Symmetry-Breaking Instability of Multimode Vector Solitons

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We show experimentally that the two-component multimode spatial optical vector soliton, i.e., a two-hump self-guided laser beam, exhibits in Kerr media a sharp space-inversion symmetry-breaking instability. The experiment is performed in a CS_2 planar waveguide using the orthogonal circular polarization states of light as the two components of the vector soliton.

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The complex and intriguing dynamics induced by the vector nature of numerous nonlinear physical phenomena have recently witnessed a renewed interest because of their experimental realization in emerging key areas of physics. For instance, vector phenomena such as vortices [1] and the so-called "ring monopoles" [2] have recently been demonstrated in multicomponent Bose-Einstein condensates. The concept of multicomponent soliton is another important example of vector phenomenon of current interest. Indeed, this concept is at the heart of the process of incoherent light self-trapping [3] and has recently been exploited to explain the dynamical properties of nonmiscible Bose-Einstein condensate mixtures [4].

The concept of multicomponent soliton was introduced in optics by Christodoulides *et al.* [5] in a theoretical study of nonlinear optical wave propagation in birefringent Kerr media. Taking into account the two polarization components of light, it was shown that birefringent Kerr media support optical solitons that consist of a bound state of two distinct orthogonally polarized solitons. These multicomponent solitons result from a complex balance between dispersion, nonlinear phase modulation, four-wave mixing (i.e., coherent nonlinear coupling), and birefringence. Because of their intrinsic vector nature, these new solitons were called "vector solitons" [5]. A few years later, Haelterman *et al.* showed that bound states of solitons can exist without birefringence and four-wave mixing simply on the basis of the mutual trapping induced by cross-phase modulation (i.e., incoherent coupling) between the circular polarization components of light in Kerr media [6]. In the spatial domain, these solitons can be simply interpreted as consisting of a superposition of the fundamental and the first higher-order modes of the waveguide induced by these modes themselves through self- and cross-phase modulations; that is why they can be regarded as being multimode solitons [7]. Owing to the simplicity of the underlying phenomenology, generalizations of the concept of multimode solitons to other systems and configurations have quickly been proposed. For instance, multimode solitons were proposed in systems of counterpropagating beams in Kerr media [7]. Considering the so-called threshold nonlinearity, Snyder *et al.* generalized the concept to the combination of higher-order modes [8]. After exploiting the concept of the multimode soliton to explain the existence of incoherent spatial solitons in photorefractive materials [3], Mitchell *et al.* performed the first experimental observation of the multimode soliton [9]. Note that the so-called dark-bright soliton previously observed experimentally in the works of Ref. [10] constitute a particular case of multimode soliton in the sense that the dark soliton is nothing but the first higher-order mode of the induced waveguide at cutoff. The generalization of the concept to two transverse dimensions, first proposed in Ref. [11], led to the discovery of the dipole-mode vector soliton, called so because of its characteristic antisymmetric two-lobe field distribution [12]. Theoretical and experimental studies of the stability of vector solitons revealed that this two-dimensional multimode soliton has a fundamental nature, in the sense that it constitutes a "self-generated robust basic composite structure of incoherently coupled fields" [13].

These considerations are intimately linked to the stability of the multimode vector solitons, a property that was demonstrated theoretically for saturable Kerr-like nonlinearities [14]. However, in the theoretical works of Refs. [15–17] it was shown that the multimode vector solitons in pure Kerr media are unstable. More precisely, it has been demonstrated theoretically in Ref. [17] that, when the strength of cross-phase modulation is larger than that of self-phase modulation (which is the case for most of the pure Kerr media), the multimode soliton undergoes a space-inversion symmetry-breaking instability. The present Letter is aimed at presenting the experimental observation of this instability.

The multimode soliton symmetry-breaking instability is interesting from several points of view. First, it allows us to propose a very simple physical system in which the fundamental inversion symmetry of the electromagnetic interaction can be broken. In essence, we show that the profile of a simple polarized laser beam propagating in a Kerr medium exhibits, under certain conditions, a sharp space-inversion symmetry-breaking instability. As compared to the optical system recently proposed by Torres *et al.* for the observation of such behavior [18], our system is elementary. To our knowledge, this is the first time that a spatial symmetry-breaking instability is demonstrated in laser physics. Second, the phenomenology that is at play in the instability of the bimodal vector soliton considered in our experiment has a universal character and has been predicted in various physical systems. In the context of optics, it is reminiscent of the fast mode instability of the nonlinear directional fiber coupler (the bimodal two-core waveguide) that has been thoroughly studied theoretically (see, e.g., [19,20]). Indeed, the symmetry-breaking instability of the nonlinear directional coupler comes from the instability of its antisymmetric guided mode that leads to energy transfer from one core of the coupler to the other, which is, as we shall see, what happens in the bimodal vector soliton. Since the model of the directional coupler is mathematically equivalent to that of the weakly birefringent fiber, one can state that our work is, to some extent, related to the fast-polarization-mode instability predicted in this type of fiber [21]. Note that, although clear evidence of the existence of the fast-polarization-mode instability has been obtained experimentally [22], no direct demonstration of the associated symmetry-breaking dynamics was provided in this apparently simpler system. But the interest of the symmetry-breaking instability studied here on the bimodal vector soliton goes well beyond the context of nonlinear optics. Indeed, the underlying phenomenology is found in a variety of nonlinear two-mode systems encountered in such diverse fields as solid-state physics or nonlinear matter waves. For instance, inversion symmetry-breaking has been described theoretically as a self-trapping effect in the so-called *dimer* system [23] or as the *Coulomb blockade* phenomenon in systems of coupled quantum dots [24]. In the emerging area of nonlinear matter waves, the same phenomenology has been predicted in the theory of coherent atomic tunneling between two coupled Bose-Einstein condensates [25].

To obtain the necessary condition of cross-phase modulation (CPM) stronger than self-phase modulation (SPM), we consider, for the two components of the vector soliton, the circular polarization states of light propagating in a Kerr medium with a nonlinearity induced by molecular reorientation, such as carbon disulfide (CS_2). The CPM/SPM ratio in this system attains the value of 7 [26]. This high value strongly favors the symmetry-breaking instability of the multimode vector soliton, as explained in Ref. [17]. Since we consider a pure nonsaturable Kerr nonlinearity, we have to consider light propagation with one transverse dimension in order to avoid catastrophic self-focusing. The experiment is therefore performed in a planar waveguide, and propagation of both circular polarization components is ruled by the two following coupled nonlinear Schrödinger equations:

$$\frac{\partial U}{\partial z} = i\frac{1}{2k}\frac{\partial^2 U}{\partial x^2} + i\gamma(|U|^2U + 7|V|^2U), \qquad (1)$$

$$\frac{\partial V}{\partial z} = i\frac{1}{2k}\frac{\partial^2 V}{\partial x^2} + i\gamma(|V|^2V + 7|U|^2V), \qquad (2)$$

where $U(x, z)$ and $V(x, z)$ are the transverse beam envelopes of the circular polarization components of the electromagnetic field (x and z are the transverse and longitudinal coordinates, respectively), k is the wave vector modulus in the waveguide, and γ is the nonlinear coefficient. As first shown in Ref. [6], these equations possess a one-parameter family of steady-state (or soliton) solutions consisting of a superposition of an envelope of even parity (say, U) with an envelope of odd parity (say, V). An example of such a solution is given in Fig. 1(a) in dimensionless units [for which $\gamma = 1$, $k = 1$, i.e., $\xi = x/x_0$, $\zeta = z/(kx_0^2)$, $u = \sqrt{\gamma k x_0^2}U$, and $v = \sqrt{\gamma k x_0^2}V$, where x_0 is an arbitrary constant]. U and V can be interpreted physically as being, respectively, the fundamental and the first-order modes of the bimodal waveguide they induce together through the Kerr nonlinearity, which justifies the name of multimode soliton. As can be seen in Fig. 1, the multimode soliton exhibits in general a two-hump intensity distribution, which makes it analogous to a directional coupler (two-core waveguide). Since the odd-mode is unstable in the nonlinear directional coupler due to a symmetry-breaking instability (fast mode instability) [19], one must expect that the multimode soliton of Fig. 1(a) is unstable for the same reason. This can be checked very easily by solving numerically Eq. (1). A typical result is shown in Fig. 1(b) representing the propagation of the multimode soliton of Fig. 1(a). As can be seen, the energy that is initially evenly distributed between the two cores of the induced waveguide goes abruptly from one core towards the other, resulting in the destruction of the waveguide. Interestingly, after the transient regime the field remains

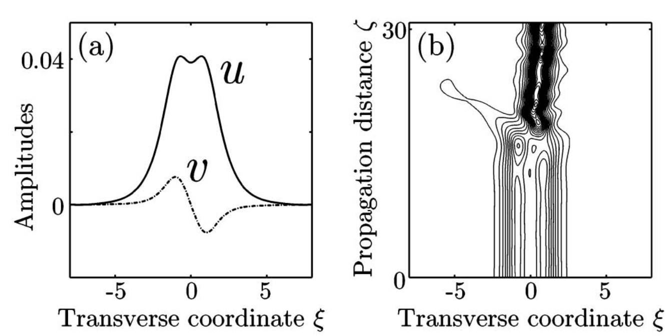

FIG. 1. (a) Envelopes u and v of the circular polarization components of the bimodal vector soliton. (b) Contour plot showing the evolution of the vector soliton in (a) when slightly perturbed by random noise.

confined in a steady-state single-hump distribution (i.e., both U and V exhibit even parity) corresponding to the fundamental elliptically polarized vector soliton [15,27].

The observation of the inversion symmetry-breaking of the bimodal vector soliton has been performed in a planar waveguide made of a 10 μm-thick CS_2 layer sandwiched between two SK5 glass plates corresponding to a refractive index step of $\Delta n = 0.04$. The difference between the propagation constants (wave vector moduli) of the TE and TM fundamental modes of this waveguide is very small and can be neglected over the propagation distance considered in the experiment, which allows us to consider that the nonlinear medium is isotropic as assumed in the theory [6]. The operation wavelength is $\lambda = 0.532$ μm so that in Eq. (1) the real parameters of the system are $k = 1.93 \times 10^7$ m^{-1} and $\gamma = 4.13 \times 10^{-6}$ W^{-1} (see, e.g., Ref. [28] for more details about the CS_2 waveguide characteristics).

The experimental setup is sketched in Fig. 2. A Gaussian laser beam coming from a 10 Hz Q-switched, modelocked (38-ps pulses), and frequency doubled YAG laser is split into two orthogonally polarized beams by a polarizing beam splitter (PBS). A Michelson interferometer setup adjusted to a zero relative phase difference is used to shape the U and V components of the bimodal vector soliton. In one arm, a $\lambda/4$ phase-step mirror introduces in the reflected beam the π phase shift necessary to shape the V component of odd symmetry. In both arms, a quarter-wave plate ensures total transmission through the PBS after reflection on the mirrors. Then, by passing through a third quarter-wave plate, the two resulting orthogonal polarizations are transformed into the required left- and right-handed circular polarizations. Finally, with a combination of spherical and cylindrical lenses, the reduced image of both beams in the plane of the mirrors is achieved at the input face of the 3 cm-long CS_2 waveguide. For their analysis the output beams then go through a quarter-wave plate and a Wollaston prism in order to measure the U- and V-wave intensity profiles separately by means of a single-shot CCD camera. Figure 3(a) shows the input beam intensity profiles in both polarization components. The V wave is characterized by a zero intensity (node) at the origin. The total intensity profile (shown in the inset) has a full width at half maximum (FWHM) of $\Delta x = 72$ μm. Figure 3(b) shows the same beams at the waveguide output at low intensity (linear regime). As can be seen, both beams diffract significantly. In particular, the U-wave intensity profile sees its FWHM increasing from $\Delta x_0 = 64$ to 87 μm, which corresponds to the broadening of a Gaussian beam over slightly more than one diffraction length, $L_D = 2.27 \Delta x_0^2 n_0 / \lambda = 2.8$ cm (where $n_0 = 1.63$ is the refractive index of CS_2). Figure 3(c) shows the output U- and V-beam profiles as well as the total intensity profile obtained when the power is raised until diffraction broadening is canceled by the nonlinearity, i.e., when the condition for the formation of the multimode soliton is reached. This condition is characterized by powers of 2.9 and 0.9 kW for the U and V waves, respectively. As can be seen in Fig. 3(c), although we observe a clear reshaping of wave U that now exhibits two humps [in agreement with Fig. 1(a)], both the output U and V waves exhibit the same width as the input waves, confirming in this way, for the first time, the existence of the bimodal vector soliton in pure Kerr media. However, as mentioned above, this soliton is not stable and our scope here is to show that it suffers the inversion symmetry-breaking instability revealed by theory [17]. As a matter of fact, this instability has been very easy to demonstrate on the soliton shown in Fig. 3(c). Indeed, the experimental intensity profiles of Fig. 3(c) are obtained for only 38% of the laser shots (the 10-Hz repetition rates allow us to record events separately to study their statistics). The other 62% of the shots are characterized by a strongly asymmetric output in which the beam is displaced either on the left or on the right of the initial beam axis. A typical example of asymmetric output

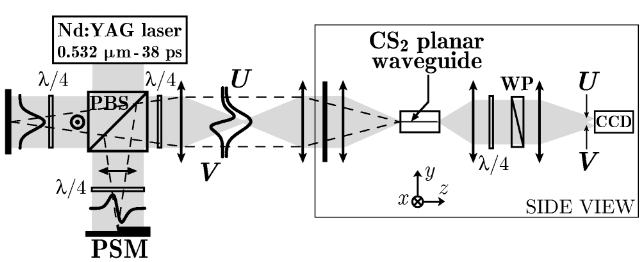

FIG. 2. Schematic of the experimental setup.

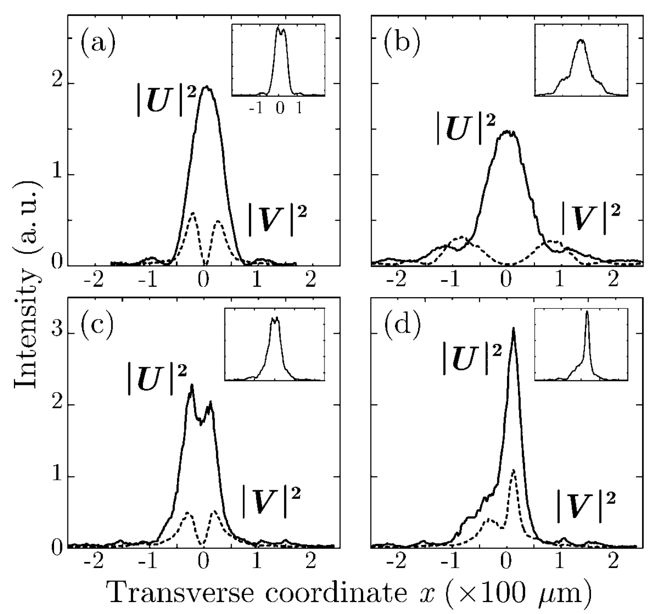

FIG. 3. Intensity profiles of the circular polarization components measured with the CCD camera: (a) Input profiles, (b) output profiles at low intensity, (c) output profiles in the soliton regime, (d) right-oriented asymmetric output profile resulting from the symmetry breaking of the soliton (the insets show the corresponding total intensity profiles).

is shown in Fig. 3(d) where we see that the V-wave envelope lost its node due to the energy transfer (tunnel effect) from one core to the other of the initial self-induced two-core waveguide [let us emphasize here that Figs. 3(c) and 3(d) are obtained with the same initial power corresponding to the soliton formation]. Note, in particular, that a beam compression occurs (the asymmetric total intensity profiles exhibit a FWHM of about 32 μm) due to the stronger energy confinement induced by the instability. Importantly, these asymmetric outputs appear to be almost evenly distributed between the left (53%) and the right (47%), which indicates that they are induced by the random noise of the laser of which we have no control. The 38% of symmetric shots correspond to situations in which the noise asymmetry is too small to induce the symmetry breaking over one diffraction length. Note that it would be, in principle, possible to control the sense of the left-right symmetry breaking by replacing noise by a well controlled initial perturbation.

In summary, we have demonstrated experimentally the existence and the symmetry-breaking instability of the bimodal vector soliton in optical Kerr media. To our knowledge, our results constitute the first experimental evidence of a spatial left-right symmetry-breaking instability in laser physics (note that the process presented here has to be distinguished from the symmetry-breaking dynamics induced by mode competition in laser systems [29]). This achievement has been possible thanks to the simplicity of the proposed system, i.e., a simple inhomogeneously polarized laser beam propagating in a Kerr cell. Because the mechanism that underlies this instability has a universal nature, the present study has potential ramifications in other contexts. In particular, our results suggest that symmetry breaking can be observed in a binary mixture of miscible Bose-Einstein condensates (BECs) set in the bimodal vector soliton configuration, i.e., a configuration in which one BEC has a symmetric bell-shaped envelope that confines another BEC exhibiting an antisymmetric envelope. This configuration has the advantage of requiring no double-well potential (contrary to what is proposed in Ref. [25]) since the necessary two-lobe field distribution of the BEC is automatically confined by the other one.

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