Forcing and Velocity Correlations in a Vibrated Granular Monolayer

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The role of forcing on the dynamics of a vertically shaken granular monolayer is investigated. Using a flat plate, surprising negative velocity correlations are measured. A mechanism for this anticorrelation is proposed with support from both experimental results and molecular dynamics simulations. Using a rough plate, velocity correlations are positive, and the velocity distribution evolves from a Gaussian at very low densities to a broader distribution at high densities. These results are interpreted as a balance between stochastic forcing, interparticle collisions, and friction with the plate.

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Granular gases, systems of large numbers of macroscopic grains in rapid motion and interacting through dissipative collisions, appear in a wide range of industrial applications and natural phenomena. Energy must be supplied externally to compensate for the inelastic collisions, so granular gases are necessarily systems out of equilibrium. As a result, they may display dramatic nonequilibrium effects such as non-Gaussian velocity distributions [1–6] and long-range spatial velocity correlations [7–10]. Non-Gaussian velocity distributions are a direct demonstration of the inapplicability of the Gibbs distribution, and significant velocity correlations indicate the absence of "molecular chaos," which is a crucial approximation normally used to solve the Boltzmann equation and to calculate other fundamental quantities in kinetic theory. Recent theoretical work has focused on the nonequilibrium steady state obtained when the energy supplied by spatially homogeneous random external forcing is balanced by the dissipation due to the collisions such that the average energy of the system remains constant [4,7,8]. Non-Gaussian velocity distributions and algebraically decaying velocity correlations arise as a direct consequence of the energy injection. Non-Gaussian velocity distributions and velocity correlations have been observed in a number of experiments [1–3,10], but in each case the forcing is sufficiently different from that of the theoretical models that a direct comparison is difficult. In this Letter, we provide a direct demonstration of the determining role that the forcing plays on the spatial velocity correlations in a homogeneously forced granular gas. We will also show how the framework of kinetic theory can provide a coherent description of the origin of the observed velocity distributions and correlations.

We have investigated a quasi-2D granular system consisting of a layer of a large number of spherical particles partially covering a vertically driven horizontal plate. Two plates with different surface properties were used in the experiments: a smooth circular plate (20 cm in diameter) made of black anodized aluminum, and a rough hexagonally shaped plate (30 cm between opposite corners). The roughness of the latter is provided by a close-packed lattice of blackened steel balls (1.19 mm diameter) glued to a flat plate. The granular gas is made of uniform stainless steel spheres with diameter $\sigma = 1.59$ mm (smooth plate) or $\sigma = 3.97$ mm (rough plate). On the smooth plate, the layer is constrained from above by an antistatic coated Plexiglas lid, 1.7σ above the plate. Using an electromagnetic shaker, the plate is driven sinusoidally, and the granular layer is brought to a nonequilibrium steady state. The strength of the shaking $\Gamma = A\omega^2/g$ is directly measured with a fast response accelerometer (A is the amplitude, ω the frequency of the plate oscillation, and g the acceleration due to gravity).

A high resolution camera (Pulnix TM1040) placed above the plate records the bright spots on the tops of the balls produced by a stroboscopic LED array. Instantaneous horizontal velocities $\mathbf{v}_i$ are obtained from the displacements between strobe pulses. The strobe is synchronized with the camera so that the first pulse occurs at the end of the exposure of one frame, and the next pulse occurs at the beginning of the subsequent frame. The time interval between pulses (~ 1 ms) is chosen to be significantly smaller than the mean collision time. The width of both pulses is typically less than 0.05 ms. Displacements of the centers of the bright spots are measured with 0.1 pixel accuracy. To eliminate systematic errors in the velocity correlations, snapshots are taken at a fixed phase relative to the driving signal corresponding to the maximum average position of the layer.

The general behavior of the granular layer on the smooth plate has been reported previously [1], and a rich phase diagram depending on both Γ and ω was found. Here we focus mainly on the fluidized regime, obtained for $\Gamma \geq 1$. All results presented in this Letter were obtained with $\Gamma = 1.5$ and $\omega/2\pi = 60$ Hz.

The longitudinal and transverse velocity correlations, $C_{\|}$ and $C_{\perp}$, respectively, are calculated by

$$C_{\|,\perp}(r) = \sum_{i\neq j}^{N_r} v_i^{\|,\perp} v_j^{\|,\perp} / N_r,$$

where the sum runs over the N_r pairs of particles separated by a distance r, $v_i^{\|}$ is the projection of $\mathbf{v}_i$ along the line connecting the centers of particles i and j, and $v_i^{\perp}$ is the projection perpendicular to that line.

Figure 1 shows $C_{\|}$ (inset: $C_{\perp}$) normalized by the granular temperature $T = \langle \mathbf{v}_i^2 \rangle / 2$ obtained on the smooth plate for $\rho = 0.4$ and 0.5, and $\Gamma = 1.5$ [11]. Surprisingly, the longitudinal velocity correlations are strongly negative for all r and all accessible densities: the velocities are anticorrelated. $C_{\|}$ decays slower than exponentially, and in the range $r = [2\sigma, 16\sigma]$ can be reasonably well described by $C_{\|}(r) \propto r^{-2}$. The structure visible between 1σ and 2σ is stronger at high density, and its origin is not understood. The transverse component $C_{\perp}$ is much smaller and has a shorter range than $C_{\|}$ (note the difference in vertical scale). Nearly identical correlations were observed without the lid in place. We have performed molecular dynamics simulations that closely match the conditions of the experiment [12] and find qualitatively similar correlations.

The existence of long-range velocity correlations in a randomly forced granular gas has been explained theoretically using a hydrodynamic approach [7]. The velocity correlations result from the balance between the excitation of hydrodynamic modes by the random external forcing and their dissipation through diffusion. At shorter range, velocity correlations arise from an increased probability of recollision due to the forcing [8]. This suggests that the correlations could be quite sensitive to the particular mechanism of energy injection, and we believe that this is the origin of the dramatic difference between the negative correlations observed in our experiment and the positive correlations observed in the randomly forced model systems [4,7–9]. In those systems, energy is injected through white noise forcing of each particle independently, while on the smooth plate energy is injected only into the vertical motion of the spheres and is then transferred to the horizontal motion through interparticle collisions. As a result of this transfer, collisions will result, on average, in an increase in the magnitude of the relative horizontal velocities $\mathbf{v}_r$ of pairs of particles. This effect can be measured directly in the molecular dynamics simulations, and at $\rho = 0.5$ we find that the average value of $\langle \mathbf{v}_r^2 \rangle$ for pairs of particles leaving a collision is 13% higher than for pairs entering a collision. Experimental support for this scenario can be found in Fig. 2, which shows a calculation of $\langle \mathbf{v}(0)\mathbf{v}(r) \rangle$, where the average includes only particles restricted to a narrow band along the direction of motion of the particle at the origin, either ahead (for positive r) or behind (for negative r). There is a clear asymmetry with respect to zero, showing that balls moving away from each other contribute significantly more to the velocity anticorrelation than do particles moving towards each other. The velocity anticorrelations would presumably dissipate through diffusion as the momentum gets transferred to surrounding particles through collisions; thus it is not surprising that the observed correlations extend over several mean free paths. This mechanism for the generation of anticorrelated velocities may also operate in inhomogeneously forced granular media when the average kinetic energy of the grains is anisotropic.

To test our hypothesis about the origin of the correlations, we have changed the forcing by using a rough surface, so that the vibrating plate also injects energy directly into the horizontal motion of the spheres. This energy input mechanism is closer to the white noise forcing of Ref. [7], and allows for a more direct comparison between experiment and theory [13].

The velocity correlations measured on the rough plate are shown in Fig. 3 for $\Gamma = 1.5$ [14] and two extremal densities. We find that both $C_{\|}$ and $C_{\perp}$ are positive for all measured ρ. $\langle \mathbf{v}(0) \cdot \mathbf{v}(r) \rangle$, calculated as described above, is positive everywhere and is roughly symmetric about zero. Molecular dynamics simulations of the layer on the rough plate [12] show similar correlations, and the effect of the

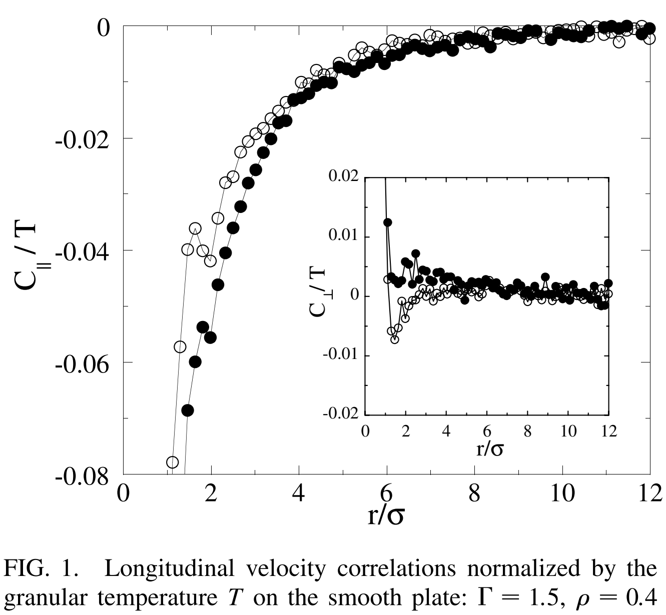

FIG. 1. Longitudinal velocity correlations normalized by the granular temperature T on the smooth plate: $\Gamma = 1.5$, $\rho = 0.4$ (•), and $\rho = 0.5$ (∘). Inset: transverse velocity correlations. Data represent averages over approximately 8000 pairs of images with about 180 balls per image.

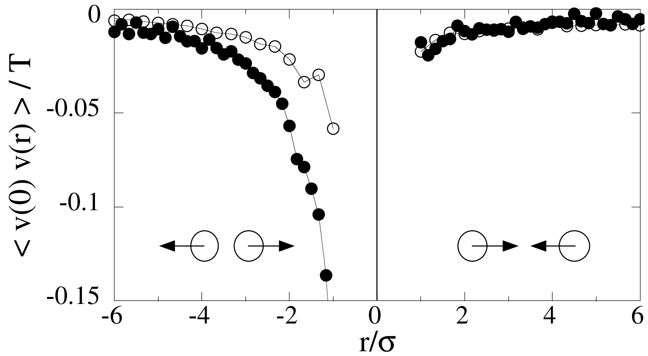

FIG. 2. $\langle \mathbf{v}(0) \cdot \mathbf{v}(r) \rangle$ (as described in the text) normalized by T for $\Gamma = 1.5$, $\rho = 0.4$ (•), and $\rho = 0.5$ (∘).

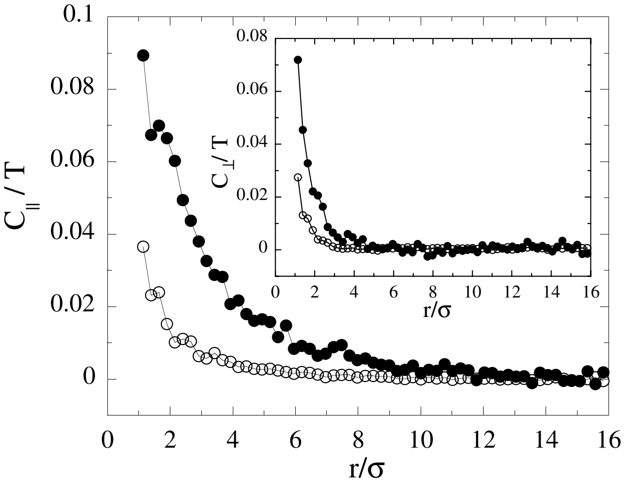

FIG. 3. Longitudinal velocity correlations normalized by the granular temperature on the rough plate: $\Gamma = 1.5$, $\rho = 0.125$ (•), and $\rho = 0.6$ (∘). Inset: transverse velocity correlations. Data are averaged over 16 000 pairs of images (∼ 80 balls per image) for $\rho = 0.125$ and 8000 pairs (∼ 420 balls) for $\rho = 0.6$.

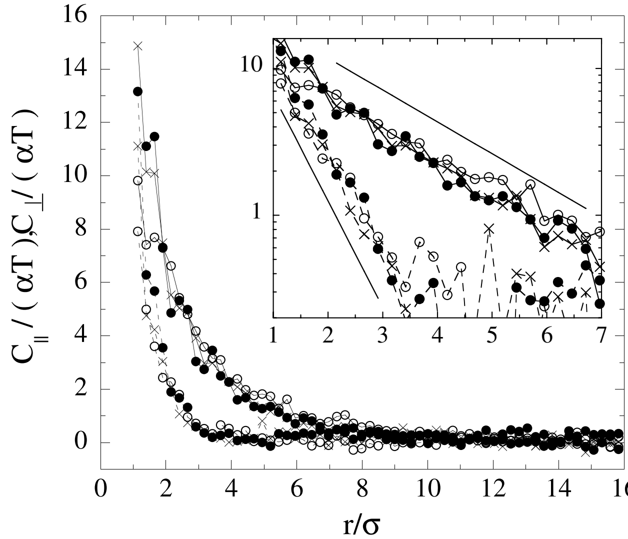

FIG. 4. Scaled longitudinal and transverse velocity correlations normalized by T. $C_{\|}/(\alpha T)$ (symbols with solid lines) and $C_{\perp}/(\alpha T)$ (symbols with dashed lines) for $\Gamma = 1.5$, $\rho = 0.125$ (∘), $\rho = 0.5$ (×), and $\rho = 0.6$ (•). Inset: Log-linear plot of the scaled velocity correlations (the straight lines are exponentials with decay lengths of 2σ and 0.6σ).

collisions is to decrease the average value of $\langle \mathbf{v}_r^2 \rangle$ by 3.7% at $\rho = 0.5$. Thus in the absence of large vertical to horizontal collisional energy transfer, the velocity anticorrelations are replaced by positive velocity correlations, presumably arising from the random forcing as described in Refs. [7,8]. The dramatic difference in the correlations between the smooth and rough plates is a clear indication that the forcing, rather than the inelastic interparticle collisions, determines the velocity correlations.

Interestingly, we find that $C_{\|}$ and $C_{\perp}$ can be roughly scaled on two different curves of the form $C_{\|,\perp} = T\alpha(\rho) f_{\|,\perp}(r)$ (Fig. 4). The functions $f_{\|}$ and $f_{\perp}$ have different r dependence, but the function $\alpha(\rho)$ is the same for both $C_{\|}$ and $C_{\perp}$. $f_{\|}$ and $f_{\perp}$ are reasonably well described by $f(r) \propto e^{-r/r_o}$, with $r_o = 2\sigma$ for $f_{\|}$ and 0.6σ for $f_{\perp}$. It is surprising that this decay length has little density dependence, despite the fact that the mean free path estimated from Enskog-Boltzmann kinetic theory varies from 2.3σ for $\rho = 0.125$ to 0.16σ for $\rho = 0.6$. A similar scaling relation is predicted in the randomly forced model at large r [7], but the scaling is the same for both $C_{\|}$ and $C_{\perp}$, and $f(r)$ decays algebraically.

An important difference between our experiment and the randomly forced model is revealed by the velocity distributions $P(v)$ of the balls on the rough plate. Figure 5 shows the normalized $P(v_x)$ obtained for $\Gamma = 1.5$, and ρ ranging from 0.014 to 0.6. At the lowest density, $P(v_x)$ is well described by a Gaussian. As ρ increases, the tails of the distribution rise significantly above a Gaussian. At high density, the velocity distribution resembles those reported previously on flat plates [1,2]. To better understand the effect of density on $P(v)$, we have followed the trajectories of individual balls using a fast camera (838 frames/s [1]), measured the time-dependent velocity autocorrelation function $C_v(t) = \langle \mathbf{v}(0) \cdot \mathbf{v}(t) \rangle$, and found that $C_v(t) \approx e^{-t/\tau(\rho)}$ at all densities. The scattering length defined by $l \equiv \sqrt{T}\tau(\rho)$ varies from about 0.1σ at $\rho = 0.6$ to about 2.5σ at $\rho = 0.014$. At the lowest density, l is much smaller than the mean free path for ball-ball scattering ($\sim 20\sigma$) obtained from kinetic theory. This suggests that it is ball-plate scattering that is responsible for the

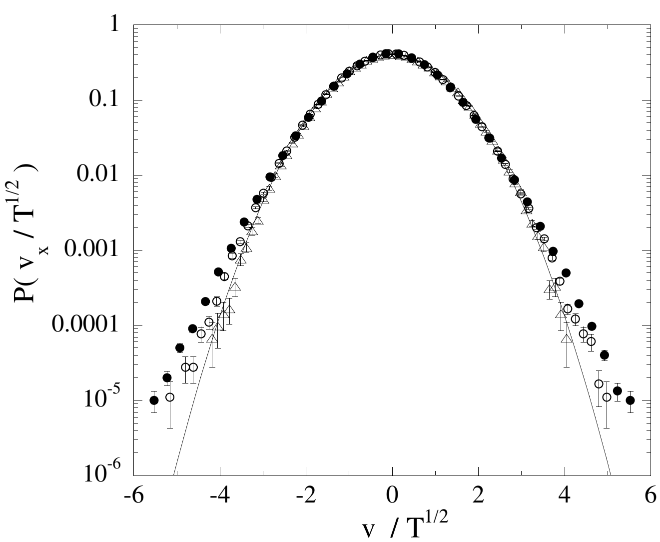

FIG. 5. Normalized velocity distributions $P(v_x)$, on the rough plate for $\Gamma = 1.5$, $\rho = 0.014$ (△), $\rho = 0.125$ (∘), and $\rho = 0.6$ (•). The solid line is a Gaussian. [Results for $P(v_y)$ are identical.]

decay of the velocity autocorrelation function, and that the scattering must therefore have a viscous component. The evolution of $P(v)$ for a single ball on the rough plate can thus be very likely modeled with an equation including only random white noise forcing and a viscous term, such as the Fokker-Plank equation for Brownian motion, which produces a Gaussian $P(v)$. When the random forcing is instead balanced by ball-ball scattering in a modified Boltzmann equation, there is an overpopulation of the tails of the distributions, $P(v) \sim \exp(-|v|^{3/2})$ [4]. This is consistent with the experimentally measured $P(v)$ at high densities, where the scattering is dominated by ball-ball collisions. At intermediate densities both scattering processes contribute, and the evolution of $P(v)$ with density can likely be described with a Boltzmann equation consisting of random white noise forcing, collisional scattering, and a density independent viscous drag. This model has been recently investigated [15], although primarily at higher inelasticities, where significant clustering is observed.

The ball-plate scattering may also explain the lack of significant density dependence of the decay length of the velocity correlations. We speculate that the density independent scattering with the plate, which unlike ball-ball collisions does not conserve momentum, is more effective at destroying the long-range velocity correlations, and therefore controls the observed decay. Support for this scenario is provided by the fact that the decay length for the exponentially decaying $C_{||}(r)$ at *all* measured densities is close to the ball-plate scattering length deduced from the individual trajectories at very low densities (2σ vs 2.5σ). However, a full quantitative understanding of the velocity correlations observed on the rough plate will require a theoretical analysis along the lines of Ref. [7], including the effect of viscous drag. Our results provide a clear experimental demonstration that the velocity distribution and correlations in a granular gas are not determined exclusively by internal dynamics (inelastic ball-ball collisions) but by a balance between internal dynamics and external forcing.

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[11] $\rho = N/N_c$, where N is the number of balls and N_c the number in a full close-packed monolayer.

[12] The interaction rules and parameters for the molecular dynamics simulation were taken from Ref. [6]. The roughness was simulated by adding a layer of close-packed spheres rigidly attached to the plate. The values for σ, ω, and A are taken from the experiment.

[13] Although the roughness of the plate is periodic, its wavelength is considerably less than the measured scattering length, and the motion of the balls on the surface is presumably chaotic, so stochastic forcing and scattering are likely to be a good approximation.

[14] At $\Gamma = 1.5$, the layer is fully fluidized. Clustering, due to trapping of the balls by the lattice, is present for values of Γ below 1.2.

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