Radiative Recombination Enhancement of Bare Ions in Storage Rings with Electron Cooling

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Experiments on ion-electron recombination in electron coolers show an enhancement of the recombination rate with respect to the standard theory. The theoretical explanation of this effect is an active field of research. Here a parameter-free model is presented in terms of the Vlasov equation. Its inherent scaling rests on two dimensionless variables and agrees with measurements. Additionally, numerical calculations yield the correct magnitude for the enhancement and trace its cause to the process of beam merging.

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Electron cooling [1] is an important method to reduce the phase space volume of ions in a storage ring by superimposing the ion beam with a comoving electron beam. But simultaneously ion-electron recombination processes are abundant. While these limit the life time of the stored ion beam, they offer the possibility to study a fundamental process relevant to many areas, such as atomic structure, astrophysics, fusion plasmas, accelerator physics, and the production of antihydrogen.

Recently, numerous experiments [2–12] have investigated the recombination of electrons with bare ions in electron coolers (a detailed description of the typical experimental setup is given, for example, in [7]). In these systems radiative recombination (RR) dominates. By tuning the velocities of the electron and ion beams, low collision energies down to a few meV can be reached. However, all available measurements show at relative velocities smaller than the longitudinal velocity spread in the electron beam a recombination rate enhancement compared to the expected RR rate between ions and an ensemble of free electrons unperturbed by the presence of time-dependent
external fields. On a quantitative level the interpretation has been impeded by the uncontrolled manner in which measurements appear to fluctuate in their dependence on parameters like the charge Z of the ion, the strength of the magnetic guiding field, and the transversal and longitudinal temperatures $T_\perp$ and $T_\parallel$, respectively, which characterize the anisotropic velocity distribution of the electron beam.

The purpose of this Letter is twofold: We show that the RR enhancement is due to the switching of the external fields upon the merging of the beams. High-lying Rydberg states are populated which opens an additional channel for RR. Moreover, the analysis in terms of the time-dependent Vlasov equation for the phase space density of the electrons allows a universal description of the apparently unsystematic experimental results in terms of a universal scaling law, as was recently proposed [5].

The experimentally observed recombination coefficient $\alpha^{\rm RR}$ is defined as the rate of recombination processes per ion divided by the bulk electron density. The recombination coefficient is related to the cross section $\sigma^{\rm RR}$ by

$$\alpha^{\rm RR} = \langle v\ \sigma^{\rm RR}(E)\rangle_{w(E)}\ . \qquad (1)$$

Herein $\langle\cdots\rangle_{w(E)}$ is an average over the energy distribution $w(E)$ of electrons incident with velocity v.

The magnetic field in the electron cooler decouples the longitudinal and transversal degrees of freedom and the temperature anisotropy $\zeta^2 = T_\parallel/T_\perp$ remains constant [13]. Only those recombination processes are accounted for by the detector in which an electron recombines below a threshold level $n_{\rm cut}$. The numerical value of $n_{\rm cut}$ depends on the field in the bending magnets of the ion beam. The established theory for the RR cross section $\sigma^{\rm RR}(E)$ is based on the work of Stobbe [14], who computed the transition matrix of free-bound processes in a hydrogen system in a flux normalized basis. It is advantageous to calculate $\sigma^{\rm RR}(E)$ in a semiclassical approximation [15] with quantum correction (Gaunt) factors [16]. Moreover, for an ion charge Z the approximation $k_{\rm B}T_\perp n_{\rm cut}^2/(Z^2{\rm Ry}) \ll 1$ (1 Ry= 13.6 eV) holds well in an electron cooler. Then the cross section of Stobbe can be simplified as

$$\sigma^{\rm RR}(E) = \sigma_0\ \frac{Z^2{\rm Ry}}{|E|}\ N_{\rm cut} \qquad (2)$$

with $\sigma_0 = 211$ b and $N_{\rm cut} = \ln(n_{\rm cut}+1) + 0.1649$.

The unperturbed electron plasma in an electron cooler is described by an anisotropic Maxwell distribution of asymptotically free electrons with $\zeta \ll 1$. By averaging over this distribution according to (1), one obtains the standard theory at zero relative velocity $v_{\rm rel}$ between electron and ion beam

$$\alpha^{\rm RR}_{\rm st} = \alpha_0\, N_{\rm cut}\ \frac{Z^2}{\sqrt{T_\perp}}\Big(1-\frac{2\zeta}{\pi}\Big) \qquad (3)$$

with $\alpha_0 = 0.3010\times10^{-12}$ eV$^{1/2}$ cm^3 s^{-1} [9,17]. These approximations are well justified in cooler experiments.

The focus on bare ions excludes all dielectronic recombination processes. There also is strong experimental evidence that three body recombination plays a subordinate role. For such processes the rate coefficient would vary substantially with the bulk electron density n_0—likewise for the mean field interaction with the surrounding electron plasma. Such a dependence on n_0 is not observed. Thus the incident electron in a recombination process can be well described in the single particle picture.

Moreover, one can discuss a direct influence of the magnetic guiding field B on σ^{RR}. Yet as long as B is small compared to the atomic field $B_{\mathrm{c}} = 2.35 \times 10^5$ T, the structure impressed by B averages out in (1) [18]. Equally, the shift of atomic levels by the Zeeman and Stark effect can be neglected in an electron cooler. In principle, stimulated emission, multiphoton processes, or recombination cascades could also occur. They all have been investigated and are negligible in an electron cooler [19–21]. Also the effects of quantum electrodynamics are small even for heavy ions [22]. So RR with the cross section (2) is dominant, and there remains the problem to explain the observed excess recombination $\Delta\alpha^{\mathrm{RR}} = \alpha^{\mathrm{RR}} - \alpha^{\mathrm{RR}}_{\mathrm{st}}$.

We propose here that the enhancement is driven by an electron distribution in (1) that includes electrons which acquire negative energy when the ions enter the electron beam. There is a transverse motional electric field due to the toroidal magnetic field at the entrance of the electron cooler. Free electrons can cross the saddle points of the ionic potential until the beams are collinear with the same mean velocity. Similar field-driven processes have been observed in pulsed field recombination [23] with the aim of producing antimatter [24]. In the cooler the loosely bound Rydberg electrons lie mostly above the atomic level n_{cut} and get separated in the bending magnet. They offer, however, an additional recombination channel, and there are contributions from free and bound electrons:

$$\alpha^{\mathrm{RR}} = \alpha^{\mathrm{RR}}_{\mathrm{free}} + \alpha^{\mathrm{RR}}_{\mathrm{bound}}. \tag{4}$$

We propose to identify $\alpha^{\mathrm{RR}}_{\mathrm{free}}$ with the prediction of standard theory (3) and $\alpha^{\mathrm{RR}}_{\mathrm{bound}}$ with the observed excess recombination $\Delta\alpha^{\mathrm{RR}}$. This differs substantially from other models [25,26] that claim the shielding of dielectric theory or multiple encounters accountable for the recombination enhancement, respectively.

In previous papers [27,28] we used particle simulation techniques and showed that Rydberg states with principal quantum numbers of a few hundred to a few thousand are populated when a slow ion enters an electron plasma. The numerical fluctuation inherent in such simulations prevented, however, reliable estimates of the electron density on the relevant atomic scale. Irrespective of the actual recombination process, the input of incident electrons is given by the classical phase space density $f(t, \boldsymbol{r}, \boldsymbol{v})$ of both free and bound electrons. Here we describe time evolution of the phase space density by the Vlasov equation

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f}{\partial \boldsymbol{r}} - \left(\boldsymbol{v} \times \boldsymbol{e}_z + \frac{Y}{r^2}\boldsymbol{e}_r\right) \cdot \frac{\partial f}{\partial \boldsymbol{v}} = 0, \tag{5}$$

as both the mean field and electron-electron collisions can be neglected. It is formulated in units of the magnetic guiding field, i.e., cyclotron frequency $\omega_{\mathrm{c}} = eB/m$ and cyclotron radius $r_{\mathrm{c}} = v_{\mathrm{th},\perp}/\omega_{\mathrm{c}}$ with $mv^2_{\mathrm{th},\perp} = k_{\mathrm{B}}T_\perp$. Then the only parameter in (5) is

$$Y = \frac{ZBe^3}{4\pi\epsilon_0 \sqrt{m}\,(k_{\mathrm{B}}T_\perp)^{3/2}}, \tag{6}$$

which can be understood as the ratio of the Landau length $r_{\mathrm{L}} = Ze^2/(4\pi\epsilon_0\, mv^2_{\mathrm{th},\perp})$ (i.e., the classical distance of the closest approach for a particle at transversal thermal velocity) and the cyclotron radius r_{c}. This governing differential Eq. (5) is provided with an initial condition depending only on the anisotropy parameter ζ. The phase space density yields the energy distribution, and in turn the recombination coefficient; see Eq. (1).

In physical units the recombination coefficient for vanishing relative energy depends on the bulk electron density n_0, the ion charge Z, the guiding field B, and both temperatures $T_\parallel$ and $T_\perp$. In units of the magnetic field, only the parameters Y and ζ occur. Thus the enhancement factor $\varepsilon^{\mathrm{RR}} = \alpha^{\mathrm{RR}}/\alpha^{\mathrm{RR}}_{\mathrm{st}}$ has the logarithmic derivatives

$$\frac{\partial \ln(\varepsilon^{\mathrm{RR}} - 1)}{\partial \ln Y} =: \mu(Y, \zeta), \tag{7}$$

$$\frac{\partial \ln(\varepsilon^{\mathrm{RR}} - 1)}{\partial \ln \zeta} =: \nu(Y, \zeta), \tag{8}$$

and scales locally as $\varepsilon^{\mathrm{RR}} - 1 \propto Y^\mu \zeta^\nu$. Consequently, we have in physical parameters locally

$$\Delta\alpha^{\mathrm{RR}} \propto Z^{2+\mu} B^\mu\, T_\perp^{-(3\mu+\nu+1)/2}\, T_\parallel^{\nu/2}, \tag{9}$$

and the scaling exponents studied separately by experiments are not independent.

Fitting of a global power law of the form (9) to the experimental observations available in literature [2–12] yields $\mu = 0.45(6)$, $\nu = -0.84(7)$, and thus $\Delta\alpha^{\mathrm{RR}} \propto Z^{2.4} B^{0.4} T_\perp^{-0.8} T_\parallel^{-0.4}$. Figure 1 displays the resulting correlation plot of measured relative enhancement $\varepsilon^{\mathrm{RR}} - 1 = \Delta\alpha^{\mathrm{RR}}/\alpha^{\mathrm{RR}}_{\mathrm{st}}$ over a global power law.

Even though in the experiments all external parameters (n_0, Z, B, $T_\perp$, and $T_\parallel$) are varied, there exists a description which solely contains the dimensionless parameters Y and ζ. The functional dependence seems to be close to a global power law. It has been noted [7] that a pure $Z^{2.8}$ scaling fails for heavy ions like Bi^{83+} and U^{92+}; in the present simultaneous scaling involving all parameters these data (points in the upper right corner of Fig. 1) also follow the systematic trend.

We also performed numerical simulations covering the range of the experimental parameters. Initially the

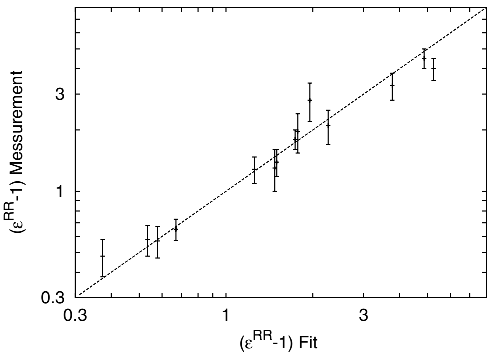

FIG. 1. Correlation between the enhancement factor fitted to a power law according to (9) and the experimental results for bare ions available in the literature [2–12].

magnetized homogenous electron gas is in a quasistationary state with an anisotropic velocity distribution [29],

$$f(\boldsymbol{v}) = \frac{1}{(2\pi)^{3/2}\,\zeta}\,\exp\left(-\frac{v_\perp^2}{2}\right)\exp\left(-\frac{v_\parallel^2}{2\zeta^2}\right), \tag{10}$$

with $\boldsymbol{v}$ in units of $v_{\mathrm{th},\perp}$. At $t \geq 0$ it is perturbed by the ionic Coulomb potential. This instantaneous approximation is justified by the dimension of the beam merging region, which is passed by the electrons on a time scale significantly shorter than the plasma and cyclotron oscillation period. Moreover, the problem now becomes axially symmetric which reduces the numerical effort significantly.

The phase space density for $t \geq 0$ is determined through the characteristics of (5) with the initial condition (10). For the implementation [30] an adaptive scheme has been proven effective, which consists of Kepler orbits and the magnetic velocity-Verlet algorithm [31]. Motivated by (4) and the observation that the RR transition matrix elements change smoothly when passing from the true continuum to the quasicontinuum of Rydberg states [32], we construct from this phase space density a measure for the enhancement factor by

$$\varepsilon^{\mathrm{RR}} := \frac{\alpha^{\mathrm{RR}}}{\alpha^{\mathrm{RR}}_{\mathrm{st}}} \approx \frac{\langle n_{\mathrm{free}} + n_{\mathrm{bound}}\rangle}{\langle n_{\mathrm{free}}\rangle}. \tag{11}$$

Therein the average $\langle\cdots\rangle$ over electron densities is calculated over a cylinder limited laterally by the cyclotron radius r_{c} and lengthwise by the longitudinal Debye length $\lambda_{\mathrm{D},\parallel} = (k_{\mathrm{B}}T_\parallel\epsilon_0/n_0e^2)^{1/2}$. This is the region from which an electron recombines according to the Stobbe cross section (2). The resulting enhancement factors depend only weakly on the size of that region. The relaxation to the new quasistationary state happens on a time scale of a few cyclotron oscillations. As the interaction time within the cooler highly exceeds the cyclotron period $2\pi/\omega_{\mathrm{c}}$, the rate coefficient α^{RR} is independent of the interaction length in the cooler. The numerical simulations show that the recombination enhancement can be globally approximated by

$$\Delta\alpha^{\mathrm{RR}} \propto Z^{2.5}\,B^{0.5}T_\perp^{-1.0}T_\parallel^{-0.3} \tag{12}$$

with an accuracy of $\delta\varepsilon^{\mathrm{RR}} \leq 0.3$ where $\delta\varepsilon^{\mathrm{RR}}$ is the difference between the actual numerical result and the fitted global power law (12).

Figure 2 shows details of the excess recombination $\Delta\alpha^{\mathrm{RR}}$ for matched beams as a function of various physical parameters. The numerical calculations are executed at the same external parameters as the measurements. The

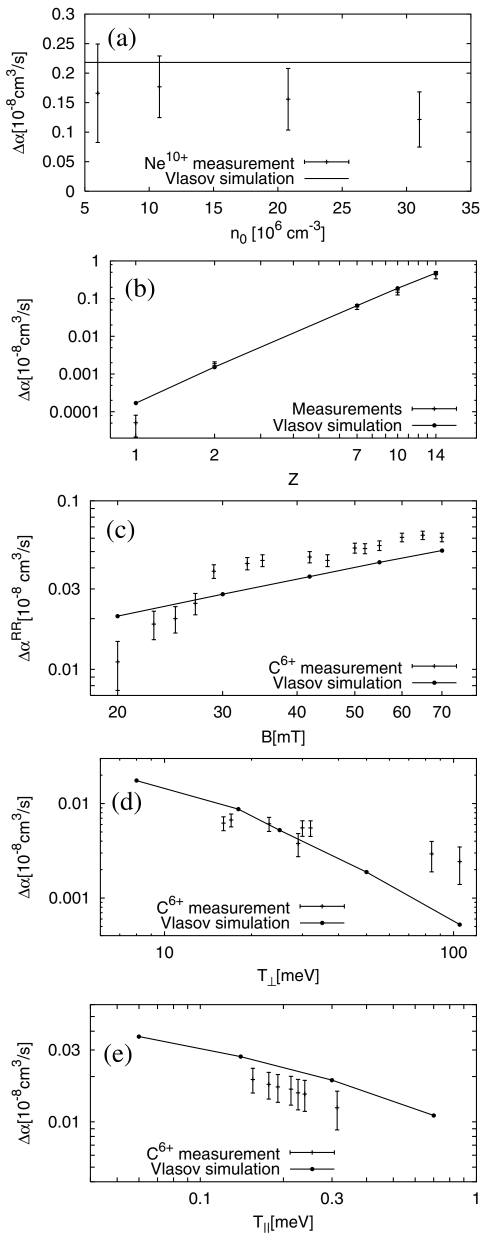

FIG. 2. Scaling of the excess recombination $\Delta\alpha^{\mathrm{RR}}$ as a function of external parameters: (a) bulk electron density n_0 [3], (b) ion charge Z [4], (c) magnetic guiding field B [5], (d) transversal temperature $T_\perp$ [9], and (e) longitudinal temperature [5]. Symbols with error bars are measurements, and solid lines numerical solutions of the Vlasov Eq. (5) evaluated for the enhancement factor from (11).

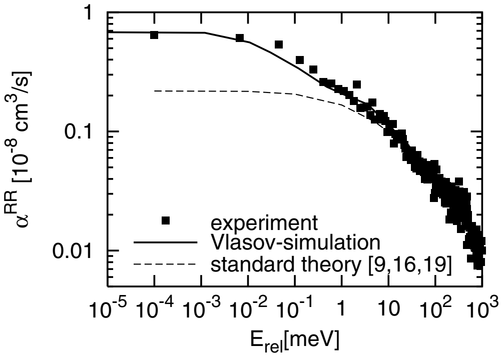

FIG. 3. Recombination rate coefficients α^{RR} as a function of the relative energy of the beams. The data points are from Ref. [4] for Si^{14+}, the dashed curve is calculated within the standard theory [9,14,17], and the solid curve results from the present Vlasov simulation.

constancy of $\Delta\alpha^{RR}$ with the electron bulk density n_0 is inherent to the Vlasov Eq. (5). The numerical scaling with the ion charge Z is in very accurate agreement with the experiment, the scaling with the magnetic guiding field is achieved for $B \geq 30$ mT; this might be due to the side effect of temperature variation in [5]. In contrast to the experiments, the numerical model shows a distinctively steeper scaling with the transversal temperature $T_\perp$. Yet the numerical slope of $\Delta\alpha^{RR}$ as a function of the longitudinal temperature $T_\parallel$ agrees well with the experiment.

In Fig. 3 we show the excess recombination $\varepsilon^{RR} - 1$ as a function of the relative velocity v_{rel} between the beams. The present model describes correctly the growth of the enhancement if v_{rel} falls below the longitudinal thermal velocity in the electron beam $v_{th,\parallel}$. No Rydberg states are occupied when the beams are merged if their velocity mismatch is larger than $v_{th,\parallel}$.

In conclusion, the classical phase space density of incident electrons can be described by the Vlasov equation for single particle propagation. This reduces the external parameter space to two dimensions. The electrons bound loosely during beam merging are identified as drivers of the rate enhancement. While the instantaneous approximation for the transient disturbance contains the main effects, an investigation under actual experimental conditions of beam merging may be worthwhile. The results give the magnitude of the excess recombination that generally agrees with the experiments. The scaling of the excess recombination with external parameters provides a unified description of the recombination experiments in different electron coolers. Here, in electron coolers, the rate enhancement is due to the transient process of beam merging. Under different conditions of recombination within moderate magnetic fields no general enhancement is to be expected. So far our model is in contrast to alternative approaches [25,26].

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