density distribution $n(y)$ considered in Fig. 1 is peaked at the neutral line. The voltage drop (12) is larger in the broad part of the layer [e.g., $x/(c/\omega_{pi}) > 0.1$ in Fig. 1 at $t/\tau_A = 0.087$] than in the narrow part [e.g., $x/(c/\omega_{pi}) < 0$ in Fig. 1 at the same time] because $n(y)$ is in the denominator. In such a situation, the incoming magnetic flux will be stored continuously at the front of flow because it cannot penetrate the low voltage region. The throat of the flow will move opposite to that of the electron flow with a velocity given by (5). In this process, the stronger periphery magnetic field is filling the space near the neutral line. This magnetic field redistribution (i.e., compression near the neutral sheet) can then lead to energy release. We have performed a number of other simulation studies using different density profiles (e.g., parabolic) and have found that these fundamental results of the paper are not altered.

Finally, we consider the situation where a localized, continuous, external flow drives the system. The simulation parameters are the same as in Fig. 1 but the size of the system in the x direction is increased to $L_x = 4c/\omega_{pi}$ so that $L_x = 10L_y$. An external flow in the y direction is imposed on the system at the bottom and top boundaries. The flow has the form $v_y = \pm v_{y0} \exp[-(x - x_0)^2/\Delta x^2]$, where we use $+v_{y0}$ at the lower boundary and $-v_{y0}$ at the top boundary. The parameters used are $v_{y0} = 1.37V_{A0}$, $\Delta x = 0.14c/\omega_{pi}$, and $x_0 = -0.5c/\omega_{pi}$. We also fix the value of the magnetic field at the bottom and top boundaries at its initial value. The results of the simulation are

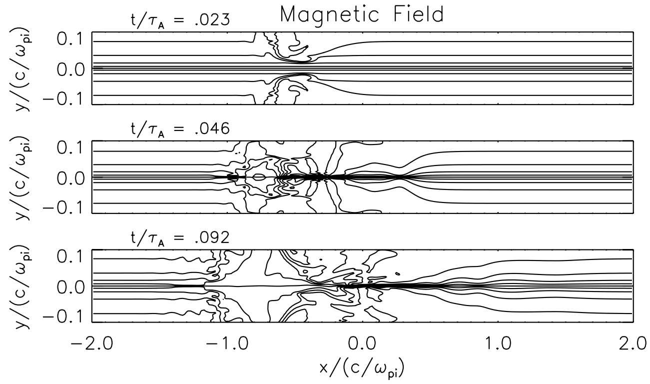

FIG. 4. Contour plot of the magnetic field as a function of time subject to a localized, incoming flow at $x/(c/\omega_{pi}) = -0.05$. Note that the axes are not drawn to scale and that the x axis is much longer than the y axis.

shown in Fig. 4. We show contour plots of the magnetic field at times $t/\tau_A = 0.023$, 0.046, and 0.092. At time $t/\tau_A = 0.023$, the magnetic field pulse caused by the external flow has impinged the neutral sheet; associated with the magnetic field pulse is a density pulse (not shown). At this early time, it is apparent that an asymmetry is developing in the x direction. A nonlinear, magnetic wave structure begins to propagate for $x > -0.5c/\omega_{pi}$ that compresses the neutral sheet; in contrast, the outer portion of the neutral sheet expands for $x < -0.5c/\omega_{pi}$. At time $t/\tau_A = 0.046$, the asymmetry has become stronger. The current is strongly enhanced at $x \simeq 0.4c/\omega_{pi}$ and there is an associated density compression. Following the strong current compression, there is a relaxation of the current layer. At time $t/\tau_A = 0.092$, a series of nonlinear pulses is seen for $x > 0$. These pulses are associated with a series of compressions and rarefactions of the plasma density as a function of time. The period of these nonlinear pulses is $t/\tau_A \simeq 0.0183$ which is comparable to the propagation time of an Alfvén wave pulse in the current layer. The initial thinning of the current layer by a Hall nonlinear wave causes a compression of the plasma at the neutral line. The plasma responds by expanding which occurs on the time scale $t \simeq V_{A0}/y_{L0}$ which is $t/\tau_A \simeq 0.023$. The plasma motion alters the plasma density profile which, in turn, alters the Hall nonlinear wave velocity. This leads to a subsequent thinning of the current sheet and the process repeats. In addition to the rapid propagation of the magnetic pulse, there is also structuring of the plasma and field in the y direction at $x/(c/\omega_{pi}) \simeq -1.0$ and $x/(c/\omega_{pi}) \simeq 0$. This is presumably a Hall Rayleigh-Taylor instability driven by the acceleration of the plasma in the x direction [7].

In summary, we have presented new analytical and numerical results of the dynamics of plasma neutral sheets in the Hall limit. A rapid, localized thinning of the current layer leads to the generation of a nonlinear, shocklike structure that propagates in the $\mathbf{B} \times \nabla n$ direction. This magnetic structure is self-supportive and can lead to a nonlocal thinning of the current layer and the release of magnetic energy. These results may have important implications on the dynamics of current layers in space plasmas such as the earth's magnetotail. For example, the earth's magnetotail may respond asymmetrically to a localized perturbation to the system; i.e., the nonlinear Hall wave will propagate in the dawn-to-dusk direction. Furthermore, the source of magnetic energy can come from the neutral layer as well as be convected from the lobe field. Finally, we have performed 3D Hall magnetic reconnection simulations. The reconnection process was initialized in a localized region along the current direction. We find that the magnetic reconnection site propagates asymmetrically: opposite to the direction of the current (which is also opposite to the $\mathbf{B} \times \nabla n$ direction). These results will be presented in a future paper.

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