Runaway Dilaton and Equivalence Principle Violations

Thibault Damour
Institut des Hautes Etudes Scientifiques, 35 route de Chartres, F-91440 Bures-sur-Yvette, France

Federico Piazza
Dipartimento di Fisica, Universita di Milano Bicocca, Piazza delle Scienze 3, I-20126 Milan, Italy

Gabriele Veneziano
Theory Division, CERN, CH-1211 Geneva 23, Switzerland,
and Laboratoire de Physique Théorique, Université Paris-Sud, 91405 Orsay, France
(Received 30 April 2002; published 5 August 2002)

In a recently proposed scenario, where the dilaton decouples while cosmologically attracted towards infinite bare string coupling, its residual interactions can be related to the amplitude of density fluctuations generated during inflation, and are large enough to be detectable through a modest improvement on present tests of free-fall universality. Provided it has significant couplings to either dark matter or dark energy, a runaway dilaton can also induce time variations of the natural "constants" within the reach of near-future experiments.

DOI: 10.1103/PhysRevLett.89.081601 PACS numbers: 11.25.Mj, 04.80.Cc, 98.80.Cq

A striking prediction of all string theory models is the existence of a scalar partner of the spin-2 graviton: the dilaton ϕ, whose vacuum expectation value determines the string coupling constant $g_s = e^{\phi/2}$ [1]. At tree level, the dilaton is massless and has gravitational-strength couplings to matter which violate the equivalence principle [2]. This is in violent conflict with present experimental tests of general relativity. It is generally assumed that this conflict is avoided because, after supersymmetry breaking, the dilaton might acquire a (large enough) mass (say, $m_\phi \gtrsim 10^{-3}$ eV so that observable deviations from Einstein's gravity are quenched on distances larger than a fraction of a millimeter). However, Ref. [3] (see also [4]) has proposed a mechanism which can naturally reconcile a *massless* dilaton with existing experimental data. The basic idea of Ref. [3] was to exploit the string-loop modifications of the (four-dimensional) effective low-energy action (we use the signature $-+++$)

$$S = \int d^4x\sqrt{\tilde{g}}\left(\frac{B_g(\phi)}{\alpha'}\tilde{R} + \frac{B_\phi(\phi)}{\alpha'}[2\tilde{\Box}\phi - (\tilde{\nabla}\phi)^2] - \frac{1}{4}B_F(\phi)\tilde{F}^2 - \dots\right), \quad (1)$$

i.e., the ϕ dependence of the various coefficients $B_i(\phi)$, $i = g, \phi, F, \dots$, given in the weak-coupling region ($e^\phi \to 0$) by series of the form $B_i(\phi) = e^{-\phi} + c_0^{(i)} + c_1^{(i)}e^{\phi} + c_2^{(i)}e^{2\phi} + \cdots$, coming from genus expansion of string theory. It was shown in [3] that, if there exists a special value ϕ_m of ϕ which extremizes all the (relevant) coupling functions $B_i^{-1}(\phi)$, the cosmological evolution of the graviton-dilaton-matter system naturally drives ϕ towards ϕm (which is a fixed point of the Einstein-dilaton-matter system). This provides a mechanism for fixing a massless dilaton at a value where it decouples from matter ("least coupling principle"). In this Letter, we consider the case (recently suggested in [5]) where the coupling functions, at least in the visible sector, have a smooth *finite* limit for infinite bare string coupling $g_s \to \infty$. In this case, quite generically, we expect

$$B_i(\phi) = C_i + \mathcal{O}(e^{-\phi}). \quad (2)$$

Under this assumption, the coupling functions are all extremized at infinity; i.e., a fixed point of the cosmological evolution is $\phi_m = +\infty$. [See [6] for an exploration of the late-time cosmology of models satisfying (2).] We found that the "decoupling" of such a "runaway" dilaton has remarkable features: (i) the residual dilaton couplings at the present epoch can be related to the amplitude of density fluctuations generated during inflation, and (ii) these residual couplings, while being naturally compatible with present experimental data, are predicted to be large enough to be detectable by a modest improvement in the precision of equivalence principle tests (nonuniversality of the free fall, and, possibly, variation of constants). This result contrasts with the case of attraction towards a finite value ϕ_m which leads to extremely small residual couplings [7].

We assume some primordial inflationary stage driven by the potential energy of an inflaton field χ. Working with the Einstein-frame metric $g_{\mu\nu} = C_g^{-1} B_g(\phi)\, \tilde{g}_{\mu\nu}$, and with the modified dilaton field $\varphi = \int d\phi[(3/4)(B_g'/B_g)^2 + B_\phi'/B_g + (1/2)\, B_\phi/B_g]^{1/2}$, we consider an effective action of the form

$$S = \int d^4x\sqrt{g}\left[\frac{\tilde{m}_P^2}{4}R - \frac{\tilde{m}_P^2}{2}(\nabla\varphi)^2 - \frac{\tilde{m}_P^2}{2}F(\varphi)(\nabla\chi)^2 - \tilde{m}_P^4\, V(\chi, \varphi)\right], \quad (3)$$

where $\tilde{m}_P^2 = 1/(4\pi G) = 4C_g/\alpha'$, and where the dilaton

 © 2002 The American Physical Society

dependence of the Einstein-frame action is related to its (generic) string-frame dependence (1) by $F(\varphi) = B_\chi(\phi)/B_g(\phi)$, $V(\chi, \varphi) = C_g^2\, \tilde{m}_P^{-4}\, B_g^{-2}(\phi)\, \tilde{V}(\tilde{\chi}, \phi)$.

Under our basic assumption (2), $d\varphi/d\phi$ tends, in the strong-coupling limit $\phi \to +\infty$, to the constant $1/c$, with $c \equiv (2C_g/C_\phi)^{1/2}$, so that the asymptotic behavior of the bare string coupling is

$$g_s^2 = e^\phi \simeq e^{c\varphi}. \quad (4)$$

Let us consider the case where $F(\varphi) = 1$ and $V(\chi, \varphi) = \lambda(\varphi)\, \chi^n/n$ with a dilaton-dependent inflaton coupling constant $\lambda(\varphi)$ of the form

$$\lambda(\varphi) = \lambda_\infty(1 + b_\lambda\, e^{-c\varphi}), \quad (5)$$

where we assume that $b_\lambda > 0$, i.e., that $\lambda(\varphi)$ reaches a *minimum* at strong coupling, $\varphi \to +\infty$. It is shown in [8] that this simple case is representative of rather general cases of φ-dependent inflationary potentials $V(\chi, \varphi)$.

During inflation $[ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j]$, it is easily seen that, while χ slowly rolls down towards $\chi \sim 1$, the dilaton φ is monotically driven towards large values. The solution of the (classical) slow-roll evolution equations leads to

$$e^{c\varphi} + \frac{b_\lambda\, c^2}{2n}\chi^2 = \text{const} = e^{c\varphi_{\text{in}}} + \frac{b_\lambda\, c^2}{2n}\chi_{\text{in}}^2. \quad (6)$$

Using the result (6), we estimate the value φ_{end} of φ at the end of inflation by inserting for the initial value χ_{in} of the inflaton the value corresponding to the end of self-regenerating inflation [9]. We note that the latter value can be related to the amplitude $\delta_H \sim 5 \times 10^{-5}$ of density fluctuations, on the scale corresponding to our present horizon, generated by inflation, through $\chi_{\text{in}} \simeq 5\sqrt{n}\,(\delta_H)^{-2/(n+2)}$. Finally, assuming $e^{c\varphi_{\text{in}}} \ll e^{c\varphi_{\text{end}}}$, we get the estimate

$$e^{c\varphi_{\text{end}}} \sim 12.5 c^2 b_\lambda (\delta_H)^{-4/(n+2)}. \quad (7)$$

A more general study [8] of the runaway of the dilaton during inflation (including an estimate of the effect of quantum fluctuations) modifies this result by only a factor of $\mathcal{O}(1)$. It is also found that the present value of the dilaton is well approximated by φ_{end}.

Equation (7) tells us that, within our scenario, the smallness of the present matter couplings of the dilaton is quantitatively linked to the smallness of the (horizon-scale) cosmological density fluctuations. To be more precise, and to study the compatibility with present experimental data, we need to estimate the crucial dimensionless quantity

$$\alpha_A(\varphi) \equiv \partial \ln m_A(\varphi)/\partial\varphi, \quad (8)$$

which measures the coupling of φ to a massive particle of type A. The definition of α_A is such that, at the Newtonian approximation, the interaction potential between particle A and particle B is $-G_{AB} m_A m_B / r_{AB}$ where [3,4] $G_{AB} = G(1 + \alpha_A \alpha_B)$. Here G is the bare gravitational coupling constant entering the Einstein-frame action (3), and the term $\alpha_A \alpha_B$ comes from the additional attractive effect of dilaton exchange.

Let us first consider the (approximately) composition-independent deviations from general relativity, i.e., those that do not essentially depend on violations of the equivalence principle. Most composition-independent gravitational experiments (in the solar system or in binary pulsars) consider the long-range interaction between objects whose masses are essentially baryonic (the Sun, planets, neutron stars). As argued in [2,3] the relevant coupling coefficient α_A is then approximately universal and given by the logarithmic derivative of the QCD confinement scale $\Lambda_{\text{QCD}}(\varphi)$, because the mass of hadrons is essentially given by a pure number times $\Lambda_{\text{QCD}}(\varphi)$. [We shall consider below the small, nonuniversal, corrections to $m_A(\varphi)$ and $\alpha_A(\varphi)$ linked to QED effects and quark masses.] Remembering from Eq. (1) the fact that, in the string frame [where there is a fixed cutoff linked to the string mass $\tilde{M}_s \sim (\alpha')^{-1/2}$] the gauge coupling is dilaton dependent $[g_F^{-2} = B_F(\varphi)]$, we see that (after conformal transformation) the Einstein-frame confinement scale has a dilaton dependence of the form

$$\Lambda_{\text{QCD}}(\varphi) \sim C_g^{1/2} B_g^{-1/2}(\varphi) \exp[-8\pi^2 b_3^{-1} B_F(\varphi)] \tilde{M}_s, \quad (9)$$

where b_3 denotes the one-loop (rational) coefficient entering the renormalization group running of g_F. Here $B_F(\varphi)$ denotes the coupling to the SU(3) gauge fields. For simplicity, we shall assume that (modulo rational coefficients) all gauge fields couple (near the string cutoff) to the same $B_F(\varphi)$. This yields the following approximately universal dilaton coupling to hadronic matter:

$$\alpha_{\text{had}}(\varphi) \simeq \left[\ln\left(\frac{\tilde{M}_s}{\Lambda_{\text{QCD}}}\right) + \frac{1}{2}\right] \frac{\partial \ln B_F^{-1}(\varphi)}{\partial\, \varphi}. \quad (10)$$

Numerically, the coefficient in front of the right-hand side of (10) is of order 40. Consistently with our basic assumption (2), we parametrize the φ dependence of the gauge coupling $g_F^2 = B_F^{-1}$ as

$$B_F^{-1}(\varphi) = B_F^{-1}(+\infty)\,[1 - b_F\, e^{-c\varphi}]. \quad (11)$$

We finally obtain

$$\alpha_{\text{had}}(\varphi) \simeq 40 b_F c e^{-c\varphi}. \quad (12)$$

Inserting the estimate (7) of the value of φ reached because of the cosmological evolution, we get the estimate

$$\alpha_{\text{had}}(\varphi_{\text{end}}) \simeq 3.2 \frac{b_F}{b_\lambda\, c}\, \delta_H^{4/(n+2)}. \quad (13)$$

It is plausible to expect that the quantity c (which is a ratio) and the ratio b_F/b_λ are both of order unity. This then leads to the numerical estimate $\alpha_{\text{had}}^2 \sim 10\, \delta_H^{8/(n+2)}$, with $\delta_H \simeq 5 \times 10^{-5}$. An interesting aspect of this result is that the expected present value of α_{had}^2 depends rather strongly on the value of the exponent n [which entered the inflaton potential $V(\chi) \propto \chi^n$]. In the case $n = 2$ [i.e., $V(\chi) = \frac{1}{2} m_\chi^2 \chi^2$] we have $\alpha_{\rm had}^2 \sim 2.5 \times 10^{-8}$, while if $n = 4$ [$V(\chi) = \frac{1}{4} \lambda \chi^4$] we have $\alpha_{\rm had}^2 \sim 1.8 \times 10^{-5}$. Both estimates are compatible with present (composition-independent) experimental limits on deviations from Einstein's theory (in the solar system, and in binary pulsars). For instance, the "Eddington" parameter $\gamma - 1 \simeq -2\alpha_{\rm had}^2$ is compatible with the present best limits $|\gamma - 1| \lesssim 2 \times 10^{-4}$ coming from very long baseline interferometry measurements of the deflection of radio waves by the Sun [10].

Let us consider situations where the nonuniversal couplings of the dilaton induce (apparent) violations of the equivalence principle. This means considering the composition dependence of the dilaton coupling α_A, Eq. (8), i.e., the dependence of α_A on the type of matter we consider. Two test masses, made, respectively, of A- and B-type particles will fall in the gravitational field generated by an external mass m_E with accelerations differing by

$$\left(\frac{\Delta a}{a}\right)_{AB} \equiv 2\frac{a_A - a_B}{a_A + a_B} \simeq (\alpha_A - \alpha_B)\,\alpha_E. \qquad (14)$$

We have seen above that in lowest approximation $\alpha_A \simeq \alpha_{\rm had}$ does not depend on the composition of A. We need, however, now to retain the small composition-dependent effects to α_A linked to the φ dependence of QED and quark contributions to m_A. This has been investigated in [3] with the result that $\alpha_A - \alpha_{\rm had}$ depends linearly on the baryon number $B \equiv N + Z$, the neutron excess $D \equiv N - Z$, and the quantity $E \equiv Z(Z-1)/(N+Z)^{1/3}$ linked to nuclear Coulomb effects. Under the plausible assumption that the latter dependence is dominant, and using the average estimate $\Delta(E/M) \simeq 2.6$ [applicable to mass pairs such as (beryllium, copper) or (platinum, titanium)], one finds that the violation of the universality of free fall is approximately given by

$$\left(\frac{\Delta a}{a}\right) \simeq (5.2 \times 10^{-5})\alpha_{\rm had}^2 \simeq 5.2 \times 10^{-4}\left(\frac{b_F}{b_\lambda\, c}\right)^2 \delta_H^{8/(n+2)}. \qquad (15)$$

This result is one of the main predictions of our model. If we insert the observed density fluctuation $\delta_H \sim 5 \times 10^{-5}$, we obtain a level of violation of the universality of free fall (UFF) due to a runaway dilaton which is $\Delta a/a \simeq 1.3[b_F/(b_\lambda c)]^2 \times 10^{-12}$ for $n = 2$ [i.e., for the simplest chaotic inflationary potential $V(\chi) = \frac{1}{2} m_\chi^2(\phi)\,\chi^2$], and $\Delta a/a \simeq 0.98[b_F/(b_\lambda\, c)]^2 \times 10^{-9}$ for $n = 4$ [i.e., for $V(\chi) = \frac{1}{4}\lambda(\phi)\,\chi^4$]. The former case is naturally compatible with current tests (at the $\sim 10^{-12}$ level [11]) of the UFF. Values $n \geq 4$ of the exponent require (within our scenario) that the (unknown) dimensionless combination of parameters $[b_F/(b_\lambda c)]^2$ be significantly smaller than 1.

Let us also consider another possible deviation from general relativity and the standard model: a possible variation of the coupling constants, most notably of the fine-structure constant $e^2/\hbar c$ on which the strongest limits are available. Consistently with our previous assumptions we expect $e^2 \propto B_F^{-1}(\varphi)$ so that, from (11), $e^2(\varphi) = e^2(+\infty)[1 - b_F e^{-c\varphi}]$. The present logarithmic variation of e^2 (introducing the derivative $\varphi' = d\varphi/dp$ with respect to the "e-fold" parameter $dp = Hdt = da/a$) is thus given by

$$\frac{d\ln e^2}{Hdt} \simeq b_F c e^{-c\varphi}\varphi_0' \simeq \frac{1}{40}\alpha_{\rm had}\varphi_0'. \qquad (16)$$

The current value of φ', φ_0', depends on the coupling of the dilaton to the two currently dominating energy forms in the universe: dark-matter [coupling $\alpha_m(\varphi)$] and vacuum energy [coupling $\alpha_V = \frac{1}{4}\partial \ln V(\varphi)/\partial\varphi$]. In the slow-roll approximation, one finds

$$(\Omega_m + 2\Omega_V)\varphi_0' = -\Omega_m\alpha_m - 4\Omega_V\alpha_V, \qquad (17)$$

where Ω_m and Ω_V are, respectively, the dark-matter and the vacuum fractions of critical energy density [$\rho_c \equiv (3/2)\tilde{m}_P^2 H^2$]. The precise value of φ_0' is model dependent and can vary (depending on the assumptions one makes) from an exponentially small value ($\varphi' \sim e^{-c\varphi}$) to a value of order unity. In models where either the dilaton is more strongly coupled to dark matter than to ordinary matter [12] or/and plays the role of quintessence (as suggested in [6]), φ_0' can be of order unity. Assuming just spatial flatness and saturation of the "energy budget" by nonrelatistic matter and dilatonic quintessence, one can relate the value of $\varphi' = d\varphi/(Hdt)$ to Ω_m and to the deceleration parameter $q \equiv -\ddot{a}a/\dot{a}^2$: $\varphi'^2 = 1 + q - 3\Omega_m/2$. This yields the following generic, model-independent relation between the present time variation of e^2, the cosmological observables, and the level of UFF violation:

$$\frac{d\ln e^2}{Hdt} \simeq \pm\, 3.5 \times 10^{-6}\sqrt{1 + q_0 - 3\Omega_m/2}\sqrt{10^{12}\frac{\Delta a}{a}}. \qquad (18)$$

Note that the sign of the variation of e^2 is in general model dependent (as it depends on both the sign of b_F and the sign of φ_0'). Specific classes of models might, however, favor particular signs of de^2/dt. For instance, within the assumptions of [5] and [6] it is natural to expect that e^2 is currently *increasing*.

The phenomenologically interesting consequence of Eq. (18) is to predict a time variation of constants which may be large enough to be detected by high-precision laboratory experiments. Indeed, using $H_0 \simeq 66$ km/s/Mpc, and the plausible estimates $\Omega_m = 0.3$, $q_0 = -0.4$, Eq. (18) yields the numerical estimate $d\ln e^2/dt \sim \pm 0.9 \times 10^{-16}\sqrt{10^{12}\Delta a/a}$ yr^{-1}. Therefore, the current bound on UFF violations ($\Delta a/a \sim 10^{-12}$ [11]) corresponds to the level 10^{-16}yr^{-1}, which is comparable to the planned sensitivity of currently developed cold-atom clocks [13]. (Present laboratory bounds are at the 10^{-14} yr^{-1} level [13,14].) We note also that the upper limit on the variation of e^2 given by the Oklo data, i.e., $|d\ln e^2/dt| \lesssim 5 \times 10^{-17}\,\mathrm{yr}^{-1}$ [15], "corresponds" to a violation of the UFF at the level $\sim 10^{-13}$. Of course, present measurements are not accurate enough to exclude an almost exact cancellation occurring in $1 + q_0 - 3\Omega_m/2$: our work points at the relevance of establishing whether or not this can be excluded.

We have also studied [8] the variation of e^2 over cosmological times. By taking into account the constraints coming from the need to be compatible with current cosmological data, we find that there is no way, within our model, to explain a variation of e^2 as large as the recent claim [16] $\Delta e^2/e^2 = (-0.72 \pm 0.18) \times 10^{-5}$ around redshifts $z \approx 0.5$–3.5. The largest possible variation we find (reached only if the UFF violation is just below the 10^{-12} level, and if φ is rather strongly coupled to dark matter) is of order $\Delta e^2/e^2 = \pm 1.9 \times 10^{-6}$. This is only a factor of ~ 4 below the claim [16] and is at the level of their one sigma error bar. Therefore a modest improvement in the observational precision (accompanied by an improved control of systematics) will start to probe a domain of variation of constants which, according to our scenario, corresponds to an UFF violation smaller than the 10^{-12} level.

Our results suggest that the residual dilaton couplings today (as determined by a cosmological "attraction" towards the fixed point at infinite bare string coupling) are just below the level $\alpha_{\mathrm{had}}^2 \sim 2.5 \times 10^{-8}$ corresponding to a violation of the UFF at the $\Delta a/a \sim 10^{-12}$ level. This gives additional motivation for improved tests of the UFF, such as the Centre National d''Etudes Spatiales (CNES) mission MICROSCOPE [17] (to fly in 2004; planned sensitivity: $\Delta a/a \sim 10^{-15}$), and the National Aeronautics and Space Agency (NASA) and European Space Agency (ESA) mission STEP (Satellite Test of the Equivalence Principle; planned sensitivity: $\Delta a/a \sim 10^{-18}$) [18]. If our estimates are correct, these experiments should find a rather strong violation signal.

Another possible observable signal of a weakly coupled runaway dilaton is the time variation of the natural constants. Here the conclusion depends crucially on the assumptions made about the couplings of the dilaton to the cosmologically dominant forms of energy (dark-matter and/or dark energy). If these couplings are of order unity (and as large as phenomenologically acceptable), the present time variation of the fine-structure constant is linked to the violation of the UFF by the relation $d\ln e^2/dt \sim \pm 2.0 \times 10^{-16}\sqrt{10^{12}\Delta a/a}\,\mathrm{yr}^{-1}$. Such a time variation might be observable (if $\Delta a/a$ is not very much below its present upper bound $\sim 10^{-12}$) through the comparison of high-accuracy cold-atom clocks and/or via improved measurements of astronomical spectra. The discovery of such a time variation [which is possible only if $(\Omega_m\alpha_m + 4\Omega_V\alpha_V)/(\Omega_m + 2\Omega_V)$ is not too small or, in terms of more observable quantities, if $1 + q_0 - 3\Omega_m/2$ is of order 1] would then tell us that the dilaton plays an important cosmological role, either because it is strongly coupled to dark matter ($\alpha_m \sim 1$) or/and because it plays the role of quintessence ($\alpha_V \sim 1$).

G. V. acknowledges the support of a "Chaire Internationale Blaise Pascal," administered by the "Fondation de L'Ecole Normale Supérieure."

[1] E. Witten, Phys. Lett. **149B**, 351 (1984).
[2] T. R. Taylor and G. Veneziano, Phys. Lett. B **213**, 450 (1988).
[3] T. Damour and A. M. Polyakov, Nucl. Phys. **B423**, 532 (1994); Gen. Relativ. Gravit. **26**, 1171 (1994).
[4] T. Damour and K. Nordtvedt, Phys. Rev. Lett. **70**, 2217 (1993); Phys. Rev. D 48, 3436 (1993).
[5] G. Veneziano, J. High Energy Phys. **06**, 051 (2002).
[6] M. Gasperini, F. Piazza, and G. Veneziano, Phys. Rev. D **65**, 023508 (2002).
[7] T. Damour and A. Vilenkin, Phys. Rev. D **53**, 2981 (1996).
[8] T. Damour, F. Piazza, and G. Veneziano, hep-th/0205111.
[9] A. Linde, *Particle Physics and Inflationary Cosmology* (Harwood, Chur, 1990).
[10] D. E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000); see "Experimental Tests of Gravitational Theory," Chap. 14, available on http://pdg.lbl.gov/; C. M. Will, Living Rev. Rel. **4**, 4 (2001).
[11] Y. Su *et al.*, Phys. Rev. D **50**, 3614 (1994).
[12] T. Damour, G. W. Gibbons, and C. Gundlach, Phys. Rev. Lett. **64**, 123 (1990).
[13] C. Salomon *et al.*, in *Cold Atom Clocks on Earth and in Space, Proceedings of the 17th International Conference on Atomic Physics*, edited by E. Arimondo and M. Inguscio (World Scientific, Singapore, 2001), p. 23; Y. Sortais *et al.*, Phys. Scr. **95**, 50 (2001).
[14] J. D. Prestage, R. L. Tjoelker, and L. Maleki, Phys. Rev. Lett. **74**, 3511 (1995).
[15] A. I. Shlyakhter, Nature (London) **264**, 340 (1976); T. Damour and F. Dyson, Nucl. Phys. **B480**, 37 (1996).
[16] J. K. Webb *et al.*, Phys. Rev. Lett. **87**, 091301 (2001).
[17] P. Touboul *et al.*, C. R. Acad. Sci. Paris (série IV) **2**, 1271 (2001).
[18] P. W. Worden, in *Proceedings of the 7th Marcel Grossmann Meeting on General Relativity*, edited by R. T. Jantzen and G. MacKeiser (World Scientific, Singapore, 1996), pp. 1569–1573.